Organizing an Optimal Cluster-Based Ad Hoc Network Architecture by the Modified Quine-McCluskey Algorithm

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***Abstract*— An optimal cluster-based ad hoc network architecture that requires the minimum number of cluster maintenance overheads not only reduces the waste of the precious bandwidth but also saves the consumption of the limited battery power. Mathematical analyses show that the cluster maintenance overheads can be minimized by minimizing the number of generated clusters and the variance of the number of cluster members. By using the Modified Quine-McCluskey (MQM) algorithm, the number of generated clusters and the variance of the number of cluster members of the generated cluster-based network architecture are minimized. Thus, the number of overheads required to maintain the cluster architecture is minimized and the precious bandwidth and the limited battery power are saved.**

***Index Terms*— Ad hoc networks, cluster, Quine-McCluskey algorithm, maintenance overhead.**

I. Introduction

WHEN wireless nodes are in an area that is not covered by any existing infrastructure, one of the possible solutions to achieve the ubiquitous computing is to enable wireless nodes to operate in the ad hoc mode [1] and self-organize themselves into a cluster-based network architecture. One of the general approaches to build up a cluster-based network architecture is to design an algorithm to organize wireless nodes into set of clusters. Within each cluster, a node is elected as a *clusterhead* (CH) to take responsible for the resource assignments and cluster maintenances. Many related algorithms have been proposed. The *minimum connected dominating set* (MCDS) approach [2] tries to obtain an optimum configuration to be the virtual backbone of the wireless ad hoc networks. However, it is shown to be an NP-hard [3] problem. The most feasible alternative is to find an approximated heuristic algorithm to obtain a sub minimum connected dominating set. The general idea among the related literatures is to select CHs based on some attributes of the networks. For example, the node degree, the link delay, the transmission power, the mobility, ..., etc.. A detail survey of the clustering algorithms can be found in [4].

In viewing the previous works, we find that the minimization of the waste of the precious bandwidth and the limited battery power in exchanging the cluster maintenance overheads has not been well studied. Thus, based on the technique to select the optimum set of prime implicants in the Quine-McCluskey (QM) algorithm [5], we propose a Modified QM (MQM) clustering algorithm to organize the wireless ad hoc network into a cluster-based network architecture that requires the minimum number of cluster maintenance overheads.

Manuscript received August 25, 2006. The associate editor coordinating the review of this letter and approving it for publication was Prof. Panagiotis Demestichas. This work was supported in part by the National Science Council under Contract NSC 95-2219-E-002-022.

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Digital Object Identifier 10.1109/LCOMM.2007.061376.

II. Preliminaries

A. Assumptions and Definitions

We assume that the wireless ad hoc network is connected and has N nodes denoted by 1, 2, ..., N. Furthermore, we also assume that each link is bi-directional. Terminologies that are used in this letter are defined as follows. Nodes that forward messages between clusters are regarded as *gateway*. Node that is neither CH nor gateway is called *ordinary node*. Node that has only one neighbor is called *boundary node*. The only neighbor of a boundary node is called *critical node*. Node that does not belong to any cluster is with status *unchecked*. Otherwise, it is with status *checked*. All nodes are initially with the status unchecked. The *logical degree* of an unchecked node is the number of unchecked one-hop neighbors.

B. Analyses of The Cluster Maintenance Overheads

In the cluster-based network architecture, two types of cluster maintenance overheads are required in order to maintain the cluster architecture: intra-cluster and inter-cluster. All members in a cluster are required to exchange intra-cluster messages in order to maintain local cluster memberships. However, only the CHs are required to exchange inter-cluster messages to maintain the global cluster architecture. Assume that a wireless ad hoc network is organized into j clusters. Let the number of cluster members in the cluster i is m_i. In this case, the number of required intra-cluster maintenance overheads in the cluster i is m_i^2 and the number of required inter-cluster maintenance overheads in the cluster i is $(j-1)$. Thus, the total number of required cluster maintenance overheads for the cluster i is m_i^2+j-1 and the total number of required cluster maintenance overheads for the entire network is

$$OH = \sum_{i=1}^{j}(m_i^2 + j - 1) = \sum_{i=1}^{j} m_i^2 + j^2 - j. \quad (1)$$

Let $\mu = N/j$ be the mean value of the number of cluster members and $\sigma^2 = \frac{1}{j}\sum_{i=1}^{j}(m_i-\mu)^2$ be the variance of the number of cluster members. Then, (1) can be rewrite to

$$OH = \frac{N^2}{j} + j\sigma^2 + j^2 - j. \quad (2)$$

Thus, to minimize the number of cluster maintenance overheads, we need to design a clustering algorithm that can not only minimize the number of generated clusters but also minimize the variance of the number of cluster members.

III. The Modified Quine-McCluskey (MQM) Clustering Algorithm

The original *Quine-McCluskey* (QM) Algorithm [5] is commonly used in the minimization of a Boolean function with multiple variables in the logic design. The operations of the QM algorithm can be divided into two stages. Due to the limited space, the procedures of each stage are briefly described as follows. (The detail procedures and examples of the QM algorithm can be found in [5].)

Stage 1: Determine the prime implicants. The first step is to express the function as a sum of minterms. The second step is to represent each minterm by its minterm number in binary form. The third step is to obtain the prime implicants (PIs) by iteratively using the law A + !A = 1 to group minterms.

Stage 2: Finding a minimum set of PIs to express the function. The first step is to construct a PI table. In this table, columns correspond to the minterms of the Boolean function and rows correspond to the PIs obtained in Stage 1. Besides, if any column contains only a single 'x', the column is called *distinguished column* and the row in which the 'x' occurs is called *essential row*. The second step is to determine the distinguished columns and essential rows (if any exists). The third step is to draw a line through each column which contains a 'x' in any of the essential rows, since inclusion of the essential rows in the solution will guarantee that these columns contain at least one 'x'. Now, for the rest of the minterms that are yet to have a 'x', choose PIs as economically as possible to cover the remaining minterms. Finally, write out the final solutions as a set of PIs. The modifications of the original QM algorithm are stated as follows.

- Due to the properties of binary number are no longer valid, all procedures in Stage 1 are neglected.
- Instead of constructing PI table, each node maintains a list of the ID and logical degree of its two-hop neighbors.
- Due to the distributed nature of the wireless ad hoc networks, the original centralized QM algorithm is modified to a distributed algorithm so that it can be executed in each ad hoc node.
- Minterms correspond to the distinguished columns and PIs correspond to the essential rows are modified to boundary nodes and critical nodes respectively.
- Selecting each of the PIs correspond to the essential rows into the minimum PI set is modified to select critical nodes that are with the highest logical degree among their one-hop neighbors as CHs (if any exists). Ties are broken by node ID.
- Drawing a line through each column which contains a 'x' in any of the essential rows is modified to let unchecked one-hop neighbors of critical node become its cluster members.
- Choosing PIs as economically as possible to cover the remaining minterms is modified to select unchecked nodes that are with the highest logical degrees among their two-hop neighbors. Ties are broken by node ID.

Based on the above modifications, the operations of the MQM algorithm are stated as follows.

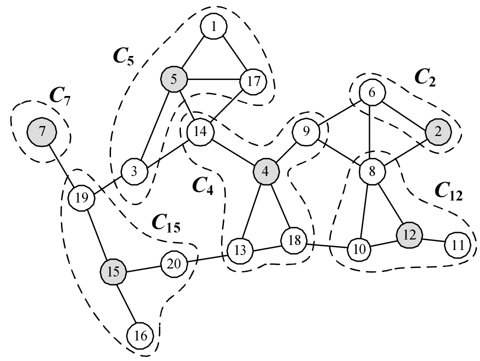

Fig. 1. The network topology.

```
For any unchecked node v
{
   if ((among the unchecked critical nodes within the one-hop
       neighbors, node v is the only critical node with the
       highest logical degree) || (among the unchecked
       one-hop critical nodes with the same highest logical
       degree, node v is with the smallest ID) || (node v is
       the only node with the highest logical degree among
       the unchecked two-hop neighbors) || (among the
       unchecked two-hop neighbors with the same highest
       logical degree, node v is with the smallest ID))
   {
      updates status to checked;
      regards itself as a CH;
      broadcasts an invite packet to all neighbors;
   }
   On receiving invite packet(s) sent from the one-hop
    neighboring node(s)
   {
      updates status to checked;
      if (more than one such packets are received)
      {
         if (senders are with the same logical degree)
            joins the one with the smallest ID (say node u);
         else
            joins the sender with the highest logical degree
             (say node u);
         regards itself as a gateway node;
      }
      else
         regards itself as an ordinary node;
      broadcasts a join packet to join the selected cluster;
 }
 On receiving a join packet sent from the one-hop neighbor
    decreases the logical degree by 1;
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Example: Consider a wireless ad hoc network with 20 nodes as shown in Fig. 1. First of all, nodes 12 finds that it is the only critical node among its one-hop neighbors and critical node 15 finds that its ID is smaller than the neighboring critical node 19 that is with the same logical degree. Therefore, node 12 and 15 update status to checked, regard themselves as CHs and broadcast invite packets to their one-hop neighbors. After receiving invite packets, unchecked nodes 8, 10 and 11 join the cluster organized by critical node 12; unchecked nodes 16, 19 and 20 join the cluster organized by critical node 15. On receiving join packets, the unchecked neighbors of the newly checked nodes 8, 10, 19 and 20 (i. e. nodes 2, 3, 6, 7, 9, 13 and 18) decrease their logical degrees. Next, node 4 finds that it is with the highest logical degree among its two-hop neighbors. So, it updates status to checked, regards itself as a CH and broadcasts an invite packet. On receiving the invite packets, unchecked nodes 9, 13, 14 and 18 will join and become its cluster members. Also, unchecked nodes 3, 5, 6, and 17 (neighbors of the newly checked nodes 9 and 14) will decrease their logical degrees. For the rest of unchecked nodes,

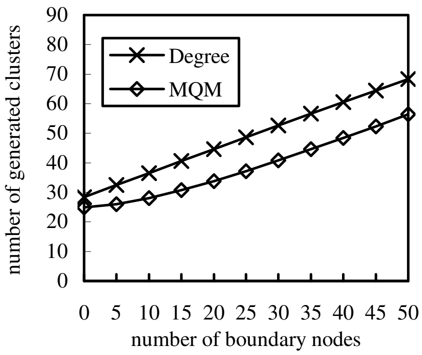

Fig. 2. The number of generated clusters.

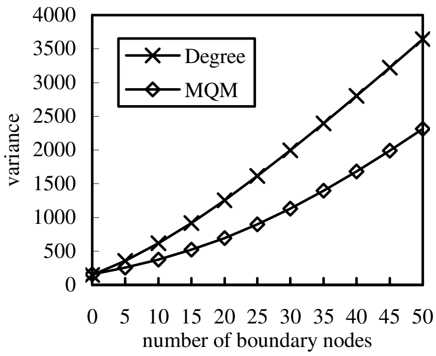

Fig. 3. The variance of the number of cluster members.

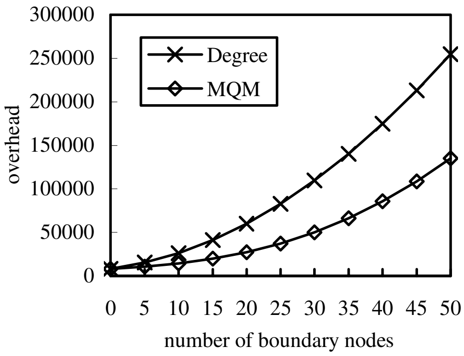

Fig. 4. The number of cluster maintenance overheads.

node 7 is with logical degree 0, node 5 is with the highest logical degree 3 among its two-hop unchecked neighbors and node 2 is with the smaller ID than its only unchecked neighbor node 6. Thus, they will regard themselves as CHs and invite unchecked neighbors to join. Now, unchecked node 7 will be the only member in its cluster, unchecked nodes 1, 3 and 7 will be the cluster members of node 5 and unchecked node 6 will be the cluster member of node 2. Thus far, all nodes are clustered and the algorithm terminates.

IV. Simulation Results

We verify the performance of the proposed MQM algorithm by conducting extensive simulations. In our simulations, we assume the size of the service area is 2000m×2000m, the number of nodes N is 300 and the transmission range for each node is 300m. We run the simulation 10,000 times and average the collected data. In each simulation, we first randomly deploy the non-boundary nodes into a connected sub-network. Then, for each boundary node, a node in the connected sub-network is randomly selected to be its only neighbor (i. e., the critical node). For the performance comparisons, the MQM and the Degree-based [6][7] clustering algorithms are used to cluster each of the generated network topology. As stated in Section II, our objective is to design a clustering algorithm that can organize a cluster-based network architecture in which the required number of cluster maintenance overheads is minimized. As derived in (2), the number of cluster maintenance overheads mainly depends on the number of generated clusters and the variance of the number of cluster members. The simulation results for the number of generated clusters and the variance of the number of cluster members are shown in Fig. 2 and Fig. 3 respectively. Since the original QM algorithm is designed to obtain the minimum set of PIs, the proposed MQM algorithm generates the minimum number of clusters as shown in Fig. 2. Furthermore, due to unchecked node is selected as a CH only if it is the critical node with the highest logical degree among its one-hop neighbors or it is the node with the highest logical degree among its two-hop neighbors, the number of clusters that are generated by the boundary node and the difference of the number of cluster members between clusters are minimized. Therefore, as shown in Fig. 3, the variance of the number of cluster members is minimized. Consequently, the number of cluster maintenance overheads as shown in Fig. 4 is minimized and the generated cluster-based network architecture is optimal.

V. Conclusion

To reduce the waste of precious bandwidth and the limited battery power in exchanging the cluster maintenance overheads, we propose a distributed Modified Quine-McCluskey (MQM) algorithm to organize the wireless ad hoc network into an optimal cluster-based network architecture that requires the minimum number of cluster maintenance overheads. Simulation results show that by minimizing the number of generated clusters and the variance of cluster members, the organized cluster-based network architecture requires the minimum number of cluster maintenance overheads. Thus, the optimal cluster-based network architecture is organized.

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