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ARE EXPECTED INFLATION RATES AND EXPECTED REAL RATES NEGATIVELY CORRELATED? A LONG-RUN TEST OF THE MUNDELL-TOBIN HYPOTHESIS

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Abstract

Some empirical evidence suggests that the expected real interest and expected inflation rates are negatively correlated. This hypothesis of negative correlation is sometimes known as the Mundell-Tobin hypothesis. In this article we reinvestigate this negative relation from a long-term point of view using cointegration analysis. The data on the historical interest rate on T-bills and the inflation rate indicate that the Mundell-Tobin hypothesis does not hold in the long run for the United States, the United Kingdom, and Canada. We also obtain similar results using the real interest rate on index-linked gilt traded in the United Kingdom.

JEL Classifications: C32, G12, G15

I. Introduction

As Mishkin (1992) points out, the relation between the level of interest rates and inflation is one of the most studied topics in financial economics. Most of the studies concentrate on the test of the Fisher effect (Fisher 1930), which states that the nominal interest rate fully reflects the expected inflation rate. Several studies demonstrate a negative correlation between the expected inflation rate and the expected real rate (Fama and Gibbons 1982; Mishkin and Simon 1995). Such a

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negative relation between the expected inflation rate and the expected real interest rate is sometimes referred to as the Mundell-Tobin hypothesis. Some researchers (Carmichael and Stebbing 1983) even argue that the nominal interest rate is constant, and thus, the real interest rate adjusts to the changes in the expected inflation rate. This phenomenon (of perfect negative correlation) is sometimes known as the inverted Fisher hypothesis.

Several theoretical explanations are offered for this negative correlation. Mundell (1963) and Tobin (1965) argue that higher inflation leads to a portfolio shift out of nominal assets and into real assets, thus pushing down the return on real assets. Fama and Gibbons (1982) relate the expected real rate to the productivity in the economy and use capital expenditure process and the money market equilibrium to explain the negative correlation. Others, such as Stulz (1986), use the liquidity premium concept.

This negative correlation has important investment implications. Because the real interest rates are usually represented by real returns on financial assets such as T-bills, the negative correlation implies that such financial assets cannot be considered as a good inflation hedge.

The empirical studies that find the negative correlation do not use cointegration analysis to distinguish between the long-run (stationary) relation and the spurious relation among nonstationary time series. Many studies perform cointegration analysis in testing the Fisher effect (Moazzami 1991; Mishkin 1992). However, to our knowledge, none performs cointegration analysis to investigate the negative relation between the expected inflation series and the expected real rate series.

In this article we perform cointegration tests to analyze the negative relation for the United States, the United Kingdom, and Canada. Cointegration analysis requires the estimation of the expected inflation and expected real interest rate series. We use three approaches in estimating these two series. The first two approaches, which are model based, use the Fisher equation and returns on T-bills to estimate the two series. The third approach uses the observed real yield on the index-linked gilt and is not based on any specific model such as the Fisher equation. Cointegration analysis is appropriate because both the expected inflation and expected real rate series are found to be nonstationary. When using the returns on T-bills, we find that the sample correlation between the two series is negative and significant for all three countries. However, the two series are not cointegrated; that is, a long-run negative relation does not exist between these two series. This indicates that T-bills can be regarded as a long-term inflation hedge. When using the index-linked gilt, we find that the sample correlation is positive but not statistically significant. Furthermore, the real interest rate is not cointegrated with the inflation rate. This also supports the long-run independence of the real interest rate and inflation rate.

II. The Models

We consider three approaches in testing the long-run relation between the expected inflation rate and the expected real interest rate. Because the expected inflation rate and the expected real interest rates are not directly observed, we need to use some specific models to estimate these two series. The three approaches differ from one another in terms of how these two series are estimated. Two of these approaches use the Fisher equation and the last approach uses the observed real yield to maturity on index-linked gilt traded in the United Kingdom to estimate the expected real interest rate.

Fisher Equation with Mean-Reverting Expected Real Interest Rate

The generalized version of the Fisher equation can be written as:[1]

$$R_t = r_t + \pi_t^e + 0.5\text{var}_t\,(\pi_t) - \gamma\,\text{cov}_t\,(\Delta c_t, \pi_t)\,, \tag{1a}$$

where

R_t = nominal interest rate for period t;
r_t = expected real interest rate for period t;
π_t^e = expected inflation rate for period t;
π_t = actual inflation rate for period t;
Δc_t = rate of change of consumption for period t; and
γ = coefficient of relative risk aversion.

To simplify the Fisher equation, the last two terms on the right-hand side of equation (1a)—that is, $0.5\,\text{var}_t\,(\pi_t) - \gamma\,\text{cov}_t\,(\Delta c_t, \pi_t)$—are replaced by a constant that leads to the following version of the Fisher equation:[2]

$$R_t = r_t + \pi_t^e + \phi, \tag{1b}$$

where $\phi = 0.5\,\text{var}_t(\pi_t) - \gamma\,\text{cov}_t(\Delta c_t, \pi_t)$ represents the risk premium.

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[1] See Shome, Smith, and Pinkerton (1988) and Evans and Wachtel (1990, 1992) for a more complete discussion of the equation.

[2] This is consistent with the fact that Shome, Smith, and Pinkerton (1988) find neither the conditional variance ($\text{var}_t(\pi_t)$) nor the covariance ($\text{cov}_t(\Delta c_t, \pi_t)$) to be significantly different from zero. Crowder and Hoffman (1996) use similar argument in replacing the last two terms by a constant. Fama and Gibbons (1982), Mishkin (1995), and Mishkin and Simon (1995) use the simplified version of the Fisher equation with ϕ equal to zero.

The nominal interest rates on financial assets are known at the beginning of a period. Therefore, this variable can be regarded as an observable variable. However, the other two variables are not directly observable. We assume the following relation between the observed inflation rate π_t and the expected inflation rate π_t^e:

$$\pi_t = \pi_t^e + u_t, \tag{2}$$

where u_t (the unexpected component of the observed inflation) is assumed to be an independently and normally distributed process with mean zero and variance σ^2; that is, $u_t \sim i.i.d.N(0,\sigma^2)$. For equation (2) to hold, it is sufficient that the investors' expectations of inflation are rational. As to the expected real rate, we assume that it follows a mean-reverting process as follows:

$$r_t - r_{t-1} = \alpha(\rho - r_{t-1}) + v_t, \tag{3}$$

where the process v_t is also assumed to be a zero mean independent normal process with variance equal to $\lambda\sigma^2$, that is, $v_t \sim i.i.d.N(0,\lambda\sigma^2)$. Furthermore, v_t is assumed to be independent of u_t. In this case, ρ is the long-run expected real interest rate and α measures the speed of adjustment. This model of the expected real interest rate is a general model that can characterize wide varieties of interest rate processes. If $\alpha = 0$, the process characterized by equation (3) is a random-walk process. With $\alpha = 1$, the model allows the real rate to be a constant mean process, and with $\alpha = 1$ and $\lambda = 0$ the process represented by equation (3) becomes a constant real rate process. The mean-reversion model of interest rate is popular among researchers (Cox, Ingersoll, and Ross 1985; Hull and White 1990).

Equations (1b)–(3) can be rewritten as follows:

$$\pi_t = -r_t + R_t - \phi + u_t, \tag{4}$$

$$r_t = \alpha\rho + (1 - \alpha)r_{t-1} + v_t. \tag{5}$$

Because both the parameter ϕ and the expected real interest rate series are unobservable, they cannot be separately identified. Therefore, we consider the following model that will replace the set of equations (4) and (5):

$$\pi_t = -r_t^* + R_t + u_t, \tag{6}$$

$$r_t^* = \alpha\rho^* + (1 - \alpha)r_{t-1}^* + v_t, \tag{7}$$

where $r_t^* = r_t + \phi$ and $\rho^* = (\rho + \phi)$. We resolve the problem of identification by estimating $(\rho + \phi)$ rather than estimating ρ and ϕ separately. Because our main objective is to analyze the cointegrating relation between the expected inflation and expected real rate series, this does not affect our result.

The parameters of the model ($\theta = (\alpha, \rho^*, \lambda, \sigma^2)$) can be estimated using the Kalman filtering technique that involves the use of the following recursive equations:

$$\hat{r}^*_{t+1|t} = \alpha\rho + (1-\alpha)\, r^*_{t|t-1} + K_{t|t-1}\eta_t, \tag{8}$$

$$\eta_t = \pi_t + \hat{r}^*_{t|t-1} - R_t, \tag{9}$$

$$K_{t|t-1} = (1-\alpha)\, P_{t|t-1}\,(-1)\left[P_{t|t-1} + \sigma^2\right]^{-1}, \tag{10}$$

$$P_{t|t-1} = [(1-\alpha) + K_{t|t-1}]P_{t|t-1}\,(1-\alpha) + \lambda\sigma^2, \tag{11}$$

$$h_{t|t-1} = P_{t|t-1} + \sigma^2, \tag{12}$$

where $\hat{r}^*_{t+1|t}$ denotes the prediction of r^*_{t+1} given the information available up to and including period t, η_t represents the innovation process, and $K_{t|t-1}$ is known as the Kalman gain matrix. Similarly, $P_{t|t-1}$ and $h_{t|t-1}$ are known as the estimation error covariance matrix and innovation covariance matrix, respectively. The innovation process η_t is a serially independent normally distributed process with variance given by $h_{t|t-1}$. Thus, the log likelihood function (apart from the irrelevant constant) is given by:

$$L\,(\theta) = \sum_{t=1}^{T} L(t,\theta) = -\frac{1}{2}\sum\left[\ln|h_{t|t-1}| + \frac{\eta_t^2}{h_{t|t-1}}\right]. \tag{13}$$

The maximum likelihood estimates can be obtained using the iterative technique suggested by Berndt et al. (1974). This involves the following iterative procedure:

$$\theta_n = \theta_{n-1} + [Q(\theta_{n-1})]^{-1}\,\frac{\partial L(\theta_{n-1})}{\partial\theta}, \tag{14}$$

where

$$Q\,(\theta) = \sum_{t=1}^{T}\left[\frac{\partial L(t,\theta)}{\partial\theta}\right]\left[\frac{\partial L(t,\theta)}{\partial\theta}\right]',$$

and θ_n is the estimated vector at the n^{th} iteration. Various hypotheses can be tested using the consistent estimator of the covariance matrix given by $[Q(\theta)]^{-1}$. The Kalman filter technique also provides the estimate of the real rate and inflation series. Once these series are estimated, we use the unit root and cointegration tests on these series to find the long-run relation between these series.

If the real interest rate is a nonstationary process, the Kalman filter estimators do not have the standard properties (e.g., consistency, asymptotic normality, etc.[3]). We address this issue in two ways. First, empirical distributions of the Kalman filter estimators are obtained using simulation with 5,000 replications. Alternatively, we assume that the real interest rate follows a random-walk process rather than a mean-reverting process. Once we assume a random-walk process, we do not need to use Kalman filter. This leads us to the second approach, which is discussed next.

Fisher Equation with Random Walk Expected Real Interest Rate

In this model, the relation between the observed inflation rate π_t and the expected inflation rate π_t^e is still given by equation (2); however, we assume that the real interest rate follows a random-walk process given by:

$$r_t = r_{t-1} + \upsilon_t, \tag{15}$$

where the process υ_t is assumed to be a zero mean normal process with variance equal to $\lambda_1^2\sigma^2$; that is, $\upsilon_t \sim i.i.d.N(0, \lambda_1^2\sigma^2)$. Furthermore, we use a different version of the Fisher equation given by:

$$\pi_t = -r_t + \beta R_t - \phi + u_t. \tag{16}$$

Note that equation (16) becomes equation (4) if we impose a restriction that β is equal to one. Several researchers use this version of the Fisher equation when testing the Fisher hypothesis (Fama and Gibbons 1982; Mishkin and Simon 1995). As before, we can simplify the model as follows:

$$\pi_t = -r_t^* + \beta R_t + u_t, \tag{17}$$

$$r_t^* = r_{t-1}^* + \upsilon_t, \tag{18}$$

where $r_t^* = r_t + \phi$. The parameters of this model as well as the expected inflation and expected real rate series can be estimated using the exact maximum likelihood estimation technique proposed by Shrestha (1988).[4]

Observed Real Interest Rate on Index-Linked Gilt

The two previous approaches use the Fisher equation and some assumptions regarding the process the real interest rate follows to estimate the expected real

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[3]We thank an anonymous referee for pointing this out.

[4]Note that when we replace *a* with 1 in Shrestha's (1988) model, we end up with the model represented by equations (17) and (18).

interest rate process. However, such complicated models are not required if we use the real yields to maturity on the index-linked gilts traded in the United Kingdom to estimate the real interest rate process. In this article we assume that the relation between the expected real interest rate r_t and the real yield to maturity on the index-linked gilt r_{gt} is given by:

$$r_{gt} = r_t + \zeta_t, \tag{19}$$

where ζ_t is a white noise process. In other words, the expected real interest rate and real yield to maturity differ by a stationary white noise process. Again, we assume that the relation between the observed inflation rate π_t and the expected inflation rate π_t^e is given by equation (2).

It is clear from equation (19) that if the expected real interest rate is a random-walk process, the yield to maturity on the index-linked gilt will also be a random-walk process. Similarly, it is clear from equation (2) that if the expected inflation rate follows a random-walk process, the observed inflation rate will follow a random-walk process. Furthermore, a stationary relation between the expected series (e.g., expected inflation rate and expected real rate) implies a stationary relation between their observed counterparts (e.g., observed inflation rate and observed yield to maturity on the index-linked gilt). Therefore, if a long-run (cointegrating) relation does not exist between the observed inflation rate and the yield to maturity on index-linked gilt, we can conclude that the long-run relation between the expected real rate and expected inflation rate does not exist.

III. Long-Run Test Procedures

Once the expected inflation and expected real rate series are estimated, we need to perform the long-run (cointegrating) analysis. This involves the unit root and cointegration tests. To test for the presence of unit root in the expected real rate r_t, we estimate the following two regression equations:

$$\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \sum_{j=1}^{p} \gamma_j \Delta r_{t-j} + \varepsilon_t, \tag{20}$$

$$\Delta r_t = \delta_0 + \delta_1 r_{t-1} + \delta_2 t + \sum_{j=1}^{p} \gamma_j \Delta r_{t-j} + \varepsilon_t. \tag{21}$$

Then, the unit root hypothesis can be performed using the following guidelines:

	Null hypothesis	*Test statistic*
(a)	$\alpha_1 = 0$	$N\hat{\alpha}_1$
(b)	$\alpha_1 = 0$	t-ratio
(c)	$\alpha_0 = \alpha_1 = 0$	F-test Φ_1
(d)	$\delta_1 = 0$	$N\hat{\alpha}_1$
(e)	$\delta_1 = 0$	t-ratio
(f)	$\delta_0 = \delta_1 = \delta_2 = 0$	F-test Φ_2
(g)	$\delta_1 = \delta_2 = 0$	F-test Φ_3

Critical values are obtained from Dickey and Fuller (1981). In this article, the Phillips and Perron (1988) test, which allows a more general structure of the error term, is used to test for the unit root. The unit root for the expected inflation series is similarly carried out.

The cointegration test can be performed using either the two-step procedure suggested by Engle and Granger (1987) or the maximum likelihood estimator suggested by Johansen and Juselius (1990). In this article we use both types of cointegration tests.

IV. Empirical Results

Fisher Equation with Mean-Reverting Expected Real Interest Rate

The model given by equations (6) and (7) is estimated for the United States, the United Kingdom, and Canada using interest rate on three-month T-bills and the Consumer Price Index (CPI) inflation rate. The data are obtained from the International Monetary Fund's International Financial Statistics. The rates used are quarterly rates and cover from the first quarter of 1964 to the fourth quarter of 1994. All the rates are continuously compounded quarterly rates. The summary statistics for the nominal interest rates and observed inflation rates are given in Table 1.

TABLE 1. Summary Statistics on Nominal Interest Rate, Observed Inflation Rate and Observed Real Interest Rate (Percent per Quarter).

Variable	Mean	Std. Dev.	Minimum	Maximum
U.S. interest rate	1.6799	0.6938	0.7205	3.7919
U.K. interest rate	2.3419	0.8099	0.9448	4.1733
Canadian interest rate	2.0777	0.9116	0.7612	5.2651
U.S. inflation rate	1.2752	0.8433	−0.9485	3.9105
U.K. inflation rate	1.8995	1.6046	−1.4399	7.2932
Canadian inflation rate	1.3479	0.9130	−0.8413	3.7264
U.S. real rate	0.4048	0.7871	−1.3441	2.7440
U.K. real rate	0.4424	1.4888	−4.7140	3.5148
Canadian real rate	0.7298	0.9752	−1.8954	2.9480

TABLE 2. Coefficient Estimates.

	United States		United Kingdom		Canada	
Parameter	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
Panel A. Fisher Equation with Mean-Reverting Expected Real Interest Rate						
α	0.0739	0.0331	0.0429	0.0357	0.0557	0.0279
ρ^*	0.4071	0.4339	0.4985	0.6363	0.7535	0.5091
σ^2	0.2318	0.1414	1.1866	0.1732	0.3631	0.1230
λ	0.2143	0.0312	0.0626	0.0425	0.1547	0.0549
Panel B. Fisher Equation with Random Walk Expected Real Interest Rate						
β	0.5081	0.1566	0.9364	0.2124	0.3115	0.1229
λ_1	0.4663	0.0957	1.0967	0.0316	0.5626	0.0480
σ	0.4354	0.0361	0.2242	0.1739	0.2903	0.0477

Note: Panel A contains the results of the estimation of the parameters of the model given by equations (6) and (7) using the Kalman filtering technique. Panel B contains the results of the estimation of the parameters of the model given by equations (17) and (18).

The parameter estimates of the mean-reverting model are reported in Table 2, Panel A. All the estimates are as expected. As mentioned before, we cannot use the standard *t*-test to test for the random walk for the expected real interest rate series. To perform the test, empirical distributions based on the hypothesis that the expected real rate follows a random-walk process are generated. A brief discussion on the empirical distribution is provided in the Appendix. The empirical distributions are summarized in Table 3, Panel A. The 95% confidence interval for α is equal to (–0.0.13, 0.0996). The estimates of α for all three countries lie inside the confidence interval, indicating that the null hypothesis of random walk for the expected real interest rate cannot be rejected. This is also consistent with the unit root test performed on the expected real interest rate series. The result of the unit root test is discussed later.

Note that the estimate of λ is low for the United Kingdom. This low estimate, together with the insignificant α, may indicate that the expected real interest rate for the United Kingdom is constant. Because the conventional standard error (reported in Table 2) cannot be used here, empirical distributions based on the hypothesis that the expected real rate is constant (i.e., $\alpha = 0$ and $\lambda = 0$) are generated. The empirical distributions are summarized in Table 3, Panel B. The 95% confidence interval for λ is equal to (0.0, 0.0013). Because the estimate of 0.0626 lies outside this interval, the expected real interest rate is not constant for the United Kingdom. Therefore, we conclude that the expected real interest rate follows a random-walk process for all three countries.

When the parameters of the model are estimated using the Kalman filter, the real interest rate and the expected inflation rate series are also simultaneously

TABLE 3. Empirical Distribution Based on 5,000 Replications.

Percentile	α	ρ^*	λ	σ^2
Panel A. Distribution Under the Null Hypothesis of Random Walk Real Interest Rate				
0.5%	−0.0180	−2.1872	0.2652	0.0029
1.0%	−0.0159	−1.9297	0.3036	0.0034
2.5%	−0.0130	−1.5790	0.4013	0.0042
5.0%	−0.0108	−1.2455	0.4715	0.0050
95.0%	0.0718	2.3890	3.0232	0.0141
97.5%	0.0996	2.7112	3.8496	0.0152
99.0%	0.1435	3.1333	5.2989	0.0162
99.5%	0.1821	3.3541	6.2556	0.0170
Theoretical value	0.0000	0.5000	1.0000	0.0100
Mean	0.0170	0.5132	1.3305	0.0095
Std. dev.	0.0517	1.1055	0.9311	0.0043
Skewness	15.9772	0.0960	2.9742	19.7944
Standardized kurtosis	362.7502	−0.1533	14.1616	735.8063
Panel B. Distribution Under the Null Hypothesis of Constant Real Interest Rate				
0.5%	−0.0365	0.4661	0.0000	0.0068
1.0%	−0.0305	0.4742	0.0000	0.0070
2.5%	−0.0178	0.4791	0.0000	0.0074
5.0%	0.0006	0.4828	0.0000	0.0078
95.0%	0.0012	0.5163	0.0001	0.0124
97.5%	0.0020	0.5197	0.0013	0.0129
99.0%	0.0704	0.5242	0.0119	0.0134
99.5%	0.1311	0.5273	0.0386	0.0138
Theoretical value	0.0000	0.5000	0.0000	0.0100
Mean	0.0028	0.4999	0.0008	0.0099
Std. dev.	0.0296	0.0105	0.0126	0.0014
Skewness	17.2301	−0.2288	30.4560	0.1764
Standardized kurtosis	374.5887	0.8951	1181.7734	0.5948

estimated.[5] The estimates of the correlation coefficients between the real rate and inflation rate are –0.5088, –0.7170, and –0.3365 for the United States, the United Kingdom, and Canada, respectively. These negative and significant correlations are consistent with the correlation of –0.9 reported by Mishkin and Simon (1995) for Australia. The evidence at this point seems to support the Mundell-Tobin hypothesis of negative relations between the expected real rate and expected inflation rate for the three economies considered. However, as mentioned earlier, such negative

[5]Actually, the Kalman filtering provides the estimate of the real interest rate plus the constant risk premium series (i.e., $r_t^* = r_t + \phi$). However, the unit root test and cointegration test performed on r_t^* is valid for r_t because a constant term added to a series does not alter the stochastic nature of the series. This is also true when the correlation between the expected real rate and expected inflation rate series is computed.

TABLE 4. Unit Root Test for the Expected Real Rate and the Expected Inflation Rate from the Mean-Reverting Model.

	Test				
Country	(c) Φ_1	(d) $N\hat{\alpha}_1$	(e) t-ratio	(f) Φ_2	(g) Φ_3
Expected real rate					
Unites States	1.974	−8.322	−2.033	1.380	2.068
United Kingdom	1.188	−8.174	−2.113	1.523	2.278
Canada	1.513	−12.230	−2.620	2.350	3.506
Expected inflation rate					
Unites States	4.165	−15.206	−2.859	2.796	4.188
United Kingdom	2.527	−9.648	−2.283	2.079	3.119
Canada	3.069	−11.748	−2.472	2.338	3.508
Critical value $N = 100$, 5%	4.71	−20.7	−3.45	4.88	6.49

Note: The test statistics are calculated based on the estimation of equations (20) and (21) for the expected real interest rate and the expected inflation rate series.

relations between two series may not imply the negative long-run relations if the two series are nonstationary. The main objective of this article is to test whether the Mundell-Tobin hypothesis holds in the long run. This involves the unit root test and the cointegration analysis.

To perform the unit root test, equations (20) and (21) are estimated using the estimated real rate and expected inflation rate series. The test statistics are calculated following Phillips and Perron (1988).[6] The results are summarized in Table 4.

First, consider the F-test statistics Φ_3. For all three countries, the null hypothesis of a unit root and no trend cannot be rejected for either the real rate or the inflation rate. The tests (d) and (e) also support the null hypothesis of a unit root for all three countries and both series. To test for nonzero drift, we perform the Φ_2 test. None of the expected real rate and expected real rate series seems to have nonzero drift. These results of random walk without drift are confirmed by the Φ_1 test based on equation (20). These results of random walk for the expected real rate are consistent with the random-walk test ($\alpha = 0$) performed earlier.

To make sure the expected real rate and expected inflation rate series do not have more than one unit root, the same tests are performed on the first difference of the series. Although not reported, the results indicate that all of the first differenced series are stationary. This confirms that each of the six series has a single unit root.

Because each of the series is a unit-root series, it is clear that the negative correlation may represent a spurious negative relation or a long-run equilibrium

[6]When we performed the usual Dickey-Fuller tests, we obtained the same results.

TABLE 5. Engle-Granger Cointegration Test with the Expected Real Rate as the Dependent Variable.

Test	United States	United Kingdom	Canada	Critical Value at 5%
Panel A. Fisher Equation with Mean-Reverting Expected Real Interest Rate				
(a) $N\hat{\alpha}$	−6.7185	−7.3053	−5.0221	−20.50
(b) t-ratio	−1.8473	−2.0137	−1.5983	−3.37
(c) $N\hat{\alpha}$	−6.1924	−11.760	−9.3748	−21.50
(d) t-ratio	−1.6920	−2.2134	−2.1773	−3.42
Panel B. Fisher Equation with Random Walk Expected Real Interest Rate				
(a) $N\hat{\alpha}$	−7.2595	−7.3047	−7.5979	−20.50
(b) t-ratio	−1.9333	−2.0545	−2.0057	−3.37
(c) $N\hat{\alpha}$	−7.5915	−7.1932	−12.1340	−21.50
(d) t-ratio	−1.9517	−1.6631	−2.4472	−3.42

Note: The tests (a) and (b) are the tests with no trend in cointegrating regression and tests (c) and (d) are those with trend in cointegrating regression. The critical values are obtained from Phillips and Ouliaris (1990).

relation. Thus, to determine whether the negative relation is a long-run stationary relation (among nonstationary variables), we use the same estimated series and test for the existence of the cointegrating relation between the real rate and expected inflation.

First, the cointegration test as suggested by Engle and Granger (1987) with the expected real rate as the dependent (left-hand side) variable is performed and the results are summarized in Table 5, Panel A. The results show that the real rate and the expected inflation rate are not cointegrated. The same conclusion of no cointegration is obtained using the expected inflation rate as the dependent variable in the cointegrating regression. Therefore, we can conclude that the negative correlation between the real rate and the expected inflation rate is not a stationary long-run relation.

To verify the result, we perform the Johansen and Juselius (1990) tests. The results of the trace test and lambda max test are summarized in Table 6, Panel A, where the critical values are obtained from Osterwald-Lenum (1992). Both the trace test and the lambda max test indicate there is no cointegration between the expected real interest rate and the expected inflation rate. This is the same result obtained using the Engle-Granger test.

Fisher Equation with Random Walk Expected Real Interest Rate

The same data (on T-Bills and CPI inflation) are used to estimate the random-walk model given by equations (17) and (18) for the United States, the United Kingdom, and Canada. The parameter estimates are summarized in Table 2, Panel B. The sample correlations between the real rate and the inflation rate are

TABLE 6. Johansen-Juselius Cointegration Tests of the Expected Real Rate and the Expected Inflation Rate.

No. of Cointegrating Relation	United States	United Kingdom	Canada	Critical Value at 5%
Panel A. Fisher Equation with Mean-Reverting Expected Real Interest Rate				
Trace test				
0	18.475	12.631	15.196	19.96
1	5.017	4.282	2.635	9.24
Lambda max test				
0	13.458	8.349	12.561	15.67
1	5.017	4.282	2.635	9.24
Panel B. Fisher Equation with Random Walk Expected Real Interest Rate				
Trace test				
0	10.822	15.469	14.792	19.96
1	4.698	3.262	3.118	9.24
Lambda max test				
0	6.124	12.207	11.674	15.67
1	4.698	3.262	3.118	9.24
Panel C. Observed Real Interest Rate on Index-Linked Gilt				
Trace test				
0		17.009		19.96
1		7.099		9.24
Lambda max test				
0		9.910		15.67
1		7.099		9.24

–0.8522, –0.7390, and –0.9140 for the United States, the United Kingdom, and Canada, respectively. Note that these correlations between the real rate and the inflation rate are considerably more negative than the correlations from the mean-reverting model.

The unit root tests, reported in Table 7, indicate that the expected inflation rate and the expected real rate consist of a single unit root. The Engle-Granger cointegration tests reported in Table 5, Panel B, indicate that the negative relation is

TABLE 7. Unit Root Test for the Expected Inflation Rate from the Random Walk Model.

Test	U.S. Expected Inflation Rate	U.K. Expected Inflation Rate	Candian Expected Inflation Rate	Critical Value $N = 100$, 5%
(c) Φ_1	2.3664	1.2550	0.9209	4.71
(d) $N\hat{\alpha}_1$	−8.0512	−6.3197	−4.1360	−20.70
(e) t-ratio	−2.1758	−1.8582	−1.4834	−3.45
(f) Φ_2	1.9809	1.6187	1.6040	4.88
(g) Φ_3	2.9543	2.4237	2.3984	6.49

not a long-run cointegrating relation. These results are supported by the trace test and lambda max test reported in Table 6, Panel B.

Observed Real Interest Rate on Index-Linked Gilt

We use the real yield to maturity on 2,011 index-linked gilt and inflation in CPI from October 1986 to October 1996 to perform the analysis discussed in section II.[7] The sample correlation between the real rate (on index-linked gilt) and the inflation rate is 6.185%, which is not significantly different from zero. The unit root tests (not reported) indicate that both series consist of a single unit root. However, both the trace test and the lambda max test (see Table 6, Panel C) indicate there is no cointegration between the real rate and the inflation rate. Note that the expected real rate is assumed to differ from the observed real rate series by a stationary error (see equation (19)). Similarly, the expected inflation rate is assumed to differ from the observed inflation rate by a stationary error as described by equation (2). Therefore, the lack of cointegration between the observed inflation rate and real rate implies there is no long-run relation between the expected inflation rate and the expected real rate. This is consistent with our earlier results.

The results from all three approaches indicate there is no cointegrating relation between the expected real interest rate and the expected inflation rate. Therefore, the Mundell-Tobin hypothesis does not seem to hold in the long run.

V. Summary and Conclusions

In this article, we use three approaches to analyze the cointegrating relation between the expected real interest rate and the expected inflation rate. The first approach is based on the Fisher equation, in which the expected real interest rate is assumed to follow a mean-reverting process. The Kalman filter technique is used to estimate the parameters of the model. The second approach is also based on the Fisher equation, in which the expected real interest rate is assumed to follow a random-walk process. The parameters are estimated using the exact maximum likelihood method proposed by Shrestha (1988). The third approach is based on the observed real yield on index-linked gilt traded in the United Kingdom. The first two approaches use the quarterly data on T-bills and CPI inflation for the United States, the United Kingdom, and Canada. The last approach uses monthly data on the real yield to maturity on 2,011 index-linked gilt and the CPI inflation rate.

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[7] The real yield to maturity used here is based on 2,011 index-linked gilt traded in the United Kingdom. We would like to thank Q. C. Chu for providing us with the data.

We find that the expected real rate of interest is not constant over time. Furthermore, the empirical results suggest that the expected real rate consists of a single unit root. The same is true for the expected inflation rate; that is, each of the expected inflation series for all three economies follows a unit-root process.

Based on the first two approaches, we find negative and significant sample correlations between the real rate and the expected inflation rate series for all three countries. However, these two series are not cointegrated for all three countries. This implies that the negative relation is not a long-run stationary relation; hence, the Mundell-Tobin hypothesis does not hold in the long run. Therefore, T-bills can be considered as a long-run inflation hedge.

When the real interest rate on index-linked gilt is used, the sample correlation between the expected real rate and the expected inflation rate is positive but insignificant. Again, the cointegration analysis indicates no cointegrating relation between these two series, indicating that the Mundell-Tobin hypothesis does not hold true in the long run.

Appendix

Here we briefly describe the simulation procedure used to generate the empirical distribution. In the simulation, we need to generate three unit root processes representing nominal interest rate R_t, expected real interest rate r_t, and expected inflation rateπ_t^e that satisfy the Fisher equation. The three unit root series are generated using the following three-variable dynamic stochastic system:

$$\begin{aligned} x_{1t} &= u_{1t} \\ x_{1t} - x_{2t} &= u_{2t} \\ x_{1t} + x_{2t} - x_{3t} &= u_{3t}, \end{aligned} \tag{A.1}$$

where the series u_{1t}, u_{2t}, and u_{3t} are generated by:

$$\begin{aligned} (1 - L)u_{1t} &= \varepsilon_{1t} \\ (1 - L)u_{2t} &= \varepsilon_{2t} \\ u_{3t} &= \varepsilon_{3t}, \end{aligned} \tag{A.2}$$

Finally, the innovation series ε_{1t}, ε_{2t}, and ε_{3t} are generated as independent and normal processes, $\varepsilon_{it} \sim N(0, 0.01)$, $i = 1, 2, 3$. To make the series have a positive mean, 3.5 is added to the nominal interest rate x_{2t} and 2.5 is added to the inflation rate. For each replication and given sample size T, $T + 100$ observations are generated with the initial values of u_{1t}, u_{2t}, and u_{3t} being set equal to zero. Then, the first

100 observations are discarded to reduce the effect of the initial conditions. The remaining T observations are used to estimate the parameters of the model using the Kalman filtering technique as discussed in the text. The entire process is repeated 5,000 times.

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