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Theory of the Ambipolar Model Applied to the Quantum Efficiency of Photocurrents in Semiconductors

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A theoretical analysis is made for a P-polarized electromagnetic surface wave propagating along a plane-parallel ambipolar semiconductor plate. Fourier transform analysis is used to derive a general formula for the determination of the dependence of the electron and hole concentration distribution function, the photocurrents, and the quantum efficiency of the excess carrier generation function in the steady state. We give a complete two dimensional analysis of the problem, taking surface recombination and the vertical diffusion current into account. We present the analysis and discuss the excess carrier photocurrent and quantum efficiency in the two cases for which the surface recombination velocity $S_p \to 0$, and $S_p \to \infty$.

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PACS. 73.25.+i – Surface conductivity and carrier phenomena.

I. Introduction

The phenomenon of the generation of excess carriers by optical excitation in semiconductors has drawn wide interest, because of its basic physical aspects and its numerous applications. The ambipolar model is a widely used approach to explain the distribution and transport of excess carriers in semiconductors. This approach was first_ developed by Roosbroeck [1] for his analytical studies of the high level injection problem. Ishaque [2] developed the ambipolar model to describe the radiation induced photocurrent of a reverse biased p-n junction. Chatterjee [3] demonstrated mathematically the modulated photocurrent in the presence of a bias monochromatic light and found the possibility of an apparent quantum efficiency greater than unity. (Q.E.> 1).

In this work, we have used a Fourier transform analysis [4] to solve the ambipolar transport equation and obtain general formulas for the excess carrier concentration, flux density, current, and the quantum efficiency. We give a complete two dimensional analysis of the problem [5], taking the surface recombination and vertical diffusion into account.

II. Electromagnetic fields

In Fig. 1, the sample is in the form of a rectangular solid and the thickness ℓ is much less than the other dimensions x, y. We assume that the P-polarized electromagnetic **wave**

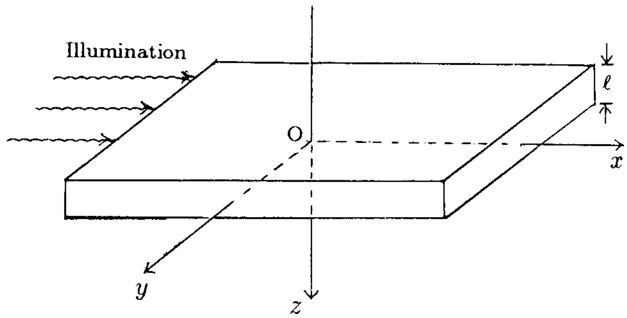


FIG. 1. The coordinate system and semiconductor film position for the analysis. The origin is chosen at the center of the sample. The dimensions of the sample in the x and y directions are assumed to be very large compared to the thickness ℓ. The incident P-polarized electromagnetic wave is propagating along the semiconductor surface in the x direction.

is incident along the semiconductor surface in the x direction. Since the incident P-polarized electromagnetic wave is propagating in the x direction, the y component of the electric field $E_y = 0$.

The electromagnetic fields in medium 1 (air) and medium 2 (semiconductor) will be of the form [6]

$$E_j(r,t) = (E_{jx}, O, E_{jz}) \exp i(k_{jx}x + k_{jz}z - \omega t),$$

$$H_j(r,t) = (0, H_{jy}, O) \exp i(k_{jx}x + k_{jz}z - wt).$$

In medium 1 (air), the subscript $j = 0$. In medium 2 (semiconductor), the subscript $j = m$.

In the general case, the wave vector k is complex,

$$k_{mx} = k_{x1} + ik_{x2}, \quad k_{mz} = k_{z1} \ t \ ik_{z2}.$$

In the semiconductor, the dielectric constant is

$$\varepsilon_m \text{ -- } \varepsilon_1 \ t \ i\varepsilon_2,$$

where ε_1 is the static dielectric constant,

$$\varepsilon_2 = \frac{4\pi\sigma}{\omega}, \quad \sigma = \frac{\sigma_0}{1 - i\omega\Im}, \quad \sigma_0 = \frac{e^2 N \Im}{m_e}. \tag{1}$$

σ is the frequency dependent conductivity, $\Im$ is the excess carrier lifetime, N is the number of excess carriers per unit volume, and m_e is the mass of excess carrier.

Using Maxwell's equations and the usual Maxwell boundary conditions, assuming $k_{x1} \gg k_{x2}$, $k_{z1} \gg k_{z2}$, $\varepsilon_1 \gg \varepsilon_2$, and neglecting the second order terms, we have the following relations:

$$k_{x2} = \frac{w}{2C} \frac{\varepsilon_2}{\varepsilon_1^{1/2}(1 + \varepsilon_1)^{3/2}}, \tag{2}$$

$$k_{z2} = \frac{\omega}{2C}\frac{\varepsilon_2(\varepsilon_1 + 2)}{(1 + \varepsilon_1)^{3/2}}.$$

III. Steady state photoconductivity

III-l. Excess carrier concentration

We calculate the steady state photoconductive response of a uniform semiconductor. Since in the steady state condition $\frac{\partial P}{\partial t}$ = 0, the ambipolar transport equation takes the form

$$\nabla^2 P(x,z) - \frac{1}{L_p^2}P(x,z) = \frac{-1}{D_p}g_s(x,z), \tag{4}$$

where $P(x,z)$ is the excess carriers concentration, L_p is the diffusion length $(L_p = \sqrt{D_p \Im})$, D_p is the diffusion coefficient, and $g_s(x,z)$ is the generation rate of excess carriers. We define

$$g_s(x,z) = \begin{cases} Fe^{-2\eta x}e^{-2\xi z}(0 \leq x < \infty) & (5) \\ Fe^{2\eta x}e^{-2\xi z} \quad (-\infty < x \leq 0), & (6) \end{cases}$$

where F, η and ξ are parameters. We will determine these parameters later.

In order to solve the ambipolar transport equation, a Fourier transform analysis is used,

$$\begin{cases} P_+(x,z) = \int_0^\infty P_+(s,z)e^{-sx}ds, \\ e^{-2\eta x} = \int_0^\infty f(s)e^{-sx}ds, \end{cases} \tag{7}$$

$$\begin{cases} P_-(x,z) = \int_{-\infty}^0 P_-(s,z)e^{sx}ds, \\ e^{2\eta x} = \int_{-\infty}^0 f(s)e^{sx}ds, \end{cases} \tag{8}$$

where f(s) = $\delta(s -$ 277). (9)

We expand the excess carriers concentration $P_\pm(s,$ z) in terms of a superposition of cosine functions.

$$P_\pm(s, z) = \sum_{n=0}^{\infty} A,(s) \cos \beta_n z. \tag{10}$$

Substituting equations (5), (7), (9) in to equation (4), using equation (10) and letting

$$M = \int_{-\ell/2}^{\ell/2} e^{-2\xi z}\cos\beta_n z dz,$$

we get $P_+(x,z)$.

From equation $(6),(8),(9)$, using the same method, we get $P_-(x,z)$. They have the form

$$\begin{cases} P_+(x,z) = e^{-2\eta x}\sum_{n=0}^{\infty}\phi_n M\cos\beta_n z, & (11)\\ P_-(x,z) = e^{2\eta x}\sum_{n=0}^{\infty}\phi_n M\cos\beta_n z, & (12)\end{cases}$$

where

$$\phi_n = \frac{-F}{D_p\ell\left(4\eta^2-\beta_n^2-\frac{1}{L_p^2}\right)}, \tag{13}$$

$$M = \frac{2}{4\xi^2+\beta_n^2}\left[2\xi\sinh\xi\ell\cdot\cos\frac{1}{2}\beta_n\ell+\beta_n\cosh\xi\ell\cdot\sin\frac{1}{2}\beta_n\ell\right], \tag{14}$$

The surface boundary conditions are

$$-D_p\left[\frac{\partial P_+(x,z)}{\partial z}\right]_{z=\frac{\ell}{2}} = S_pP_+(x,z)|_{z=\frac{\ell}{2}}, \tag{15}$$

$$D_p\left[\frac{\partial P_-(x,z)}{\partial z}\right]_{z=\frac{-\ell}{2}} = S_pP_-(x,z)|_{z=\frac{-\ell}{2}}, \tag{16}$$

where S_p is the surface recombination velocity. In order to satisfy the surface boundary conditions $(15),(16)$, **we** must have

$$\cot\frac{1}{2}\beta_n\ell = \frac{\beta_nD_p}{S_{rp}}. \tag{17}$$

111-2. Excess carrier currents

The time averaged energy flux of an electromagnetic wave in a semiconductor is given by the real part of the complex poynting vector:

$$\langle\vec{S}_m\rangle = \frac{C}{8\pi}Re\,(E_m \text{ x } H_m^*). \tag{18}$$

Using the electromagnetic fields in the semiconductor and equation (18), we get

$$\text{-V}\ .\ \langle\vec{S}_m\rangle = \frac{C}{4\pi}Re\left[(-k_{x2}A_{m2}B_m^* \text{ t } k_{z2}A_{m1}B_m^*)\,e^{-2k_{x2}x}e^{-2k_{z2}z}\right], \tag{19}$$

where -V $\cdot\langle\vec{S}_m\rangle$ is the electromagnetic energy per unit volume per unit time lost in the semiconductor and A_{m1},A_{m2},B_m are the amplitudes of E_{mx},F_{mz},H_m respectively.

Since $g_s(x,z)$ is the generation rate of excess carriers per unit volume per unit time for every photon absorbed, one electron-hole pair is created in the semiconductor. We define

$$\left[\frac{-\nabla\cdot\langle\vec{S}_m\rangle}{\hbar\omega}\right]\alpha e^{-\alpha\ell} = Fe^{\pm 2\eta x}e^{-2\xi z}, \tag{20}$$

where α is the absorption coefficient and ℓ is the thickness of the sample.

From equations (19),(20), we get

$$\eta = Re[k_{x2}], \qquad \xi = Re[k_{z2}], \tag{21}$$

$$F = \frac{1}{4\pi\hbar\omega}Re\left[-k_{x2}A_{m2}B_m^* + k_{z2}A_{m1}B_m^*\right]\alpha e^{-\alpha\ell}. \tag{22}$$

Applying Maxwell equations, and equations (1),(2),(3),(22), we get

$$F = \frac{1}{8\pi\hbar\omega}\left(1+\frac{1}{\varepsilon_1}\right)\frac{\omega_p^2}{\omega^2\Im}|A_{01}|^2\alpha e^{-\alpha\ell}, \tag{23}$$

where A_{01} is the amplitude of the incident electric field E_{ox}, and w is the angular frequency of the incident electromagnetic wave, while $\omega_p^2 = \frac{4\pi Ne^2}{m_e}$ is the plasma frequency.

The diffusion flux density of excess carriers can be calculated from the following relations:

$$\begin{cases} J_x^+(x,z) = -D_p\dfrac{\partial P_+(x,z)}{\partial x}, \\ J_x^-(x,z) = D_p\dfrac{\partial P_-(x,z)}{\partial x}, \end{cases} \qquad \begin{cases} J_z^+(x,z) = -D_p\dfrac{\partial P_+(x,z)}{\partial z}, \\ J_z^-(x,z) = D_p\dfrac{\partial P_-(x,z)}{\partial z}. \end{cases}$$

Using equations (11), (12) and the above relations, we get the diffusion flux density of excess carriers

$$\begin{cases} J_x^+(x,z) = 2D_p\eta e^{-2\eta x}\displaystyle\sum_{n=0}^{\infty}\phi_n M\cos\beta_n z, \\ J_x^-(x,z) = 2D_p\eta e^{2\eta x}\displaystyle\sum_{n=0}^{\infty}\phi_n M\cos\beta_n z, \end{cases} \tag{24}$$

$$\begin{cases} J_z^+(x,z) = D_p e^{-2\eta x}\displaystyle\sum_{n=0}^{\infty}\beta_n\phi_n M\sin\beta_n z, \\ J_z^-(x,z) = -D_p e^{2\eta x}\displaystyle\sum_{n=0}^{\infty}\beta_n\phi_n M\sin\beta_n z. \end{cases} \tag{25}$$

The excess carrier currents are defined by

$$I_x = e\ell_y\ell_z\left[\int_{-\infty}^{0}J_x^-(x,z)dx + \int_0^{\infty}J_x^+(x,z)dx\right], \tag{26}$$

$$I_z = e\ell_y\ell_z\left[\int_{-\infty}^{0}J_z^-(x,z)dx - \int_0^{\infty}J_z^+(x,z)dx\right], \tag{27}$$

where e is the charge of the electron and hole, and ℓ_y and ℓ_z are the width and thickness of the sample. Substituting equations (24), (25) in to equations (26) and (27), we obtain the currents at $z \rightarrow \frac{\ell_z}{2}$,

$$\boldsymbol{I,} = 2e\ell_y\ell_z \ D_p \sum_{n=0}^{\infty} \phi_n M \cos \frac{1}{2}\beta_n \ell_z, \tag{28}$$

$$\mathrm{I,} = -\frac{1}{\eta} e\ell_y\ell_z D_p \sum_{n=0}^{\infty} \beta_n \phi_n M \ \sin \frac{1}{2}\beta_n \ell_z. \tag{29}$$

Using equations (13),(14),(23), (28) and (29), we get the currents

$$I_x = \frac{e\ell_y\ell_z L_p^2}{4\pi\hbar\omega}\left(1+\frac{1}{\varepsilon_1}\right)\frac{\omega_p^2}{\omega^2\Im}|A_{01}|^2\alpha e^{-\alpha\ell_z}, \tag{30}$$

$$I_z = \frac{eC\ell_y\ell_z L_p}{8\pi\hbar\omega}\frac{(\varepsilon_1+1)^{5/2}}{\varepsilon_1^{1/2}}|A_{01}|^2\alpha e^{-\alpha\ell_z}. \tag{31}$$

111-3. **Quantum efficiency**

The time averaged energy flux of an electromagnetic wave in the spacer layer is given by the real part of the complex poynting vector

$$\langle \vec{S}_o \rangle = \frac{C}{8\pi} Re\,[E_0 \times H_0^*]. \tag{32}$$

$z\langle\vec{S}_0\rangle$ponent of

$$\langle \vec{S}_{0z} \rangle = \frac{C}{8\pi} Re\left[A_{01} \cdot B_0^* e^{-2k_{ox2}x}\right], \tag{33}$$

where A_{01} and B_0 are the amplitude of the electric field E_{ox} and magnetic field *Ho*, respectively.

The total number of photons per unit time incident upon the surface of the semiconductor is

$$N_p = \int_{-\frac{\ell_y}{2}}^{\frac{\ell_y}{2}} dy \int_{-\infty}^{\infty} dx \left[\frac{\langle \vec{S}_{oz} \rangle}{\hbar\omega}\right]. \tag{34}$$

From equations (33), (34) and the boundary conditions, we find the total number of photons per unit time to be

$$N_p = \frac{\ell_y C^2}{4\pi\hbar\omega}\frac{\varepsilon_1^{1/2}(\varepsilon_1+1)^2}{\Im\omega_p^2}|A_{01}|^2. \tag{35}$$

The Quantum efficiency is defined by

$$Q_E = \frac{[I/e]}{N_p}. \tag{36}$$

Using equations (30),(31), (35) and (36), we get the Quantum efficiency in two cases. Case (1), when the surface recombination velocity is very small ($S_p \to 0$), the current $I_z \to 0$, so the Quantum efficiency is

$$Q_{EX} = \frac{\ell_z L_p^2 \omega_p^4}{C^2\omega^2} \frac{1}{\varepsilon_1^{3/2}(\varepsilon_1+1)} \alpha e^{-\alpha \ell_z} \tag{37}$$

Case (2). When the surface recombination velocity is very large ($S_p \to \infty$), the current $I_x \to 0$, so the Quantum efficiency is

$$Q_{EZ} = \frac{\ell_z L_p \omega_p^2 \Im}{2\,c} \frac{(\varepsilon_1+1)^{1/2}}{\varepsilon_1} \alpha e^{-\alpha \ell_z} \tag{38}$$

These are the ratio of the number of excess carriers per unit volume per unit time to the number of incident photons per unit volume per unit time.

IV. Discussion and conclusions

When an electromagnetic wave is incident on a semiconductor, the generated electron-hole pairs are excited in a thin surface layer and their subsequent decay is expected to be largely influenced by surface recombination and diffusion effects.

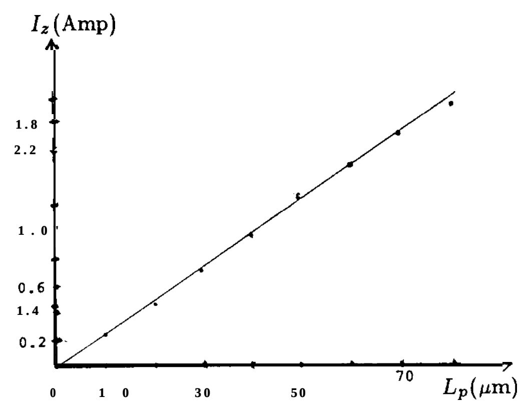
Now we apply the above theory to a few typical cases. There are two limiting cases that are of practical importance. We shall give numerical estimates for the currents I_x, I_z, and the Quantum efficiency Q_{EX}, Q_{EZ}.

We take the thickness and width of the sample $\ell_z = 5 \times 10^{-2}$ cm, and $\ell_y = 3$ cm, the diffusion length $L_p = 1 \times 10^{-3} \sim 8 \times 10^{-3}$ cm, the absorption constant $\alpha = 250$ l/cm, the incident electric field $E_0 = 15$ stat volt/cm, the plasma frequency $\omega_p^2 = 1 \times 10^{25}$ $1/\mathrm{sec}^2$, the excess carriers life time $\Im = 6 \times 10^{-8}$ sec, the angular frequency of the incident electromagnetic wave w = 1.7×10^{15} rad/sec.

Case (1). Low surface recombination velocity ($S_p \to 0$). From the surface boundary condition, if $S_p \to 0$, it gives $\frac{\partial P}{\partial z} \to 0$, corresponding to the condition of no net diffusive flow to the surface. As $S_p \to 0$, from equation (17), $\frac{1}{2}\beta_n \ell \to n\pi$, sin $\frac{1}{2}\beta_n \ell \to 0$, therefore $I_z \to 0$. Now from equation (30), (37) andusing the above numerical value, we find the excess carrier current to be $I_x = 6 \times 10^{-15}$ Amp $\sim 4 \times 10^{-13}$ Amp, and the Quantum efficiency is $Q_{EX} = 3.5 \times 10^{-15} \sim 2.25 \times 10^{-13}$.

Case (2). High surface recombination velocity ($S_p \to \infty$). From equation (17), if $S_p \to \infty$, cot $\frac{1}{2}\beta_n \ell \to 0$, $\frac{1}{2}\beta_n \ell \to \frac{1}{2}(2n+1)\pi$, cos $\frac{1}{2}\beta_n \ell \to 0$, therefore $I_x \to 0$, using the numerical value and equation (31), (38) we find the excess carrier current **I,** = 0.25 Amp $\sim$ 2.0 Amp and the Quantum efficiency is $Q_{EZ} = 0.28 \sim 1.12$. We plot the numerical value of **I,** and Q_{EZ} in Fig. 2, and Fig. 3.

When the surface recombination velocity $S_p \to \infty$, if the diffusion length $L_p > 8 \times 10^{-3}$ cm, there is the possibility of an apparent Quantum efficiency greater than unity ($Q_{EZ} > 1$).



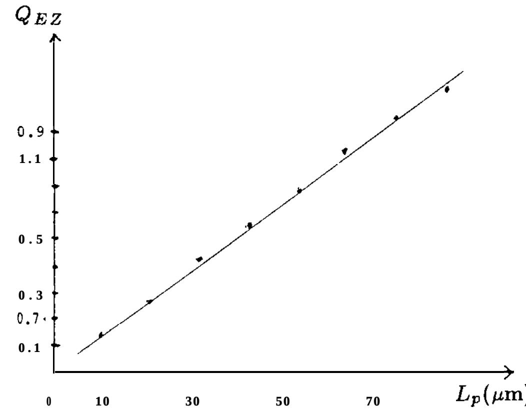


FIG. 2. The excess carrier currents I_z
function of diffusion length . of excess carrier diffusion length L_p.

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