Mathematics for physicists

Lecturer: Prof. Ven-Chung Lee

(Dated: October 15, 2005)

Some important equations in physics

Maxwell's equations of electric field $\vec{E}$ and magnetic field $\vec{B}$, $\vec{E} = \vec{E}\left(\vec{r}, t\right), \vec{B} = \vec{B}\left(\vec{r}, t\right)$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon} \quad , \quad \vec{\nabla} \cdot \vec{B} = 0 \quad , \quad \vec{\nabla} \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad , \quad \vec{\nabla} \times \vec{B} = \mu \left[\vec{j} + \frac{\partial \left(\epsilon \vec{E} \right)}{\partial t} \right]$$

$\rho \rightarrow$ charge density , $\vec{j} \rightarrow$ current density

$\epsilon, \mu \rightarrow$ characteristic parameters of the medium

Wave equation

$$\nabla^2 \psi = \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2}$$

wave function $\psi = \psi\left(\vec{r}, t\right) \rightarrow A e^{i\left(\vec{k} \cdot \vec{r} - \omega t\right)}$ plane wave function

$$\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \quad , \quad k = \frac{2\pi}{\lambda} \ , \ \omega = 2\kappa\nu \ , \ c = \lambda \cdot \nu$$

$c \rightarrow$ phase velocity of the wave

Schrödinger wave equation

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\left(\vec{r}\right) \psi$$

$i \equiv \sqrt{-1}$, $\psi = \psi\left(\vec{r}, t\right)$ wave function of the matter wave

$V\left(\vec{r}\right)$ potential energy, $\hbar \rightarrow$ Planck constant, $m \rightarrow$ mass

Newton's equation of motion

$$\vec{F} = m\vec{a} = m \frac{d^2 \vec{r}}{dt^2} = \frac{d\vec{p}}{dt} = -\vec{\nabla} V\left(\vec{r}\right)$$

$$\vec{p} = m\vec{v} \text{ momentum }, \quad \vec{r} = \vec{r}(t) \text{ position vector}$$

$$\vec{F} \to \text{Force }, \quad \vec{\nabla}V(\vec{r}) = \hat{\imath}\frac{\partial V}{\partial x} + \hat{\jmath}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}$$

for example the gravitation force

$$\vec{F}(\vec{r}) = -\frac{GMm\vec{r}}{r^3} \quad , \quad r \equiv |\vec{r}(t)|$$

and the gravitational potential energy

$$V(\vec{r}) = -\frac{GMm}{r}$$

What's the meaning of the above mentioned quantities and operations?

$$\frac{\partial}{\partial t}, \frac{\partial}{\partial x} \to \text{partial differential}$$

$$\vec{\nabla} \cdot \vec{E} \to \text{divergence of } \vec{E} \text{ vector}$$

$$\vec{\nabla} \times \vec{E} \to \text{Curl of } \vec{E} \text{ vector}$$

$$\vec{\nabla}V \to \text{gradient of scalar potential energy}$$

$$\frac{d}{dt} \to \text{total differential}$$

$$i = \sqrt{-1} \to \text{imaginary number}$$

$$e^{\beta x} \to \text{exponential function}$$

$$\vec{E} = E_x \cdot \hat{\imath} + E_y \cdot \hat{\jmath} + E_z \cdot \hat{k} \to \text{vector in 3 dimensional space}$$

$$\vec{E} = \vec{E}(\vec{r}, t) \qquad \text{Function of multiple variables } \vec{r} \text{ and } t$$

Contents of the course:

Number, especially the imaginary number and complex number $z = a + ib$

Algebra, for example the Binominal theorem:$(a+b)^n = \sum_{m=0}^{n} \frac{n!}{m!(n-m)!} a^{n-m} b^m$

Functions, Logarithms $g(t) = \alpha \cdot \ln \beta t$, exponentials $f(t) = \alpha \cdot e^{\beta t}$

Trigonometry, especially the Euler formula $e^{i\theta} = \cos\theta + i\sin\theta$ and the phasor $z = |z|\, e^{i\theta}$

Differential and Integral Calculus.

Vector Analysis.