Proceedings of the 41st IEEE
Conference on Decision and Control
Las Vegas, Nevada USA, December 2002

Adaptive Backstepping Speed/Position Control with Friction Compensation for Linear Induction Motor

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Abstract

In this paper, we will propose a nonlinear adaptive controller and an adaptive backstepping controller for linear induction motor to achieve speed/position tracking. A nonlinear transformation is proposed to facilitate controller design. Besides, the very unique end effect of the linear induction motor is also considered and is well taken care of in our controller design. We also consider friction dynamics effect and employ observer-based compensation which cope with friction force. Stability analysis based on Lyapunov theory is also performed to guarantee that the controller design here is stable. Also, the computer simulations and experiments are done to demonstrate the performance of our various controller design.

Keyword: *Adaptive Backstepping control, Motion Control, Linear Induction motor, Friction Compensato*

Nomenclature

$V_{qs}(V_{ds})$	q-(d-) axis input stator voltage	$i_q(i_d)$	q-(d-) axis input stator current
$R_s(R_r)$	Primary (secondary) resistance	$L_s(L_r)$	Primary (secondary) inductance
$\lambda_{qr}(\lambda_{dr})$	q-(d-) axis rotor flux	v_r	Linear speed of the primary
p_r	Position of the primary	M_m	Primary mass
B	Viscous friction coefficient	F_e	Electromagnetic force
P	Mechanical load force	L_m	Mutual inductance
K_f	Force constant($=3PL_m\pi/2\tau L_r$)	$a_2=\beta R_r/L_r, a_3=L_m R_r/L_r, a_4=R_r/L_r$	
$D=L_sL_r-L_m^2, p=P\frac{\pi}{\tau}, \beta=L_m/D, c=L_r/D, a_1=(R_sL_r)/D+\beta L_m R_r/L_r$			

I. Introduction

Nowadays, linear induction motors(LIM) are now widely used in many industrial applications including transportation, conveyor systems, actuators, material handling, pumping of liquid metal, and sliding door closers, etc. with satisfactory performance. The most obvious advantage of linear motor is that it has no gears and requires no mechanical rotary-to-linear converters. The linear electric motors can be classified into the following: D.C. motors, induction motors, synchronous motors and stepping motors, etc. Among these, the LIM has many advantages such as simple structure replacement of the gear between motor and motion devices, reduction of mechanical losses and the size of motion devices, silence, high starting thrust force, and easy maintenance, repairing and replacement.

For high precision motion performance, the friction problem is one of the significant limitations.

In the early works, Yamamura has first discovered a particular phenomenon of the end effect on LIM [1]. A control method, decoupling the control of thrust and the attractive force of a LIM using a space vector control inverter, was presented in [2], i.e. by selecting voltage vectors of PWM inverters appropriately.

Although the parameters of the simplified equivalent circuit model of an LIM can be measured by conventional methods (no-load and locked secondary tests), due to limited length of the machine the realization of the no-load test is almost impossible. Thus, the applicability of conventional methods for calculating the parameters of the equivalent model is limited. In order to measure the parameters, application of the finite element (FE) method for determining the parameters of a two-axis model of a three-phase linear induction motor has been proposed in [3]. Another method is proposed by removing the secondary [4].

To resolve the unique end effect problem, speed dependent scaling factors are introduced to the magnetizing inductance and series resistance in the d-axis equivalent circuit of the rotary induction motor (RIM) [5] to correct the deviation caused by the "end effect". On the other hand, there is a thrust correction coefficient introduced by [6,7] to calculate an actual thrust to compensate for the end effect. A related method to deal with the problem is that an external force corresponding to the end effect is introduced into the RIM model to provide a more accurate modeling of an LIM under consideration of end effect as shown in [8]. In another work [9], extra compensating-winding was proposed to compensate such problem.

Although the end effect is an important issue of the LIM control, but there are still many works in the literature without considering it, such as [10-16]. In this paper, we will take this as an important issue which can not be ignored. By the way, for the sake of the contact area of bearing in LIM is much larger than that of rotary induction motor (RIM), hence the friction term cannot be neglected. When accounting for the high speed applications, especially for the affects of the "end effect" and the friction mentioned above, we cannot over-emphasize the importance of "friction".

On the other hand, for high precision motion performance, the friction problem is one of the significant limitations. Because friction can lead to tracking errors, limit cycles, and undesired stick-slip motion[19,22]. To modeling a suitable friction model to predict and compensate for the friction, C. Canudas de Wit has propose a LuGre model[19]. In that model includes the Stribeck effect, hysteresis, spring-like characteristics for stiction, and varying break-away force. Furthermore, the adaptive scheme addressed in [20,21] is extended to handle non-uniform parametric variations of the friction force. In this paper, a nonlinear adaptive controller with adaptive friction compensation is proposed and the tracking performance is achieved.

II. PROBLEM FORMULATION

We consider the following assumptions to simplify the analysis:

(A.1) Three phases are balanced;

(A.2) The magnetic circuit is unsaturated;

(A.3) It is without end effect (we will relax this assumption later in controller design), then the dynamics of the entire system can be rearranged into the following more compact form

$$\begin{aligned}
\dot{i}_q &= -a_1 i_q + a_2\lambda_q - \beta p v_r \lambda_d + cV_{qs} \\
\dot{i}_d &= -a_1 i_d + \beta p v_r \lambda_q + a_2\lambda_d + cV_{ds} \\
\dot{\lambda}_q &= a_3 i_q - a_4\lambda_q + p v_r \lambda_d \\
\dot{\lambda}_d &= a_3 i_d - a_4\lambda_d - p v_r \lambda_q \\
M_m \dot{v}_r &= K_f(\lambda_d i_q - \lambda_q i_d) - \overline{F}_L - F_r
\end{aligned} \tag{1}$$

In this paper, we try to design the speed and position controller for the linear induction motor. All the parameters are assumed known except the payload. However, some knowledge about the payload structure is available, which is expressed in terms of and we use a second-order differential equation as

$$\overline{F}_L = M_L\dot{v}_r + b_{L0} + b_{L1}v_r + b_{L2}v_r^2 . \tag{2}$$

Furthermore, the friction force F_r in (1) is modeled by the LuGre friction model [19] with friction force variation:

$$\frac{dl}{dt} = v_r - \frac{|v_r|}{n(v_r)} l \tag{3}$$

$$F_r = \zeta_0 l + \zeta_1 \frac{dl}{dt} + \zeta_2 v_r \tag{4}$$

where l is the friction state that physically stands for the average deflection of the bristles between two contact surface. The friction force parameters ζ_0, ζ_1, ζ_2 can be physically explained as the stiffness of b ristles, damping coefficient, and viscous coefficient, respectively. In our design, we assume that these three parameters are unknown positive constants. A parameterization of $n(v_r)$ that has been proposed to describe the stribeck effect [19], i.e.,

$$n(v_r) = F_c + (F_s - F_c)e^{-(\frac{v_r}{v_s})^2} \tag{5}$$

where F_c, F_s and v_s are the Coulomb friction value, stiction force value, and the Stribeck velocity, respectively.

III. OBSERVER AND NONLINEAR ADAPTIVE CONTROLLER DESIGN

3.1 Analysis of mechanical load and end effect

The fundamental difference between a rotary induction motor and a LIM is the finite length of the magnetic and electric circuit of the LIM along the direction of the traveling field. The open magnetic circuit causes an initiation of the so-called longitudinal end effects [5].

For a LIM, the end effect with the load force plus friction effect can be represented as a function of the speed v_r, which can be normally simplified into the form

$$\begin{aligned}
F_L + F_r &= \sum_{n=0}^{2} b_n' v_r^n + M_e\dot{v}_r + F_L' + \zeta_0 l + \zeta_1(v_r - \frac{|v_r|}{n(v_r)} l) + \zeta_2 v_r \\
&= M_e\dot{v}_r + b_0' + b_1' v_r + b_2' v_r^2 + M_L\dot{v}_r + b_{L0} + b_{L1}v_r + b_{L2}v_r^2 + \zeta_0 l + (\zeta_1 + \zeta_2)v_r - \zeta_1\frac{|v_r|}{n(v_r)} l \\
&= M_L\dot{v}_r + b_0 + b_1 v_r + b_2 v_r^2 + \zeta_0 l - \zeta_1\frac{|v_r|}{n(v_r)} l
\end{aligned}$$

where F_L is denoted as the mechanical payload accounting for end effect and can be expressed in a compact form as $F_L = \Theta V_r^T$ with the unknown constant parameters $\Theta = [M_L \;\; b_0 \;\; b_1 \;\; b_2]$, and a known function vector $V_r^T = [\dot{v}_r \;\; v_r^0 \;\; v_r^1 \;\; v_r^2]$. The joint mass $M = M_m + M_L$ is therefore also unknown, which leads to the total mechanical load with motor itself as $F = \Theta^T V_r$, where $\Theta^T = [M \; b_0 \; b_1 \; b_2]$.

To proceed further, we introduce some additional assumptions as shown below:

(A.4) $x_2 = \lambda_q^2 + \lambda_d^2 > 0$,

(A.5) The desired speed should be a bounded smooth function with known first and second order time derivatives, then further simplify the dynamics shown in (1) by introducing a nonlinear coordinate transformation given as follows[23, 17]:

$$\begin{aligned}
x_1 &= i_q^2 + i_d^2 \\
x_2 &= \lambda_q^2 + \lambda_d^2 \\
x_3 &= i_q\lambda_q + i_d\lambda_d \\
x_4 &= i_q\lambda_d - i_d\lambda_q \\
x_5 &= v_r
\end{aligned}$$

Remarks: The transformation is trying to make the secondary flux norm, the electric force and the rotor speed as individual variables x_2, x_4 and x_5, respectively, and certainly the nonlinear transformation is not unique. Initially, we adopt the stator voltage inputs as $cV_{ds} = \frac{-\lambda_q}{\sqrt{\lambda_d^2+\lambda_q^2}}V, cV_{qs} = \frac{\lambda_d}{\sqrt{\lambda_d^2+\lambda_q^2}}V$ [23,17], with such transformation, then the dynamical equations shown in (1) can thus be transformed into the following dynamic model:

$$\begin{aligned}
\dot{x}_1 &= -2a_1x_1 + 2a_2x_3 + \tfrac{2x_4}{\sqrt{x_2}}V \\
\dot{x}_2 &= -2a_4x_2 + 2a_3x_3 \\
\dot{x}_3 &= a_3x_1 + a_2x_2 - (a_1 + a_4)x_3 + px_5x_4 \\
\dot{x}_4 &= -px_5x_3 - \beta px_5x_2 - (a_1 + a_4)x_4 + \sqrt{x_2}V \\
M\dot{x}_5 &= K_f x_4 - \sum_{n=0}^{2} b_n x_5^n - \zeta_0 l + \zeta_1\tfrac{|x_5|}{n(x_5)} l
\end{aligned} \tag{6}$$

To control the system (6), we develop the position controller to achieve the goal $p_r \rightarrow p_d$ as introduced in the following section.

3.2 Two Nonlinear Observer Design for Friction effect

In this paper, we consider dynamic friction effect and present it by a LuGre model. But we know the friction state l is not measurable. In order to handle different nonlinearities of l present in the system dynamics, we employ two nonlinear observers to estimate the immeasurable state l and replace l with its estimates $\hat{l}_0$ and $\hat{l}_1$ [20,21], of which the dynamics are respectively given by

$$\frac{d\hat{l}_0}{dt} = x_5 - \frac{|x_5|}{n(x_5)}\hat{l}_0 + \eta_0$$
$$\frac{d\hat{l}_1}{dt} = x_5 - \frac{|x_5|}{n(x_5)}\hat{l}_1 + \eta_1 \tag{7}$$

where η_0, η_1 are compensation terms that are yet to be determined in later design. The corresponding observation errors can be computed as

$$\frac{d\tilde{l}_0}{dt} = -\frac{|x_5|}{n(x_5)}\tilde{l}_0 + \eta_0$$
$$\frac{d\tilde{l}_1}{dt} = -\frac{|x_5|}{n(x_5)}\tilde{l}_1 + \eta_1$$

where $\tilde{l}_0 = l - \hat{l}_0$ and $\tilde{l}_1 = l - \hat{l}_1$ are estimation errors.

3.3 Adaptive Position Controller with Friction Compensation Design

Now, we introduce another state

$$x_6 = p_r \tag{8}$$

to facilitate investigation of the development of a position controller. Then, define the tracking errors as follows:

$$e_p = p_r - p_d \triangleq e_6 \tag{9}$$

Normally, while the position tracking error is driven to zero, the speed is also regulated to zero. Thus, we naturally define a joint error signal S as follows:

$$S = \dot{e}_p + ae_p = e_5 + ae_6,$$

where a is a positive scalar gain, and note the case with $a = 0$ will be degenerated back speed tracking problem. We can obtain the error dynamics equation as:

$$M\dot{S} = K_f x_4 - \sum_{n=0}^{2} b_n x_5^n - M(\dot{v}_d - ae_5) - \zeta_0 l + \eta_1 \frac{|x_5|}{n(x_5)} l$$
$$= K_f x_4 - \sum_{n=0}^{2} b_n x_5^n - M(\dot{v}_d - ae_5) - \zeta_0(\tilde{l}_0 + \hat{l}_0) + \zeta_1 \frac{|x_5|}{n(x_5)}(\tilde{l}_1 + \hat{l}_1)$$

.Based on this equation, we will propose a position tracking controller and the following theorem summarizes the design procedure and the resulting control effect.

In order to show the boundedness of all the parameter estimates and the tracking errors e_4, e_5, we choose a Lyapunov like function V_e as shown below:

$$V_e = \frac{1}{2}[MS^2 + e_4^2 + \tilde{b}_0^2 + \tilde{b}_1^2 + \tilde{b}_2^2 + \tilde{M}^2 + \tilde{\zeta}_0^2 + \tilde{\zeta}_1^2 + \tilde{l}_0^2 + \tilde{l}_1^2] \tag{10}$$

According to the suggested parameter adaptive laws as, namely,

$$\dot{\hat{b}}_0 = -S, \quad \dot{\hat{b}}_1 = -Sx_5, \quad \dot{\hat{b}}_2 = -Sx_5^2, \quad \dot{\hat{M}} = -S(\dot{v}_d - ae_5),$$
$$\dot{\hat{\zeta}}_0 = -S\hat{l}_0, \quad \dot{\hat{\zeta}}_1 = -S\frac{|x_5|}{n(x_5)}\hat{l}_1$$

and the friction observer compensation terms are defined by:

$$\eta_0 = -S, \quad \eta_1 = -\frac{|x_5|}{n(x_5)}S$$

if one designs the auxiliary signal x_{4d} as

$$x_{4d} = \frac{1}{K_f}[\sum_{n=0}^{2}\hat{b}_n x_5^n + \hat{M}(\dot{v}_d - ae_5) + \hat{\zeta}_0\hat{l}_0 - \hat{\zeta}_1\frac{|x_5|}{n(x_5)}\hat{l}_1 - \rho_1 S],$$

then the time derivative of the function V_e becomes

$$\dot{V}_e = -\rho_1 S^2 + e_4[-(a_1 + a_4)x_4 - \beta p x_2 x_5 - p x_3 x_5 + \sqrt{x_2}V - \dot{x}_{4d} + K_f S]$$
$$-\zeta_0 \tfrac{|x_5|}{n(x_5)}\tilde{l}_0^2 - \zeta_1 \tfrac{|x_5|}{n(x_5)}\tilde{l}_1^2 \tag{11}$$

Now, design the actual input

$$V = \frac{1}{\sqrt{x_2}}[(a_1 + a_4)x_4 + \beta p x_2 x_5 + p x_3 x_5 + \dot{x}_{4d} - \rho_2 e_4 - K_f S],$$

then it apparently leads to the result that

$$\dot{V}_e = -\rho_1 S^2 - \rho_2 e_4^2 - \zeta_0 \tfrac{|x_5|}{n(x_5)}\tilde{l}_0^2 - \zeta_1 \tfrac{|x_5|}{n(x_5)}\tilde{l}_1^2 \le 0$$

where $\rho_1, \rho_2 > 0$, ζ_0, ζ_1 are positive constants and the friction characteristic function $n(x_5)$ is chosen to be a positive function, which readily implies boundedness of all parameter estimates as well as of both signals x_4 and x_5. Since $\dot{V}_e$ in (11) is nonpositive, we conclude that all the error signals in V_e and, in particular, x_5 and x_{4d} are bounded, which in turn implies that x_4 and hence $\dot{x}_5$ (from system (6)) are both bounded. So that the estimation errors $\tilde{l}_0, \tilde{l}_1 \in L_\infty$ and all parametric error $\tilde{\zeta}_0, \tilde{\zeta}_1 \in L_\infty$. Because ζ_0, ζ_1 are unknown positive constants and $\tilde{l}_0 = l - \hat{l}_0$, $\tilde{l}_1 = l - \hat{l}_1$, the parameter estimates $\hat{\zeta}_0, \hat{\zeta}_1 \in L_\infty$. From the friction dynamics in (3) and the bounded speed x_5, the bounded friction state l is concluded, which further implies the observer states $\hat{l}_0, \hat{l}_1$ are bounded. We thus conclude that all the internal signals are kept bounded. Now, since I_s is bounded, then guarantees all signals x_i, $i = 1,..,5.$, are then guaranteed to be bounded.

By the power formula, $P_s = a_5\ x_4\ x_5 = 3\ V_s I_s$, which can be shown bounded from the above. We now show that I_s will be bounded via argument of contradiction. Say, I_s eventually grows unbounded, then V_s and, hence, V will diminish eventually. However, if I_s does grow unbound, then it implies that V will tend to $px_5x_3/\sqrt{x_2}$ eventually. However, from the dynamics of x_2 in (6), we have x_2 and x_3 grow at the same rate, which readily says that V will also grow unbounded. This obviously leads to a contradiction and therefore l_s is bounded.

Furthermore, we can show that $\dot{x}_{4d}$ is bounded, and hence $\dot{e}_4$ and $\dot{S}$ are also bounded, which implies the convergence of e_4 and S due to Barbalat's Lemma. Therefore, the control scheme with the properly designed input V will drive the output p_r to the desired p_d asymptotically. □

3.4 Consideration of Uncertainty Inductance

From the previous LIM dynamics, the parameters a_1, a_4, β, c and K_f depend on the inductance, but as we know the mutual inductance is hard to identify due to its intricate structure and undesirable end effect. In particular,

$$a_1 + a_4 = \left(\frac{R_s L_r + R_r L_m^2 / L_r}{L_s L_r - L_m^2}\right) + \frac{R_r}{L_r} \triangleq a_{10} + a_{40} + \alpha,$$

$$c = c_0 + \sigma$$

where α and σ are uncertainty terms of $(a_1 + a_4)$ and variance c, respectively. We rewrite the dynamic equations (3) as followings:

$$\dot{x}_1 = -2a_1 x_1 + 2a_2 x_3 + \frac{2(c_0+\sigma)x_4}{\sqrt{x_2}} V$$

$$\dot{x}_2 = -2a_4 x_2 + 2a_3 x_3$$

$$\dot{x}_3 = a_3 x_1 + a_2 x_2 - (a_{10} + a_{40} + \alpha) x_3 + p x_5 x_4$$

$$\dot{x}_4 = -p x_5 x_3 - \beta p x_5 x_2 - (a_{10} + a_{40} + \alpha) x_4 + (c_0 + \sigma)\sqrt{x_2} V$$

$$\frac{M}{K_f}\dot{x}_5 = x_4 - \sum_{n=0}^{2}\frac{b_n}{K_f} x_5^n - \frac{\zeta_0}{K_f} l + \frac{\zeta_1}{K_f}\frac{|x_5|}{n(x_5)} l$$

$$\dot{x}_6 = x_5 \tag{12}$$

and design the control input

$$V_{qs} = \frac{\lambda_d}{\sqrt{\lambda_q^2 + \lambda_d^2}} V, \; V_{ds} = \frac{-\lambda_q}{\sqrt{\lambda_q^2 + \lambda_d^2}} V$$

To facilitate subsequent investigation, we define several variables as follows:

$$\tilde{\alpha} = \alpha - \hat{\alpha}, \tilde{\beta} = \beta - \hat{\beta},$$

$$d_n = \frac{b_n}{K_f}, \; H = \frac{M}{K_f}, \xi_0 = \frac{\zeta_0}{K_f}, \text{and } \xi_1 = \frac{\zeta_1}{K_f}$$

where $\hat{\alpha}$ is the estimate of α, $\hat{\beta}$ is the estimate of β.

In order to show the boundedness of all the parameter estimators and the tracking errors e_4, S, we choose a Lyapunov like function V_e as shown below:

$$V_e = \frac{1}{2}[HS^2 + e_4^2 + \tilde{d}_0^2 + \tilde{d}_1^2 + \tilde{d}_2^2 + \tilde{H}^2 + \tilde{\xi}_0^2 + \tilde{\xi}_1^2 + \tilde{l}_0^2 + \tilde{l}_1^2 + \tilde{\alpha}^2 + \tilde{\beta}^2] \tag{13}$$

whose time derivative can be evaluated as follows.

If we employ friction observer (7) and design the parameter adaptive laws as

$$\dot{\hat{d}}_0 = -S, \; \dot{\hat{d}}_1 = -Sx_5, \; \dot{\hat{d}}_2 = -Sx_5^2, \; \dot{\hat{H}} = -S(\dot{v}_d + ae_5)$$

$$\dot{\hat{\alpha}} = -e_4 x_4, \; \dot{\hat{\beta}} = -e_4 p x_5 x_2, \; \dot{\hat{\xi}}_0 = -S\hat{l}_0, \; \dot{\hat{\xi}}_1 = -S\frac{|x_5|}{n(x_5)}\hat{l}_1$$

and the friction observer compensation terms are defined by:

$$\eta_0 = -S, \; \eta_1 = -\frac{|x_5|}{n(x_5)} S$$

along with the proper design of x_{4d} as

$$x_{4d} = [\sum_{n=0}^{2}\hat{d}_n x_5^n + \hat{H}(\dot{v}_d - ae_5) + \hat{\xi}_0\hat{l}_0 - \hat{\xi}_1\frac{|x_5|}{n(x_5)}\hat{l}_1 - \rho_1 S],$$

then the time derivative of the Lyapunov function V_e becomes

$$\dot{V}_e = -\rho_1 S^2 + e_4[g(x, S) + (c_0 + \sigma)\sqrt{x_2} V] - \xi_0 \tfrac{|x_5|}{n(x_5)}\tilde{l}_0^2 - \xi_1 \tfrac{|x_5|}{n(x_5)}\tilde{l}_1^2$$

After we substitute the properly designed input V as:

$$V = \frac{1}{c_0\sqrt{x_2}}\{-g(x, S) - \eta\,\mathrm{sgn}(e_4)\}$$

where $\mathrm{sgn}()$ is the sign function, then the time derivative $\dot{V}_e$ can be simplified as

$$\dot{V}_e = -\rho_1 S^2 - \xi_0 \tfrac{|x_5|}{n(x_5)}\tilde{l}_0^2 - \xi_1 \tfrac{|x_5|}{n(x_5)}\tilde{l}_1^2 - \left(\frac{c_0+\sigma}{c_0}\right)e_4[\eta\,\mathrm{sgn}(e_4) + \left(\frac{c_0}{c_0+\sigma}\right)g(x,S)\}$$

$$\le -\rho_1 S^2 - \xi_0 \tfrac{|x_5|}{n(x_5)}\tilde{l}_0^2 - \xi_1 \tfrac{|x_5|}{n(x_5)}\tilde{l}_1^2 - \left(\frac{c_0+\sigma}{c_0}\right)[\eta - \left(\frac{c_0}{c_0+\sigma}\right)|g(x,S)|]|e_4|$$

If η is chosen to satisfy $\eta \ge |g(x, S)| + k$ for some $k>0$, then we have

$$\dot{V}_e \le -\rho_1 S^2 - \xi_0 \tfrac{|x_5|}{n(x_5)}\tilde{l}_0^2 - \xi_1 \tfrac{|x_5|}{n(x_5)}\tilde{l}_1^2 - \rho_2 |e_4|$$

for some $\rho_2 > 0$, which again implies boundedness of all internal signals and convergence of the position tracking error.

IV. Adaptive Backstepping Controller Design

In the previous section, we have proposed an adaptive controller for the LIMs, which will require acceleration signals of the motor. Although this signal can be obtained through numerical differencing and digital filtering, it is more susceptible to noise. In order to avoid such problem, we thus propose the following nonlinear backstepping position controller without need of acceleration signal in this section.

Theorem 1. *Consider a linear induction motor whose dynamics are governed by system (3) under the assumptions (A.4). Given a friction observer (7) third-time differentiable smooth desired position trajectory p_d with $p_d, \dot{p}_d, \ddot{p}_d$ and $\dddot{p}_d$ being all bounded, then the following control input can achieve the control objective $p_r \to p_d$ (i.e. $x_6 = p_r$. will follow p_d asymptotically) with the control input*

$$V_{qs} = \frac{\lambda_d}{\sqrt{\lambda_q^2 + \lambda_d^2}}\frac{V}{c}, \; V_{ds} = \frac{-\lambda_q}{\sqrt{\lambda_q^2 + \lambda_d^2}}\frac{V}{c},$$

and

$$V = \frac{1}{\sqrt{x_2}}[g_2(x) + \hat{\Theta}_2 W_2 - K_f z_1 - \rho_2 z_2],$$

with adaptation law

$$\dot{\hat{\Theta}} = \dot{\hat{\Theta}} = -\Gamma_1 z_1 W, \ \dot{\hat{\Theta}}_2 = \dot{\hat{\Theta}}_2 = -\Gamma_2 z_2 W_2,$$

$$\dot{\hat{\xi}}_0 = -z_1 \hat{l}_0, \ \dot{\hat{\xi}}_1 = -z_1 \frac{|x_5|}{n(x_5)} \hat{l}_1$$

and the friction observer compensation terms are defined by:

$$\eta_0 = -z_1, \ \eta_1 = -\frac{|x_5|}{n(x_5)} z_1$$

where $\Gamma_1, \Gamma_2 > 0$, *and* $z_1 = S$, $z_2 = x_4 - \alpha_1$,

$$\alpha_1 = -\rho_1 MS + \frac{1}{K_f}\Theta^T W - \frac{a}{K_f} Me_5 + \hat{\xi}_0 \hat{l}_0 - \hat{\xi}_1 \frac{|x_5|}{n(x_5)} \hat{l}_1$$

for some $\rho_1, \rho_2 > 0$, *and*

$$g_2(x) = px_3 x_5 + \beta x_2 x_5 + (a_1 + a_4 - a)x_4 - \rho_1(K_f x_4 + \kappa J e_6)$$

$$, \Theta_2^T W_2 = (\rho_1 + \frac{a}{K_f})\Theta^T W + \frac{1}{K_f}\Theta^T \dot{W} = (\rho_1 + \frac{a}{K_f})\Theta^T W + \frac{1}{K_f}\Theta' W'$$

with the parameter vector Θ' *as well as the known function vector* W' *satisfying* $\Theta^T \dot{W} = \Theta'^T W'$.

Proof:

Step 1. Choose a different stabilizing function α_2 as follows

$$\alpha_1 = -\rho_1 MS + \frac{1}{K_f}\hat{\Theta}^T W - \frac{a}{K_f} Me_5 + \hat{\xi}_0 \hat{l}_0 - \hat{\xi}_1 \frac{|x_5|}{n(x_5)} \hat{l}_1 \quad (14)$$

where $\hat{\Theta}$ denotes the on-line parameter estimate. And, redefine the new error variables $z_1 = S$, $z_2 = x_4 - \alpha_2$.

Evaluate the time derivative of the Lyapunov function candidate

$$V_1 = \frac{1}{2}Mz_1^2 + \frac{1}{2\Gamma_1}\tilde{\Theta}^T\tilde{\Theta} + \frac{1}{2}\tilde{\xi}_0^2 + \frac{1}{2}\tilde{\xi}_1^2 + \frac{1}{2}\tilde{l}_0^2 + \frac{1}{2}\tilde{l}_1^2, \quad (15)$$

along the solution trajectories to obtain

$$\dot{V}_1 = -\rho_1 K_f M z_1^2 + K_f z_1 z_2 + \tilde{\Theta}^T(\frac{1}{\Gamma_1}\dot{\hat{\Theta}} + z_1 W) + \tilde{\xi}_0(\dot{\hat{\xi}}_0 - z_1\hat{l}_0) + \tilde{\xi}_1(\dot{\hat{\xi}}_1 - z_1\frac{|x_5|}{n(x_5)}\hat{l}_1)$$

$$-\xi_0 \tilde{l}_0(\eta_0 + z_1) + \xi_1 \tilde{l}_1(\eta_1 + \frac{|x_5|}{n(x_5)} z_1) - \xi_0 \frac{|x_5|}{n(x_5)} \tilde{l}_0^2 - \xi_1 \frac{|x_5|}{n(x_5)} \tilde{l}_1^2 \quad (16)$$

Devise the adaptation law as

$$\dot{\hat{\Theta}} = \dot{\hat{\Theta}} = -\Gamma_1 z_1 W, \ \dot{\hat{\xi}}_0 = -z_1 \hat{l}_0, \ \dot{\hat{\xi}}_1 = -z_1 \frac{|x_5|}{n(x_5)} \hat{l}_1$$

$$\eta_0 = -z_1, \ \eta_1 = -\frac{|x_5|}{n(x_5)} z_1 \quad (17)$$

for some proper positive adaptation gain Γ_1, then (16) can be slightly simplified as:

$$\dot{V}_4 = -\rho_1 K_f M z_1^2 + K_f z_1 z_2 - \xi_0 \frac{|x_5|}{n(x_5)} \tilde{l}_0^2 - \xi_1 \frac{|x_5|}{n(x_5)} \tilde{l}_1^2 \quad (18)$$

Step 2. The time derivative of z_2 is now expressed as

$$\dot{z}_2 = \dot{x}_4 - \dot{\alpha}_2 = -g_1(x) - \Theta_1^T W_1 + \sqrt{x_2} V \quad (19)$$

where the function are as previously defined. Thus, we need to select a Lyapunov function candidate and design V to render its time derivative nonpositive. We want to apply the augmented Lyapunov function candidate as:

$$V_2 = V_1 + \frac{1}{2}z_2^2, \quad (20)$$

whose time derivative is found to be

$$\dot{V}_2 = -\rho_1 K_f M z_2^2 + K_f z_1 z_2 + z_2[-g_1(x) - \Theta_1 W_1 + \sqrt{x_2} V]$$

$$-\xi_0 \frac{|x_5|}{n(x_5)} \tilde{l}_0^2 - \xi_1 \frac{|x_5|}{n(x_5)} \tilde{l}_1^2, \quad (21)$$

The control law V should be able to cancel the indefinite term in (21). On the other hand, to deal with the unknown parameters Θ_2, we will try to employ the current estimates

$$\hat{\Theta}_1, \text{i.e., } V = \frac{1}{\sqrt{x_2}}[g_1(x) + \hat{\Theta}_1 W_1 - K_f z_1 - \rho_2 z_2], \quad (22)$$

From this resulting derivative

$$\dot{V}_2 = -\rho_1 K_f M z_1^2 + z_2 \hat{\Theta}_1 W_1 - \rho_2 z_2^2 - \xi_0 \frac{|x_5|}{n(x_5)} \tilde{l}_0^2 - \xi_1 \frac{|x_5|}{n(x_5)} \tilde{l}_1^2 \quad (23)$$

in order to cancel the last term in (19), we modify the Lyapunov function as below:

$$V_3 = V_2 + \frac{1}{2}z_2^2 + \frac{1}{2}\tilde{\Theta}_1^T \tilde{\Theta}_1, \quad (24)$$

and the time derivative of V_6 hence is

$$\dot{V}_3 = -\rho_1 K_f M z_1^2 + \tilde{\Theta}_1^T(z_2 W_2 + \frac{1}{\Gamma_2}\dot{\hat{\Theta}}_1) - \xi_0 \frac{|x_5|}{n(x_5)} \tilde{l}_0^2 - \xi_1 \frac{|x_5|}{n(x_5)} \tilde{l}_1^2 \quad (25)$$

Now, the term with $\tilde{\Theta}_3$ can be eliminated completely with the update law

$$\dot{\hat{\Theta}}_2 = \dot{\hat{\Theta}}_2 = -\Gamma_2 z_2 W_2 \quad (26)$$

for some positive adaptation gain Γ_2, which thus yields

$$\dot{V}_3 = -\rho_1 K_f M z_1^2 - \rho_2 z_2^2 - \xi_0 \frac{|x_5|}{n(x_5)} \tilde{l}_0^2 - \xi_1 \frac{|x_5|}{n(x_5)} \tilde{l}_1^2 \quad (27)$$

which guarantees boundedness of all parameter estimates $\hat{\theta}$, $\hat{\theta}_1$ and z_1, z_2, and $z_1 \in L^2 \cap L^\infty$. To show boundedness of the rest of states, we can rearrange the dynamical equations from system (6) as shown below[17]:

$$X = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2a_1 & 0 & 2a_2 \\ 0 & -2a_4 & 2a_3 \\ a_3 & a_2 & -(a_1 + a_4) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{2x_4}{\sqrt{x_2}} V \\ 0 \\ px_5 x_4 \end{bmatrix} = AX + u$$

,where A can be shown to be Hurwitz. After reviewing definitions of x_3 and V, repectively, we found that the first entry of u will be bounded because x_2 grows no slower than x_3 if x_3 does grow unbounded (due to the second equation of (6)). As a result, u is apparently bounded, and hence X will be bounded. This then proves the boundedness of all the states. We note that $\dot{z}_1$ is also bounded, and hence by Barbalat's lemma we can conclude $\dot{z}_1 \in L_\infty$ so that

$$\lim_{t \to \infty} z_1 \to 0, \text{ i.e., } p_r \to p_d \text{ as } t \to \infty. \qquad \square$$

V. EXPERIMENTAL RESULTS

In order to compare controller without friction compensation with controller with friction, and will see that second controller has better performance. When accounting for the high speed applications the friction effect is more important. In first class, the controller can't compensate for friction effect. In the other hand, the controller with compensator that observed states and the position tracking errors do converges . All these position tracking errors will approach to zero when time goes to infinity. All the results are shown in Fig 5.1 to Fig 5.10.

Case I: Desired Position $5\sin(4)$ ***without Friction Compensator***

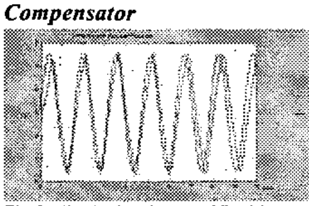
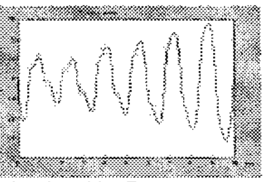

Fig 5.1 Desired and Actural Position Fig 5.2 Position Error

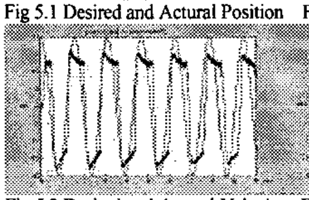
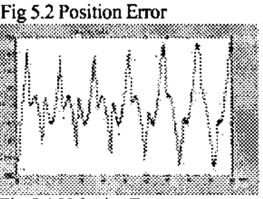

Fig 5.3 Desired and Actural Velocity Fig 5.4 Velocity Error

Case II: Desired Position $5\sin(4t)$ ***with Friction Compensator***

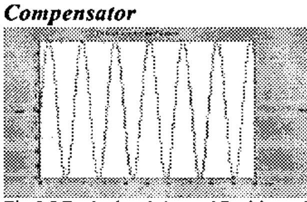
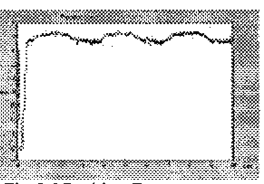

Fig 5.5 Desired and Actural Position Fig 5.6 Position Error

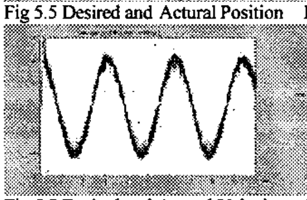
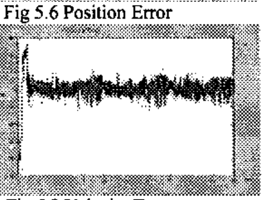

Fig 5.7 Desired and Actural Velocity Fig 5.8 Velocity Error

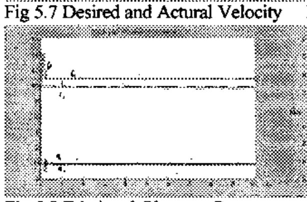
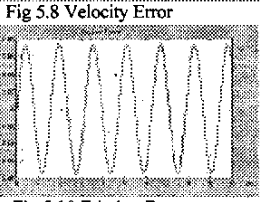

Fig 5.9 Frictional Observer Parameters Fig 5.10 Friction Force

VI. CONCLUSION

In this paper, we have proposed an adaptive backstepping controller for the linear induction motor with fifth/sixth order nonlinear dynamic model which is control by the primary voltage source. To cope with the uncertainty part of the linear induction motor, i.e., friction, end effect, payload, and inductance, we design our controller based on an appropriate nonlinear transformation. Due to inaccessibility to the flux in general. Stability analysis based on Lyapunov theory is performed to guarantee the controller design is stable. Finally, both the simulation and experimental results confirm the effectiveness of our control design.

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