Optimal Design of Two-Channel Nonuniform-Division FIR Filter Banks with -1, 0, and $+1$ Coefficients

Ju-Hong Lee, *Member, IEEE*, and Ding-Chiang Tang

***Abstract*—This paper deals with the optimal design of two-channel nonuniform-division filter (NDF) banks whose linear-phase FIR analysis and synthesis filters have coefficients constrained to -1, 0, and $+1$ only. Utilizing an approximation scheme and a WLS algorithm, we present a method to design a two-channel NDF bank with continuous coefficients under each of two design criteria, namely, least-squares reconstruction error and stopband response for analysis filters and equiripple reconstruction error and least-squares stopband response for analysis filters. It is shown that the optimal filter coefficients can be obtained by solving only linear equations. In conjunction with a proposed filter structure, a method is then presented to obtain the desired design result with filter coefficients constrained to -1, 0, and $+1$ only. The effectiveness of the proposed design technique is demonstrated by several simulation examples.**

***Index Terms*—Filter banks, FIR filters.**

I. Introduction

QUADRATURE mirror filter (QMF) banks find an important role in the areas of subband coding of speech signals [1], communication systems [2], short-time spectral analysis [3], and subband coding of image signals [4]. In these applications, a QMF bank is used to decompose a signal into subbands with equal bandwidth, and the subband signals in the analysis system are decimated by an integer equal to the number of the subbands. However, uniform-subband decomposition is not an appropriate scheme to match the requirements for the subband coding of speech and audio signals. The most appropriate decomposition must consider the critical bands of the ear. It has been mentioned in [5] that these critical bands have nonuniform bandwidths and cannot be easily constructed by conventional tree structure based on two-channel QMF banks. Therefore, it is worth exploiting the design problem of two-channel nonuniform-division filter (NDF) banks.

The basic theory regarding the principle and the related conditions of perfect reconstruction for NDF banks has been presented in [5]. Methods for designing NDF banks were also proposed in [5]. However, it is difficult to solve the resulting design problem with nonlinear constraints. In [6], a structure for NDF banks was introduced, and a design method based on the use of pseudo-QMF was presented. The main drawback of this method is that FIR filters with complex coefficients are required by the resulting NDF bank to reduce the aliasing distortion. Recently, one of the authors considered a structure for two-channel NDF banks and proposed design methods for optimally designing NDF banks based on L_1 error criteria in [7].

Although designing a NDF bank has been considered in [5]–[7], hardware implementation for the designed NDF banks generally requires large and complicated digital circuits because these NDF banks are designed with real or complex coefficients. To achieve circuit complexity reduction or to speed up filtering operation besides concern for the overall performance, it is preferable to design a NDF bank with coefficients restricted to -1, 0, and $+1$ only. However, there are practically no papers concerning the optimal design of NDF banks whose FIR analysis and synthesis filters have coefficients restricted to -1, 0, and $+1$ only in the literature.

In this paper, the NDF banks with the structure similar to [7], as shown in Fig. 1, are considered. We deal with the optimal design and realization of the two-channel NDF bank with -1, 0, and $+1$ coefficients. First, a method is developed based on an approximation scheme for designing a continuous-coefficient NDF banks with optimal reconstruction response and stopband response for its linear-phase (LP) FIR analysis and synthesis filters in the least-squares (L_2) sense. This method is further incorporated with the WLS algorithm of [8] to optimally design NDF banks with minimax (L_∞) reconstruction response and L_2 stopband response for analysis and synthesis filters. It is shown that the resulting coefficients for the LP FIR analysis and synthesis filters can be found through solving only linear equations. To obtain a design with -1, 0, and $+1$ coefficients, which achieves the optimal performance, we propose a new filter structure for realization. The coefficients -1, 0, and $+1$ are used in the oversampled domain, and the design procedure leads to finely quantized coefficients. Simulation results show that very satisfactory NDF banks can be obtained using the proposed technique.

This paper is organized as follows. Section II briefly describes the principle of two-channel NDF FIR filter banks. In Section III, we formulate the associated design problem for a two-channel NDF filter bank with L_2 reconstruction response and L_2 filter stopband response. A design method based on an approximation scheme is presented for solving the re-

Manuscript received September 23, 1997; revised July 31, 1998. This work was supported by the National Science Council under Grant NSC87-2213-E002-071. The associate editor coordinating the review of this paper and approving it for publication was Dr. Truong Q. Nguyen.

The authors are with the Department of Electrical Engineering, National Taiwan University, Taipei, Taiwan, R.O.C.

Publisher Item Identifier S 1053-587X(99)00756-4.

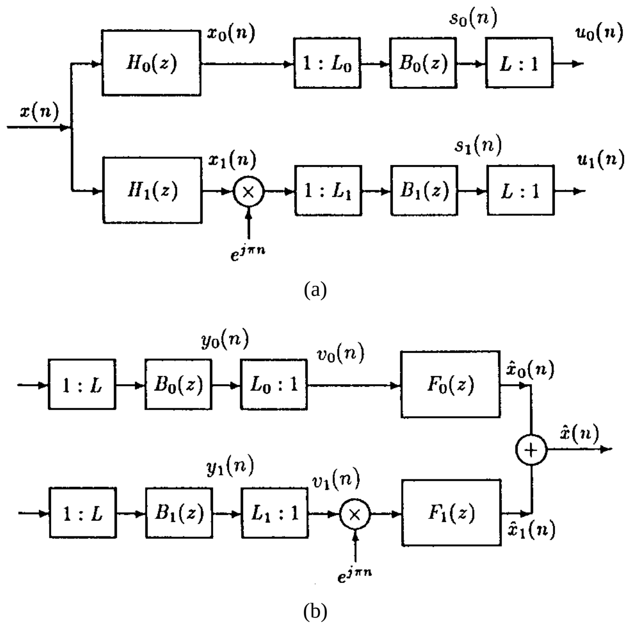


Fig. 1. Two-channel nonuniform-division maximally decimated filter bank system. (a) Analysis system. (b) Synthesis system.

sulting optimization problem. Section IV presents the method utilizing the WLS algorithm for obtaining a design with L_∞ reconstruction response and L_2 filter stopband response. In Section V, the method in conjunction with a new filter structure for obtaining a design with -1, 0, and $+1$ coefficients from the continuous designs is proposed. Several simulation examples are provided in Section VI for illustration. Finally, we conclude this paper in Section VII.

II. Two-Channel Nonuniform-Division FIR Filter Banks

Consider the two-channel NDF bank with the architecture given in [7], which is shown in Fig. 1. The analysis lowpass and highpass filters are designated by $H_0(z)$ and $H_1(z)$, respectively, whereas the synthesis lowpass and highpass filters are designated by $F_0(z)$ and $F_1(z)$, respectively. $B_0(z)$ and $B_1(z)$ are two lowpass filters responsible for achieving aliasing-free operation during the rational decimation and interpolation. It can be shown that using the modulations of multiplying $\exp(jn\pi)$ in a highpass subband channel leads to the favorable result that $B_1(z)$ can be a lowpass filter with real coefficients. The desired magnitude responses for the analysis filters $H_0(z)$ and $H_1(z)$ with passband widths equal to $L_0\pi/L$ and $L_1\pi/L$, respectively are shown in Fig. 2, where $L = L_0 + L_1$. ω_p and ω_s denote the related band-edge frequencies satisfying $\omega_p + \omega_s = 2\pi L_0/L$.

Assume that the associated magnitude responses are set to $B_0(\omega) = 1$ for $\omega \in [0, (\omega_s/L_0)]$ and $= 0$, for $\omega \in [(2\pi - \omega_s)/L_0, \pi]$, and $B_1(\omega) = 1$, for $\omega \in [0, (\pi - \omega_p)/L_1]$ and $= 0$, for $\omega \in [(\pi + \omega_p)/L_1, \pi]$, respectively. Further, assume that $H_0(z)$ and $H_1(z)$ have zero stopband response. As shown in the Appendix, the input/output relationship of the

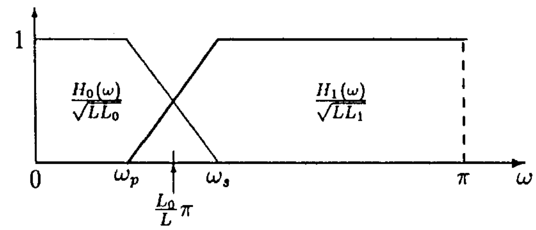


Fig. 2. Desired magnitude specifications for the analysis filters.

NDF bank in the frequency domain is given by [11]

$$\begin{aligned}\hat{X}(e^{j\omega}) = &\frac{e^{-j\omega G_0}}{LL_0}\,[X(e^{j\omega})H_0(e^{j\omega}) \\ &+ X(e^{j\omega}W_L^{L_0})H_0(e^{j\omega}W_L^{L_0}) \\ &+ X(e^{j\omega}W_L^{-L_0})H_0(e^{j\omega}W_L^{-L_0})]F_0(e^{j\omega}) \\ &+ \frac{e^{-j\omega G_1}}{LL_1}\,[X(e^{j\omega})H_1(e^{j\omega}) \\ &+ X(e^{j\omega}W_L^{L_1})H_1(e^{j\omega}W_L^{L_1}) \\ &+ X(e^{j\omega}W_L^{-L_1})H_1(e^{j\omega}W_L^{-L_1})]F_1(e^{j\omega}) \end{aligned} \tag{1}$$

where G_0 and G_1 are the resulting group delays of the upper and lower channels, respectively. $W_L = \exp(-j2\pi/L)$. Substituting $L = L_0 + L_1, F_0(e^{j\omega}) = H_0(e^{j\omega})$ and $F_1(e^{j\omega}) = -H_1(e^{j\omega})$ into (1) yields

$$\begin{aligned}\hat{X}(e^{j\omega}) = &T(e^{j\omega})X(e^{j\omega}) + A_1(e^{j\omega})X(e^{j\omega}W_L^{L_0}) \\ &+ A_2(e^{j\omega})X(e^{j\omega}W_L^{L_1})\end{aligned} \tag{2}$$

where

$$T(e^{j\omega}) = \frac{e^{-j\omega G_0}}{LL_0}\,H_0^2(e^{j\omega}) - \frac{e^{-j\omega G_1}}{LL_1}\,H_1^2(e^{j\omega}) \tag{3}$$

$$\begin{aligned}A_1(e^{j\omega}) = &\frac{e^{-j\omega G_0}}{LL_0}\,H_0(e^{j\omega})H_0(e^{j\omega}W_L^{L_0}) \\ &- \frac{e^{-j\omega G_1}}{LL_1}\,H_1(e^{j\omega})H_1(e^{j\omega}W_L^{L_0})\end{aligned} \tag{4}$$

$$\begin{aligned}A_2(e^{j\omega}) = &\frac{e^{-j\omega G_0}}{LL_0}\,H_0(e^{j\omega})H_0(e^{j\omega}W_L^{L_1}) \\ &- \frac{e^{-j\omega G_1}}{LL_1}\,H_1(e^{j\omega})H_1(e^{j\omega}W_L^{L_1}).\end{aligned} \tag{5}$$

The first term of (2) represents the response of a linear shift-invariant system $T(e^{j\omega})$ with input $X(e^{j\omega})$, whereas the other two terms represent the resulting aliasing distortion. Therefore, perfect reconstruction requires the following conditions.

PR 1: The magnitude $T(\omega)$ of $T(e^{j\omega})$ must be equal to 1, i.e., $T(\omega) = 1$, for all ω.

PR 2: The magnitude $A_1(\omega)$ of $A_1(e^{j\omega})$ must be zero, i.e., $A_1(\omega) = 0$, for all ω.

PR 3: The magnitude $A_2(\omega)$ of $A_2(e^{j\omega})$ must be zero, i.e., $A_2(\omega) = 0$, for all ω.

We note from (3) that $H_0(z)$ must be a either case 1 or case 2 LP FIR filter, whereas $H_1(z)$ must be a case 4 LP FIR filter, as shown in [9], to ensure the PR 1 condition. Let $H_0(z)$ and $H_1(z)$ have filter lengths equal to N_0 and N_1, respectively. Then, $H_0(e^{j\omega})$ can be expressed as [9]

$$H_0(e^{j\omega}) = e^{-j((N_0-1)\omega/2)}H_0(\omega) \tag{6}$$

where

$$H_0(\omega)=\begin{cases} h_0\left(\frac{N_0-1}{2}\right)+\sum_{n=0}^{(N_0-3)/2} 2h_0(n) \\ \quad \cos\left(\left(n-\frac{N_0-1}{2}\right)\omega\right), \\ \quad \text{for case 1} \\ \sum_{n=0}^{(N_0/2)-1} 2h_0(n)\cos\left(\left(n-\frac{N_0-1}{2}\right)\omega\right) \\ \quad \text{for case 2}\end{cases}$$

and $h_0(n)$ denotes the impulse response of $H_0(e^{j\omega})$. Similarly, we can express $H_1(e^{j\omega})$ as [9]

$$H_1(e^{j\omega})=je^{-j((N_1-1)\omega/2)}H_1(\omega) \tag{7}$$

where

$$H_1(\omega)=\sum_{n=0}^{(N_1/2)-1} 2h_1(n)\sin\left(\left(n-\frac{N_1-1}{2}\right)\omega\right)$$

and $h_1(n)$ denotes the impulse response of $H_1(e^{j\omega})$. Substituting (6) and (7) into (3) yields

$$T(e^{j\omega})=\frac{1}{LL_0}\,e^{-j(G_0+N_0-1)\omega}H_0^2(\omega)+\frac{1}{LL_1}\,e^{-j(G_1+N_1-1)\omega}H_1^2(\omega). \tag{8}$$

Let $D_0=G_0+N_0-1$ and $D_1=G_1+N_1-1$. Assuming $D_0=D_1$, we can neglect the LP term of (8) and express $T(\omega)$ as

$$T(\omega)=\frac{1}{LL_0}\,H_0^2(\omega)+\frac{1}{LL_1}\,H_1^2(\omega). \tag{9}$$

Next, substituting (6) and (7) into (4) and (5), we can obtain

$$A_1(\omega)=\frac{1}{LL_0}\,H_0(\omega)H_0(\omega-\omega_p-\omega_s)+\frac{1}{LL_1}\,H_1(\omega)H_1(\omega-\omega_p-\omega_s) \tag{10}$$

and

$$A_2(\omega)=\frac{1}{LL_0}\,H_0(\omega)H_0(\omega+\omega_p+\omega_s)+\frac{1}{LL_1};H_1(\omega)H_1(\omega+\omega_p+\omega_s) \tag{11}$$

respectively, where it is also assumed that the related group delay difference between the lowpass and highpass subband channels is equalized.

Following (9)–(11), we can reformulate the conditions required for perfect reconstruction as

$$\begin{cases} T(\omega)=1, & \text{for}\quad 0\le\omega\le\pi \\ H_0(\omega)=0, & \text{for}\quad \omega_s\le\omega\le\pi \\ H_1(\omega)=0, & \text{for}\quad 0\le\omega\le\omega_p \\ \frac{1}{\sqrt{LL_0}}\,H_0(\omega)=\frac{1}{\sqrt{LL_1}}\,H_1(\omega_p+\omega_s-\omega) & \\ \qquad\qquad \text{for}\quad \omega_p\le\omega\le\omega_s. & \end{cases} \tag{12}$$

Equation (12) reveals that the conditions for perfect reconstruction can be met only when $H_0(z)$ and $H_1(z)$ have infinite filter length. Therefore, the design problem of the two-channel NDF banks of Fig. 1 is finding such $H_0(z)$ and $H_1(z)$ with finite filter length that the conditions listed in (12) can be approximately met in some optimal sense.

III. Optimal Design of Two-Channel NDF Banks in the L_2 Sense

A. Problem Formulation Using an Approximation Scheme

According to the conditions listed in (12), the overall error function E to be minimized in the L_2 sense can be expressed as

$$E=E_r+\alpha_1E_{1s}+\alpha_2E_{0s}+\alpha_3E_t \tag{13}$$

where E_r is the squared reconstruction error given by

$$E_r=\int_{\omega=0}^{\pi}(T(\omega)-1)^2\,d\omega. \tag{14}$$

E_{1s} and E_{0s} denote the squared stopband errors of $H_1(z)$ and $H_0(z)$, respectively. They are given by

$$E_{1s}=\int_{\omega=0}^{\omega_p}H_1^2(\omega)\,d\omega \quad\text{and}\quad E_{0s}=\int_{\omega=\omega_s}^{\pi}H_0^2(\omega)\,d\omega \tag{15}$$

where E_t denotes the squared error related to the fourth condition of (12) and is given by

$$E_t=\int_{\omega=\omega_p}^{\omega_s}\left(\frac{1}{\sqrt{LL_0}}\,H_0(\omega)-\frac{1}{\sqrt{LL_1}}\,H_1(\omega_p+\omega_s-\omega)\right)^2d\omega. \tag{16}$$

The parameters $\alpha_i, i=1,2,3,$ represent the relative weights between $E_r, E_{1s}, E_{0s},$ and E_t. From (13), we note that the overall error function E is a function of the fourth degree in the filter coefficients. Therefore, directly minimizing E leads to a highly nonlinear optimization problem. Although many well-developed nonlinear programming algorithms can be utilized to solve (13), to obtain satisfactory design results in several iterations is not an easy task.

Based on a linearization scheme, we present a method to efficiently solve the design problem of (13). During the optimization process for finding the optimal filter coefficients, let $h_0^l(n)$ and $h_1^l(n)$ be the filter coefficients of $H_0(z)$ and $H_1(z)$, respectively, at the lth iteration. An approximation for $T(\omega)$ is defined as

$$\hat{T}(\omega)=\frac{1}{LL_0}\,H_0^l(\omega)H_0(\omega)+\frac{1}{LL_1}\,H_1^l(\omega)H_1(\omega) \tag{17}$$

where $H_0^l(\omega)$ and $H_1^l(\omega)$ denote the magnitude responses corresponding to $h_0^l(n)$ and $h_1^l(n)$, respectively. Accordingly, we have the following approximation for the overall error function:

$$\hat{E}=\int_{\omega=0}^{\pi}(\hat{T}(\omega)-1)^2\,d\omega+\alpha_1E_{1s}+\alpha_2E_{0s}+\alpha_3E_t. \tag{18}$$

Note that $\hat{E}$ of (18) is a quadratic function of the filter coefficients if $H_0^l(\omega)$ and $H_1^l(\omega)$ are two known functions of ω.

B. The Proposed Design Method

Let $\{\omega_1 = 0, \omega_2, \cdots, \omega_I = \omega_p, \cdots, \omega_J = \omega_s, \cdots, \omega_K = \pi\}$ be a grid of equidistant frequencies distributed in the range of $\omega = 0$ to $\omega = \pi$ for evaluating the magnitude response of the NDF bank and the related error functions defined as above. Assume that $H_0(z)$ is a case 2 LP FIR filter. We define the following matrices. $\boldsymbol{U}_0$ is a $K \times (N_0/2)$ matrix with the (i,j)th element given by

$$u_0(i,j) = 2\cos\left\{\left(\frac{N_0+1}{2} - j\right)\omega_i\right\} \quad 1 \le i \le K, \quad 1 \le j \le \frac{N_0}{2}. \tag{19}$$

$\boldsymbol{U}_1$ is a $K \times (N_1/2)$ matrix with the (i,j)th element given by

$$u_1(i,j) = 2\sin\left\{\left(\frac{N_1+1}{2} - j\right)\omega_i\right\} \quad 1 \le i \le K, \quad 1 \le j \le \frac{N_1}{2}. \tag{20}$$

$\boldsymbol{U}_{0p}$ is an $I \times (N_0/2)$ matrix with the (i,j)th element given by

$$u_{0p}(i,j) = 2\cos\left\{\left(\frac{N_0+1}{2} - j\right)\omega_i\right\} \quad 1 \le i \le I, \quad 1 \le j \le \frac{N_0}{2}. \tag{21}$$

$\boldsymbol{U}_{1p}$ is a $(K-J+1) \times (N_1/2)$ matrix with the (i,j)th element given by

$$u_{1p}(i,j) = 2\sin\left\{\left(\frac{N_1+1}{2} - j\right)\omega_{i+J-1}\right\} \quad 1 \le i \le K-J+1, \quad 1 \le j \le \frac{N_1}{2}. \tag{22}$$

$\boldsymbol{U}_{0s}$ is a $(K-J+1) \times (N_0/2)$ matrix with the (i,j)th element given by

$$u_{0s}(i,j) = 2\cos\left\{\left(\frac{N_0+1}{2} - j\right)\omega_{i+J-1}\right\} \quad 1 \le i \le K-J+1, \quad 1 \le j \le \frac{N_0}{2}. \tag{23}$$

$\boldsymbol{U}_{1s}$ is an $I \times (N_1/2)$ matrix with the (i,j)th element given by

$$u_{1s}(i,j) = 2\sin\left\{\left(\frac{N_1+1}{2} - j\right)\omega_i\right\} \quad 1 \le i \le I, \quad 1 \le j \le \frac{N_1}{2}. \tag{24}$$

$\boldsymbol{U}_{0t}$ is a $(J-I+1) \times (N_0/2)$ matrix with the (i,j)th element given by

$$u_{0s}(i,j) = 2\cos\left\{\left(\frac{N_0+1}{2} - j\right)\omega_{i+I-1}\right\} \quad 1 \le i \le J-I+1, \quad 1 \le j \le \frac{N_0}{2}. \tag{25}$$

$\boldsymbol{U}_{1t}$ is a $(J-I+1) \times (N_1/2)$ matrix with the (i,j)th element given by

$$u_{1t}(i,j) = 2\sin\left\{\left(\frac{N_1+1}{2} - j\right)(\omega_p + \omega_s - \omega_{i+I-1})\right\} \quad 1 \le i \le J-I+1, \quad 1 \le j \le \frac{N_1}{2}. \tag{26}$$

Moreover, let $\boldsymbol{y}$ and $\boldsymbol{z}$ be two vectors containing the independent filter coefficients as

$$\boldsymbol{y} = \left[h_0(0), h_0(1), \cdots, h_0\left(\frac{N_0}{2} - 1\right)\right]^T \quad \text{and} \quad \boldsymbol{z} = \left[h_1(0), h_1(1), \cdots, h_1\left(\frac{N_1}{2} - 1\right)\right]^T \tag{27}$$

where the superscript T denotes the transpose operation.

Utilizing the above matrix notations, the overall design problem according to (18) can be approximately reformulated in matrix form as

$$\begin{aligned}\hat{\hat{E}} = &(\boldsymbol{U}_a\boldsymbol{y} + \boldsymbol{U}_b\boldsymbol{z} - \boldsymbol{1}_K)^T(\boldsymbol{U}_a\boldsymbol{y} + \boldsymbol{U}_b\boldsymbol{z} - \boldsymbol{1}_K) \\ &+ \alpha_1(\boldsymbol{U}_{1s}\boldsymbol{z})^T(\boldsymbol{U}_{1s}\boldsymbol{z}) + \alpha_2(\boldsymbol{U}_{0s}\boldsymbol{y})^T(\boldsymbol{U}_{0s}\boldsymbol{y}) \\ &+ \alpha_3(\boldsymbol{U}_{0t}\boldsymbol{y} - \boldsymbol{U}_{1t}\boldsymbol{z})^T(\boldsymbol{U}_{0t}\boldsymbol{y} - \boldsymbol{U}_{1t}\boldsymbol{z})\end{aligned} \tag{28}$$

where $\boldsymbol{U}_a = \boldsymbol{H}_0\boldsymbol{U}_0, \boldsymbol{U}_b = \boldsymbol{H}_1\boldsymbol{U}_1, \boldsymbol{H}_0 = (1/LL_0)\ \mathrm{diag}(H_0^l(\omega_1), H_0^l(\omega_2), \cdots, H_0^l(\omega_K)), \boldsymbol{H}_1 = (1/LL_1)\ \mathrm{diag}(H_1^l(\omega_1), H_1^l(\omega_2), \cdots, H_1^l(\omega_K))$, and $\boldsymbol{1}_K$ is a column vector with size K and all entries equal to one. Therefore, finding the filter coefficient vectors $\boldsymbol{y}$ and $\boldsymbol{z}$ that minimize (28) is equivalent to solving the following linear equations:

$$\begin{bmatrix}\boldsymbol{A} & \boldsymbol{B}\\ \boldsymbol{B}^T & \boldsymbol{C}\end{bmatrix}\begin{bmatrix}\boldsymbol{y}\\ \boldsymbol{z}\end{bmatrix} = \begin{bmatrix}\boldsymbol{U}_a^T\boldsymbol{1}_k\\ \boldsymbol{U}_b^T\boldsymbol{1}_k\end{bmatrix} \quad \text{or} \quad \begin{bmatrix}\boldsymbol{y}\\ \boldsymbol{z}\end{bmatrix} = \boldsymbol{R}_0^{-1}\begin{bmatrix}\boldsymbol{U}_a^T\boldsymbol{1}_K\\ \boldsymbol{U}_b^T\boldsymbol{1}_K\end{bmatrix} \tag{29}$$

where

$$\begin{aligned}\boldsymbol{R}_0 &= \begin{bmatrix}\boldsymbol{A} & \boldsymbol{B}\\ \boldsymbol{B}^T & \boldsymbol{C}\end{bmatrix}\\ \boldsymbol{A} &= \boldsymbol{U}_a^T\boldsymbol{U}_a + \alpha_2\boldsymbol{U}_{0s}^T\boldsymbol{U}_{0s} + \alpha_3\boldsymbol{U}_{0t}^T\boldsymbol{U}_{0t}\\ \boldsymbol{B} &= \boldsymbol{U}_a^T\boldsymbol{U}_b - \alpha_3\boldsymbol{U}_{0t}^T\boldsymbol{U}_{1t}\\ \boldsymbol{C} &= \boldsymbol{U}_b^T\boldsymbol{U}_b + \alpha_1\boldsymbol{U}_{1s}^T\boldsymbol{U}_{1s} + \alpha_3\boldsymbol{U}_{1t}^T\boldsymbol{U}_{1t}.\end{aligned} \tag{30}$$

After obtaining the coefficient vectors $\boldsymbol{y}$ and $\boldsymbol{z}$, we update the coefficients of $H_0(z)$ and $H_1(z)$ at the $(l+1)$th iteration as

$$\begin{aligned}h_0^{l+1}(n) &= 0.5(h_0^l(n) + h_0(n)) \quad \text{for} \quad n = 0, 1, \cdots, \frac{N_0}{2} - 1\\ h_1^{l+1}(n) &= 0.5(h_1^l(n) + h_1(n)) \quad \text{for} \quad n = 0, 1, \cdots, \frac{N_1}{2} - 1.\end{aligned} \tag{31}$$

Our simulations show that the overall error function $\hat{\hat{E}}$ given by (28) decreases as the iteration process proceeds when using (31) to update the designed filter coefficients. It is appropriate to terminate the design process if the following criterion is satisfied:

$$\left|\frac{E^l - E^{l+1}}{E^l}\right| \le \epsilon \tag{32}$$

where E^l denotes the value of E at the lth iteration with filter coefficients $h_0^l(n)$ and $h_1^l(n)$. ϵ is a preset small positive real number. To initiate the design process, it is appropriate to set the initial filter coefficients $h_0^0(n)$ and $h_1^0(n)$ to the least-squares results, which optimally approximate the desired magnitude specifications shown in Fig. 2. That is, the initial filter coefficient vectors $\boldsymbol{y}^0$ and $\boldsymbol{z}^0$ are given by the closed-form solutions

$$\begin{aligned}\boldsymbol{y}^0 &= \sqrt{LL_0}(\boldsymbol{U}_{0p}^T\boldsymbol{U}_{0p} + \alpha_2\boldsymbol{U}_{0s}^T\boldsymbol{U}_{0s})^{-1}(\boldsymbol{U}_{0p}^T\mathbf{1}_I)\\ \boldsymbol{z}^0 &= \sqrt{LL_1}(\boldsymbol{U}_{1p}^T\boldsymbol{U}_{1p} + \alpha_1\boldsymbol{U}_{1s}^T\boldsymbol{U}_{1s})^{-1}(\boldsymbol{U}_{1p}^T\mathbf{1}_{K-J+1}).\end{aligned} \tag{33}$$

Here, we summarize the proposed method by presenting the following design procedure.

Design Procedure 1:

Step 1) Specify the design parameters: filter lengths N_0 and N_1, the bandedge frequencies ω_p and ω_s, the relative weights $\alpha_1, \alpha_2,$ and α_3, and the value of ϵ. Set the iteration number $l = 0$.

Step 2) Obtain the initial filter coefficient vectors $\boldsymbol{y}^0$ and $\boldsymbol{z}^0$ using (33). Then, compute the corresponding initial magnitude responses $H_0^0(\omega)$ and $H_1^0(\omega)$.

Step 3) Compute the coefficients $h_0(n)$ and $h_1(n)$ by solving the linear equations of (29).

Step 4) Compute the resulting filter coefficients at the $(l+1)$th iteration using the updating formulas given by (31).

Step 5) Compute the corresponding overall error function E^{l+1}. If the stopping criterion shown by (32) is satisfied, terminate the design process. Otherwise, set $l = l + 1$, and go to Step 3.

IV. Optimal Design of Two-Channel NDF Banks with L_∞ Reconstruction Response and L_2 Stopband Response

A. Problem Formulation Using a WLS Algorithm and an Approximation Scheme

In order to achieve the design with equiripple reconstruction error, the associated overall error function E can be formulated based on the WLS algorithm of [8]

$$E = \int_{\omega=0}^{\pi} W(\omega)(T(\omega)-1)^2\, d\omega + \alpha_1 E_{1s} + \alpha_2 E_{0s} + \alpha_3 E_t \tag{34}$$

where $W(\omega)$ denotes the frequency response weighting function. Equation (34) reveals that the design problem becomes the minimization of the reconstruction error in the weighted least-squares (WLS) sense and the stopband error in the least-squares sense. To obtain the design result with equiripple reconstruction error, the weighting function $W(\omega)$ must be appropriately chosen. The WLS algorithm presented in [8] provides a systematic approach for finding the suitable $W(\omega)$.

Utilizing the result of (17) obtained from approximation, we reformulate the overall error function of (34) as

$$\tilde{E} = \int_{\omega=0}^{\pi} W(\omega)(\hat{T}(\omega)-1)^2\, d\omega + \alpha_1 E_{1s} + \alpha_2 E_{0s} + \alpha_3 E_t. \tag{35}$$

B. The Proposed Design Method

Using the matrices given by (19)–(27), we can express (35) in matrix form as

$$\begin{aligned}\tilde{\tilde{E}} = &(\boldsymbol{U}_a\boldsymbol{y} + \boldsymbol{U}_b\boldsymbol{z} - \mathbf{1}_K)^T\boldsymbol{W}(\boldsymbol{U}_a\boldsymbol{y} + \boldsymbol{U}_b\boldsymbol{z} - \mathbf{1}_K)\\ &+ \alpha_1(\boldsymbol{U}_{1s}\boldsymbol{z})^T(\boldsymbol{U}_{1s}\boldsymbol{z}) + \alpha_2(\boldsymbol{U}_{0s}\boldsymbol{y})^T(\boldsymbol{U}_{0s}\boldsymbol{y})\\ &+ \alpha_3(\boldsymbol{U}_{0t}\boldsymbol{y} - \boldsymbol{U}_{1t}\boldsymbol{z})^T(\boldsymbol{U}_{0t}\boldsymbol{y} - \boldsymbol{U}_{1t}\boldsymbol{z})\end{aligned} \tag{36}$$

where $\boldsymbol{W}$ represents a diagonal matrix containing the weight values of $W(\omega)$ evaluated on the dense grid $\{\omega_1 = 0, \omega_2, \cdots, \omega_I = \omega_p, \cdots, \omega_J = \omega_s, \cdots, \omega_K = \pi\}$, i.e., $\boldsymbol{W} = \mathrm{diag}\{W(\omega_1), W(\omega_2), \cdots, W(\omega_K)\}$. Again, (36) is a quadratic function of $\boldsymbol{y}$ and $\boldsymbol{z}$. Hence, finding the filter coefficients that minimize (36) is equivalent to solving the following linear equations:

$$\begin{aligned}\begin{bmatrix}\tilde{\boldsymbol{A}} & \tilde{\boldsymbol{B}}\\ \tilde{\boldsymbol{B}}^T & \tilde{\boldsymbol{C}}\end{bmatrix}\begin{bmatrix}\boldsymbol{y}\\ \boldsymbol{z}\end{bmatrix} &= \begin{bmatrix}\boldsymbol{U}_a^T\boldsymbol{W}\mathbf{1}_K\\ \boldsymbol{U}_b^T\boldsymbol{W}\mathbf{1}_K\end{bmatrix} \quad \text{or} \quad \begin{bmatrix}\boldsymbol{y}\\ \boldsymbol{z}\end{bmatrix}\\ &= \boldsymbol{R}_0^{-1}\begin{bmatrix}\boldsymbol{U}_a^T\boldsymbol{W}\mathbf{1}_K\\ \boldsymbol{U}_b^T\boldsymbol{W}\mathbf{1}_K\end{bmatrix}\end{aligned} \tag{37}$$

where

$$\begin{aligned}\boldsymbol{R}_0 &= \begin{bmatrix}\tilde{\boldsymbol{A}} & \tilde{\boldsymbol{B}}\\ \tilde{\boldsymbol{B}}^T & \tilde{\boldsymbol{C}}\end{bmatrix}\\ \tilde{\boldsymbol{A}} &= \boldsymbol{U}_a^T\boldsymbol{W}\boldsymbol{U}_a + \alpha_2\boldsymbol{U}_{0s}^T\boldsymbol{U}_{0s} + \alpha_3\boldsymbol{U}_{0t}^T\boldsymbol{U}_{0t}\\ \tilde{\boldsymbol{B}} &= \boldsymbol{U}_a^T\boldsymbol{W}\boldsymbol{U}_b - \alpha_3\boldsymbol{U}_{0t}^T\boldsymbol{U}_{1t}\\ \tilde{\boldsymbol{C}} &= \boldsymbol{U}_b^T\boldsymbol{W}\boldsymbol{U}_b + \alpha_1\boldsymbol{U}_{1s}^T\boldsymbol{U}_{1s} + \alpha_3\boldsymbol{U}_{1t}^T\boldsymbol{U}_{1t}.\end{aligned} \tag{38}$$

After obtaining the coefficient vectors $\boldsymbol{y}$ and $\boldsymbol{z}$, we update the filter coefficients by using the updating formulas of (31) at the $(l+1)$th iteration. To achieve the design with equiripple reconstruction error, the WLS algorithm presented in [8] is utilized for adjusting $W(\omega)$ during the design process. The following procedure summarizes the proposed design method.

Design Procedure 2:

Step 1) Specify the design parameters: filter lengths N_0 and N_1, the bandedge frequencies ω_p and ω_s, the relative weights $\alpha_1, \alpha_2,$ and α_3, and the value of ϵ. Set the iteration number $l = 0$.

Step 2) Obtain the initial filter coefficient vectors $\boldsymbol{y}^0$ and $\boldsymbol{z}^0$ using (33). Then, compute the corresponding initial magnitude responses $H_0^0(\omega)$ and $H_1^0(\omega)$. Moreover, set the initial weighting matrix $\boldsymbol{W}$ to the identity matrix.

Step 3) Compute the coefficients $h_0(n)$ and $h_1(n)$ by solving the linear equations of (37).

Step 4) Compute the resulting filter coefficients at the $(l+1)$th iteration using the updating formulas given by (31).

Step 5) Compute the corresponding overall error function E^{l+1}. If the stopping criterion shown by (32) is satisfied, go to Step 6). Otherwise, set $l = l + 1$, and go to Step 3).

Step 6) Find Max(V) and Min(V), which are the maximum and minimum of $|T(\omega) - 1|$ over the extremal frequencies, respectively, according to the envelope

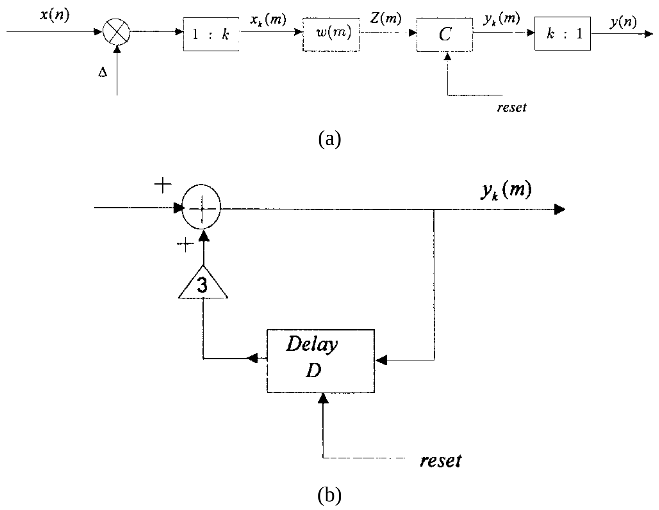


Fig. 3. Proposed new filter structure. (b) Function block C.

construction of [8]. If

$$\frac{\mathrm{Max}(V) - \mathrm{Min}(V)}{\mathrm{Max}(V)} \leq \kappa \tag{39}$$

is satisfied, then terminate the design process. Otherwise, go to Step 7).

Step 7) Adjust $W(\omega)$ according to the WLS algorithm of [8] as follows.

7.1) Construct the envelop function $Q(\omega)$ of $|T(\omega)-1|$.

7.2) Compute the updating function $v(\omega)$ according to

$$v(\omega) = \frac{K\{Q(\omega)\}^{1.5}}{\sum_{i=1}^{K}[W(\omega_i)\{Q(\omega_i)\}^{1.5}]}. \tag{40}$$

7.3) Update the frequency response weighting function $W(\omega)$ by performing $W(\omega)v(\omega)$.

Then, we update the corresponding weighting matrix $\boldsymbol{W}$, set $l = l + 1$, and go to Step 3).

V. Optimal Design of Two-Channel NDF Banks With Coefficients −1, 0, and +1

A. The Proposed Filter Structure

Consider the new filter structure shown by Fig. 3 for realizing the LP analysis and synthesis filters. The input sequence $x(n)$ is first multiplied by Δ to adjust the range of $x(n)$. The purpose of oversampling by k is to keep the error due to the operation similar to the delta modulation, as shown by the function block C at an acceptable level. The impulse response $w(m)$ has a finite number of samples with values restricted to -1, 0, and $+1$ only. The reset terminal receives a signal to clear the contents of the delay element of C when $m = nk+1$.

Let the impulse response of the filter structure be $h_d(n)$ with length N. $w(m)$ will have length $N_k = Nk + 1$. Setting $x(n)$ to an impulse sequence, we obtain

$$y_k(0) = \Delta w(0), y_k(m) = \Delta w(m) \quad \text{for} \quad m = nk + 1, n = 1, 2, \cdots, N \tag{41}$$

and

$$y_k(m) = 3y_k(m-1) + \Delta w(m) \quad for \quad m \neq nk + 1, n = 1, 2, \cdots, N. \tag{42}$$

We note that imposing the reset operation in C is equivalent to putting a constraint for $y_k(m)$, as shown by (42). Moreover, (41) and (42) can be rewritten as

$$y_k(0) = \Delta w(0), y_k(m) = \Delta \sum_{i=k\langle(m-1)/k3\rangle+1}^{m} w(i)3^{m-i} \quad \text{for} \quad m = 1, 2, \cdots, N_k - 1 \tag{43}$$

where $\langle x \rangle$ denotes the largest integer not greater than x. The sum in (43) is obtained due to the fact that in C, only the samples after the last reset have to be considered. Accordingly, the relationship between $y(n)$, which is equal to the impulse response $h_d(n)$ and $y_k(m)$, is given by

$$h_d(0) = y(0) = y_k(0) = \Delta w(0)$$
$$h_d(n) = y(n) = y_k(nk) = \Delta \sum_{m=(n-1)k+1}^{nk} w(m)3^{nk-m} \quad \text{for} \quad n = 1, 2, \cdots, N - 1. \tag{44}$$

Let $w(m) = 0$ for $m < 0$; thus, we can rewrite (44) as

$$h_d(n) = y(n) = y_k(nk) = \Delta \sum_{m=(n-1)k+1}^{nk} w(m)3^{nk-m} \quad \text{for} \quad n = 0, 1, \cdots, N - 1. \tag{45}$$

Since $w(m)$ has values restricted to -1, 0, and $+1$ only for $m = 0, 1, \cdots, N_k - 1$, we note from (45) that $h_d(n)$ satisfies the following inequalities:

$$-\frac{3^k - 1}{2} \leq \frac{h_d(n)}{\Delta} \leq \frac{3^k - 1}{2}. \tag{46}$$

For any integer P within the range of $[-(1/2)(3^k - 1), (1/2)(3^k - 1)]$, it is easy to show that there exists a unique set of $w((n-1)k+1), w((n-1)k+2), \cdots, w(nk) \in \{-1, 0, 1\}$ such that the integer P can be expressed as

$$P = \sum_{m=(n-1)k+1}^{nk} w(m)3^{nk-m}. \tag{47}$$

B. Discrete Optimization Procedure

The procedure is basically a modification of that given in [10] and described as follows.

1) Constrained Optimization: Let $\boldsymbol{h} = [h(0), h(1), \cdots, h((N_0 + N_1)/2 - 1)]^T = [\boldsymbol{y}^T; \boldsymbol{z}^T]^T = [h_0(0), h_0(1), \cdots, h_0((N_0/2) - 1), h_1(0), h_1(1), \cdots, h_1((N_1/2) - 1)]^T$. Assume that $\boldsymbol{h}^{(i)}$ represents the continuous coefficient vector excluding the ith coefficient $h(i)$, which is fixed at a discrete value, i.e., $\boldsymbol{h}^{(i)}$ is a $((N_0 + N_1)/2 - 1) \times 1$ vector with the jth entry

$h^{(i)}(j)$ as

$$\begin{aligned}\boldsymbol{h}^{(i)} &= \Big[h(0), h(1), \cdots, h(i-1), h(i+1) \\ &\quad \cdots, h\left(\frac{N_0+N_1}{2}-1\right)\Big]^T \\ &= \Big[h^{(i)}(0), h^{(i)}(1), \cdots, h^{(i)}(j) \\ &\quad \cdots, h^{(i)}\left(\frac{N_0+N_1}{2}-2\right)\Big]^T. \end{aligned} \tag{48}$$

Let $\boldsymbol{h}_1^{(i)}$ be the vector of (48) that minimizes the overall error function E of (18) when $h(i)$ is fixed at a discrete value $h_d(i)$. Let the difference between the optimal continuous value $h_c(i)$ and the discrete value $h_d(i)$ of $h(i)$ be $\Delta h(i) = h_d(i) - h_c(i)$. Based on the LMS algorithm of [10], we can obtain $\boldsymbol{h}_1^{(i)} = \boldsymbol{h}_c^{(i)} + (\boldsymbol{G}^{(i)})\Delta h(i) = [h_1^{(i)}(0), h_1^{(i)}(1), \cdots, h_1^{(i)}((N_0+N_1)/2-2)]^T$, where $\boldsymbol{h}_c^{(i)}$ denotes the coefficient vector $\boldsymbol{h}_c = [h_c(0), h_c(1), \cdots, h_c((N_0+N_1)/2-1)]^T$, which is obtained from the *Design Procedure 1* or *Design Procedure 2,* with $h_c(i)$ omitted. $\boldsymbol{G}^{(i)}$ is an $((N_0+N_1)/2-1)\times 1$ column vector given by $\boldsymbol{G}^{(i)} = \boldsymbol{R}_1^{-1}\boldsymbol{S}$, where $\boldsymbol{R}_1$ and $\boldsymbol{S}$ are obtained from the submatrices of $\boldsymbol{R}_0$ of (30) or (38). Let $\boldsymbol{R}_0$ be partitioned as

$$\boldsymbol{R}_0 = \begin{bmatrix} \boldsymbol{A}_1 & B_1 & \boldsymbol{A}_2 \\ C_1^T & D & C_2^T \\ \boldsymbol{A}_3 & B_2 & \boldsymbol{A}_4 \end{bmatrix} \tag{49}$$

where $[C_1^T\ D\ C_2^T]$ and $[B_1^T\ D\ B_2^T]^T$ are the ith row and the ith column of $\boldsymbol{R}_0$, respectively. Then

$$\boldsymbol{R}_1 = \begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{A}_2 \\ \boldsymbol{A}_3 & \boldsymbol{A}_4 \end{bmatrix} \quad \text{and} \quad \boldsymbol{S} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}. \tag{50}$$

Next, for fixed j, we put $h_1^{(i)}(j) = h_d^{(i)}(j)$. Let $\boldsymbol{h}_2^{(i,j)}$ be the coefficient vector that minimizes (18) when $h_c(i)$ and $h_1^{(i)}(j)$ are fixed at discrete values. Assume that the difference between the optimal continuous value $h_{1c}^{(i)}(j)$ and the discrete value $h_d^{(i)}(j)$ of the coefficient $h_1^{(i)}(j)$ is $\Delta h^{(i)}(j) = h_d^{(i)}(j) - h_{1c}^{(i)}(j)$. Similarly, we can obtain $\boldsymbol{h}_2^{(i,j)} = \boldsymbol{h}_1^{(i,j)} + \boldsymbol{G}^{(i,j)}\Delta h^{(i)}(j)$, where $\boldsymbol{h}_1^{(i,j)}$ represents the vector $\boldsymbol{h}_1^{(i)}$ with the coefficient $h_1^{(i)}(j)$ omitted. We obtain $\boldsymbol{G}^{(i,j)}$ by following the same procedure as above with $\boldsymbol{R}_1$ instead of $\boldsymbol{R}_0$. This process is repeated until all of the filter coefficients are chosen and fixed at discrete values. The required weighting function $W(\omega)$ for finding the WLS design solution is adjusted based on the obtained discrete filter coefficients using the systematic adjusting approach of [8].

2) Tree Search Algorithm: The algorithm used for performing the tree search is basically the same as that presented in [10]. However, some modifications are made to enhance the capability of the proposed design method. After obtaining the optimal continuous coefficient design from (29) or (37), we choose a coefficient $h_c(i)$ and fix it at M discrete values in the vicinity of $h_c(i)$. An optimization problem must be solved for each of the discrete values of $h_c(i)$ to find the corresponding $\boldsymbol{h}_1^{(i)}$. Based on the $\boldsymbol{h}_1^{(i)}$, M further optimization problems are produced when the second coefficient $h_1^{(i)}(j)$ is chosen and fixed at M discrete values. Therefore, M^2 optimization problems must be solved when $h_c(i)$ and $h_1^{(i)}(j)$ take on discrete values. To keep the required computation load manageable, we select only M of the M^2 optimization problems for further discretizing the remaining coefficients. The criterion for selecting the M optimization problems is to choose the M problems that provide the M smallest values of the overall error function or the weighted peak reconstruction error. Next, each of the M selected problems produces other M optimization problems when the third coefficient is chosen to take on M discrete values. The search process continues until all of the filter coefficients are discretized.

3) Filter Coefficient Selection: We present a criterion for dealing with the discrete coefficient constraint of (46). In general, the grid density decreases as the value of discrete coefficients increases. Thus, the effect of discretizing the small coefficients is more easily compensated by the remaining reoptimized coefficients. Hence, we discretize the coefficient with the largest relative sensitivity first at each tree stage. The relative sensitivity of a continuous coefficient $h(i)$ is defined as

$$\text{Relative Sensitivity of } h(i) = \text{Max}|g^{(i)}(j)| \tag{51}$$

where $g^{(i)}(j)$ is the jth element of the vector $\boldsymbol{G}^{(i)}$.

C. The Design Procedures

The following design procedure summarizes the optimal design with coefficients -1, 0, and $+1$ under the criterion of L_2 reconstruction response and L_2 filter stopband response.

Design Procedure 3:

- Step 1) Use the design method presented in Section III to find the optimal continuous coefficient vectors $\boldsymbol{y}$ and $\boldsymbol{z}$.
- Step 2) Choose four powers-of-two values in the vicinity of the maximum of $|h_c(n)|/((1/2)(3^k-1))$ as the values for the step size Δ.
- Step 3) For a given Δ, perform the discrete optimization procedure described in Section V-B to find the corresponding discrete coefficients $h_d(n), n = 0, 1, 2, \cdots, (N_1+N_2)/2-1$.
- Step 4) Compute the error function $\hat{\hat{E}}$ of (28) corresponding to $h_d(n)$, and adjust the matrices $\boldsymbol{U}_a$ and $\boldsymbol{U}_b$. Then, recompute the new optimal continuous coefficient vector $\boldsymbol{h}_c$ from (29).
- Step 5) Repeat Steps 3) and 4) until the overall error function $\hat{\hat{E}}$ cannot be further reduced.
- Step 6) Select the Δ that makes the overall error function smallest among the four powers-of-two values for Δ. Find the corresponding $w(m)$ by utilizing the relationship given by (45).

The optimal design with coefficients -1, 0, and $+1$ under the criterion of L_∞ reconstruction response and L_2 filter stopband response is summarized as the following design procedure:

Design Procedure 4:

Step 1) Use the design method presented in Section IV to find the optimal continuous coefficient vectors $\boldsymbol{y}$ and $\boldsymbol{z}$.

Step 2) Choose four powers-of-two values in the vicinity of the maximum of $|h_c(n)|/((1/2)(3^k-1))$ as the values for the step size Δ.

Step 3) For a given Δ, perform the discrete optimization procedure described in Section V-B to find the corresponding discrete coefficients $h_d(n), n = 0, 1, 2, \cdots, (N_1+N_2)/2-1$.

Step 4) Compute the reconstruction error function $|T(\omega_i)-1|, i = 1, 2, \cdots, K$ corresponding to $h_d(n)$, and adjust the matrices $\boldsymbol{U}_a, \boldsymbol{U}_b$, and $\boldsymbol{W}$ using the WLS algorithm of [8]. Then, recompute the new optimal continuous coefficient vector $\boldsymbol{h}_c$ from (37).

Step 5) Repeat Steps 3) and 4) until the peak reconstruction error cannot be further reduced.

Step 6) Select the Δ that makes the overall error function smallest among the four powers-of-two values for Δ. Find the corresponding $w(m)$ by utilizing the relationship given by (45).

VI. Simulation Examples

In this section, we present several simulation examples of designing two-channel NDF banks with LP FIR filters having coefficients of -1, 0, and $+1$ only for illustration. These designs are performed on a personal computer with Pentium CPU using MATLAB programming language. For all design examples, the number K of frequency grid points used is set to 8 $\times \max(N_0, N_1)$. Moreover, the ripple ratio r for the design problem shown by (24) is set to 0.25. The values of κ and ϵ used for terminating the design process are set to 10^{-6} and 10^{-3}, respectively. The number M for discretizing filter coefficients and the k for oversampling are set to 3 and 10, respectively. The performance for each of the designed filter banks is evaluated in terms of the peak reconstruction error (PRE) in decibels, the normalized peak stopband ripple (NPSR) in decibels, and the stopband ripple energies (SRE) of the designed $H_0(z)$ and $H_1(z)$. They are defined as

$$\begin{aligned}
\text{PRE} &= \max\{|20\log_{10} T(\omega)|\} \quad \text{for} \quad \omega \in [0,\pi] \\
\text{NPSR}_0 &= \max\left\{20\log_{10}\frac{|H_0(\omega)|}{\sqrt{LL_0}}\right\} \quad \text{for } \omega \in [\omega_s,\pi] \\
\text{SRE}_0 &= \int_{\omega_s}^{\pi} H_0^2(\omega)\,d\omega \\
\text{NPSR}_1 &= \max\left\{20\log_{10}\frac{|H_1(\omega)|}{\sqrt{LL_1}}\right\} \\
&\qquad \text{for} \quad \omega\in[0,\omega_p], \quad \text{and} \\
\text{SRE}_1 &= \int_{0}^{\omega_p} H_1^2(\omega)\,d\omega. \qquad (52)
\end{aligned}$$

Example 1: We use the design specifications shown by Table I and *Design Procedure 3.* Table II shows the significant design results for both of continuous and discrete coefficients. The resulting step size Δ and discrete coefficients of the designed analysis filters are listed in Table III. Fig. 4 plots the corresponding magnitude responses in decibels of $H_0(\omega)/\sqrt{LL_0}$ and $H_1(\omega)/\sqrt{LL_1}$ and the overall magnitude response $T(\omega)$ in decibels of the designed NDF banks with -1, 0, and $+1$ coefficients (dashed-line) and with the optimal continuous coefficients (solid-line), respectively. We note that the satisfactory performances of the designed NDF banks with the optimal continuous coefficients and coefficients of -1, 0, and $+1$ only are very close.

TABLE I
Design Specifications for Examples 1 and 2

	Case 1
N_0	32
N_1	32
ω_p	0.3π
ω_s	0.5π
L_0	2
L_1	3

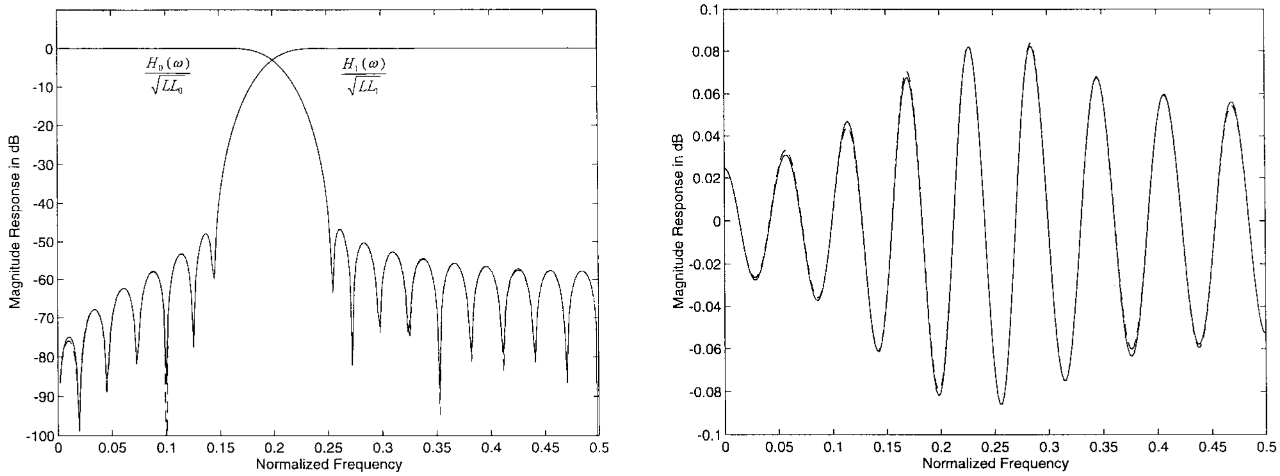
TABLE II
Significant Design Results for Example 1

	Continuous Design	Design with Coefficient -1 , 0 , +1
PRE(dB)	0.08578966114005	0.08576981765324
NPSR(dB) of $H_0(\omega)$	-43.02033400486856	-42.97317108014493
NPSR(dB) of $H_1(\omega)$	-40.73807913981903	-40.69279544025814
SRE of $H_0(\omega)$	0.00005155677951	0.00005157294680
SRE of $H_1(\omega)$	0.00004290781008	0.00004331931948

Example 2: The design specifications shown by Table I and *Design Procedure 4* are used for this design. Table IV lists the significant design results for both of continuous and discrete coefficients. Table V lists the resulting step size Δ and discrete coefficients of the designed analysis filters. Fig. 5 depicts the corresponding magnitude responses in decibels of $H_0(\omega)/\sqrt{LL_0}$ and $H_1(\omega)/\sqrt{LL_1}$ and $T(\omega)$, in decibels, of the designed NDF banks with -1, 0, and $+1$ coefficients (dashed-line) and with the optimal continuous coefficients (solid-line), respectively. Again, we observe that the designed NDF banks show very close satisfactory performances.

VII. Conclusion

This paper has presented a technique for the optimal design of two-channel nonuniform-division filter (NDF) banks with linear-phase FIR filters having -1, 0, and $+1$ coefficients only. First, we formulate the design problem with continuous coefficients for each of two optimal criteria, namely, least-squares reconstruction response and filter stopband response and equiripple reconstruction response and least-squares filter stopband response. The WLS algorithm of [8] has been utilized to achieve the design of equiripple reconstruction



(a) (b)

Fig. 4. Magnitude responses for Example 1. (a) $H_0(\omega)/\sqrt{LL_0}$ and $H_1(\omega)/\sqrt{LL_1}$. (b) $T(\omega)$.

TABLE III
(a) ANALYSIS FILTER COEFFICIENTS FOR THE CONTINUOUS DESIGN OF EXAMPLE 1. (b) ANALYSIS FILTER COEFFICIENTS FOR THE DESIGN WITH COEFFICIENTS -1, 0, $+1$ OF EXAMPLE 1

n	$h_0(n)$	$h_1(n)$	n
0	0.00788452000896	0.01092797492227	31
1	0.00487351524964	-0.00123583243954	30
2	-0.01361844584460	-0.02052606414568	29
3	-0.01858308019068	-0.01818025282684	28
4	0.01331293949847	0.01861362930646	27
5	0.04093439317290	0.05056808192357	26
6	0.00222270910263	0.01806467613523	25
7	-0.06696499994361	-0.06683018119809	24
8	-0.04438256016257	-0.09461198585060	23
9	0.08258093340173	0.01850229686699	22
10	0.12448049556955	0.17456381755339	21
11	-0.06113021074883	0.14071561444490	20
12	-0.26295149544782	-0.16933679754048	19
13	-0.06501883862383	-0.46492907536118	18
14	0.60424872641439	-0.17065258141526	17
15	1.23550604938688	2.03902241448382	16

(a)

Δ= 1.220703125000000e-004

n	$h_{0d}(n)$	$h_{1d}(n)$	n
0	64Δ	89Δ	31
1	40Δ	-10Δ	30
2	-111Δ	-168Δ	29
3	-152Δ	-149Δ	28
4	109Δ	153Δ	27
5	335Δ	414Δ	26
6	18Δ	147Δ	25
7	-549Δ	-547Δ	24
8	-364Δ	-775Δ	23
9	676Δ	152Δ	22
10	1020Δ	1430Δ	21
11	-500Δ	1152Δ	20
12	-2154Δ	-1387Δ	19
13	-533Δ	-3809Δ	18
14	4950Δ	-1397Δ	17
15	10122Δ	16704Δ	16

(b)

TABLE IV
SIGNIFICANT DESIGN RESULTS FOR EXAMPLE 2

	Continuous Design	Design with Coefficient -1,0,+1
PRE(dB)	0.07329003138699	0.08203811034700
NPSR(dB) of $H_0(\omega)$	-43.91400068048565	-43.98217256385478
NPSR(dB) of $H_1(\omega)$	-42.76780122845712	-42.83190428181179
SRE of $H_0(\omega)$	0.00005105358859	0.00005115848108
SRE of $H_1(\omega)$	0.00006471354472	0.00006410765891

behavior. In conjunction with a new filter structure for realizing the analysis filters, an efficient method to obtain an optimal design with coefficients restricted to -1, 0, and $+1$ only has been presented. The effectiveness of the proposed technique has been demonstrated by several design examples.

APPENDIX

Here, we derive the input/output relationship given by (1) of the NDF bank shown in Fig. 1. From the architecture of Fig. 1, we have

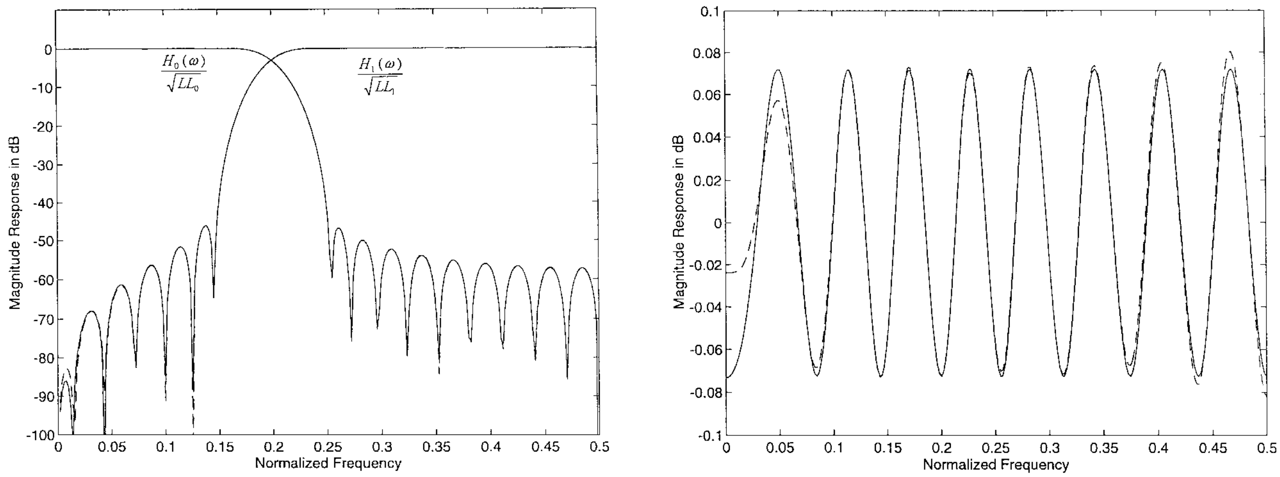
$$X_0(z) = H_0(z)X(z) \quad \text{and} \quad X_1(z) = H_1(z)X(z). \tag{53}$$

After interpolating and filtering, we obtain

$$\begin{aligned} S_0(z) &= H_0(z^{L_0})X(z^{L_0})B_0(z) \quad \text{and} \\ S_1(z) &= H_1(z^{L_1}e^{-j\pi})X(z^{L_1}e^{-j\pi})B_1(z). \end{aligned} \tag{54}$$

Performing decimation yields

$$\begin{aligned} U_0(z) &= \frac{1}{L}\sum_{i=0}^{L-1} S_0(z^{(1/L)}W_L^i) \\ &= \frac{1}{L}\sum_{i=0}^{L-1} H_0(z^{(L_0/L)}W_L^{iL_0}) \\ &\quad \cdot X(z^{(L_0/L)}W_L^{iL_0})B_0(z^{(1/L)}W_L^i) \\ U_1(z) &= \frac{1}{L}\sum_{i=0}^{L-1} S_1(z^{(1/L)}W_L^i) \\ &= \frac{1}{L}\sum_{i=0}^{L-1} H_1(z^{(L_1/L)}W_L^{iL_1}e^{-j\pi}) \\ &\quad \cdot X(z^{(L_1/L)}W_L^{iL_1}e^{-j\pi})B_1(z^{(1/L)}W_L^i). \end{aligned} \tag{55}$$



(a) (b)

Fig. 5. Magnitude responses for Example 2. (a) $H_0(\omega)/\sqrt{LL_0}$ and $H_1(\omega)/\sqrt{LL_1}$. (b) $T(\omega)$.

TABLE V

(a) ANALYSIS FILTER COEFFICIENTS FOR THE CONTINUOUS DESIGN OF EXAMPLE 2. (b) ANALYSIS FILTER COEFFICIENTS FOR THE DESIGN WITH COEFFICIENTS -1, 0, $+1$ OF EXAMPLE 2

n	h0(n)	h1(n)	n
0	1.23520953211558	2.03867154550689	31
1	0.60424460430862	-0.17131285848437	30
2	-0.06509087697145	-0.46535604712587	29
3	-0.26345421639602	-0.16904673815428	28
4	-0.06167384455702	0.14139059424412	27
5	0.12440476040468	0.17503451707952	26
6	0.08262897954677	0.01820755584870	25
7	-0.04492439075652	-0.09528178793365	24
8	-0.06799251942106	-0.06719154931303	23
9	0.00144744327705	0.01840095516399	22
10	0.04049445181084	0.05098552028284	21
11	0.01245492969932	0.01872834676818	20
12	-0.02017341509891	-0.01814012720125	19
13	-0.01518199532098	-0.02032904023171	18
14	0.00416460988121	-0.00254020344555	17
15	0.00792414161275	0.01140952312328	16

(a)

Δ = 1.220703125000000e-004

n	h0d(n)	h1d(n)	n
0	10121Δ	16702Δ	31
1	4952Δ	-1404Δ	30
2	-531Δ	-3813Δ	29
3	-2156Δ	-1384Δ	28
4	-503Δ	1158Δ	27
5	1022Δ	1434Δ	26
6	680Δ	149Δ	25
7	-365Δ	-781Δ	24
8	-554Δ	-550Δ	23
9	15Δ	151Δ	22
10	335Δ	417Δ	21
11	105Δ	155Δ	20
12	-163Δ	-150Δ	19
13	-123Δ	-165Δ	18
14	35Δ	-23Δ	17
15	65Δ	95Δ	16

(b)

In the synthesis system, after interpolating and filtering, we have

$$\begin{aligned}Y_0(z) =& U_0(z^L)B_0(z)\\ =& \frac{1}{L}\sum_{i=0}^{L-1} H_0(z^{L_0}W_L^{iL_0})X(z^{L_0}W_L^{iL_0})B_0(zW_L^i)B_0(z)\\ Y_1(z) =& U_1(z^L)B_1(z)\\ =& \frac{1}{L}\sum_{i=0}^{L-1} H_1(z^{L_1}W_L^{iL_1}e^{-j\pi})\\ &\cdot X(z^{L_1}W_L^{iL_1}e^{-j\pi})B_1(zW_L^i)B_1(z).\end{aligned} \tag{56}$$

Performing the decimation provides

$$\begin{aligned}V_0(z) =& \frac{1}{L_0 L}\sum_{i=0}^{L-1}\sum_{k=0}^{L_0-1} H_0(zW_{L_0}^{kL_0}W_L^{iL_0})X(zW_{L_0}^{kL_0}W_L^{iL_0})\\ &\cdot B_0(z^{1/L_0}W_{L_0}^k W_L^i)B_0(z^{1/L_0}W_{L_0}^k)\\ V_1(z) =& \frac{1}{L_1 L}\sum_{i=0}^{L-1}\sum_{k=0}^{L_1-1} H_1(zW_{L_1}^{kL_1}W_L^{iL_1}e^{-j2\pi})\\ &\cdot X(zW_{L_1}^{kL_1}W_L^{iL_1}e^{-j2\pi})B_1(z^{1/L_1}W_{L_1}^k W_L^i e^{-j\pi/L_1})\\ &\cdot B_1(z^{1/L_1}W_{L_1}^k e^{-j\pi/L_1}).\end{aligned} \tag{57}$$

Next, multiplying $v_1(n)$ by $e^{i\pi n}$, performing filtering by $F_i(z), i = 0, 1$, and taking the summation yields

$$\begin{aligned}\hat{X}(z) =& \hat{X}_0(z) + \hat{X}_1(z)\\ =& \frac{1}{LL_0}\sum_{i=0}^{L-1}\sum_{k=0}^{L_0-1} H_0(zW_L^{iL_0})X(zW_L^{iL_0})\\ &\cdot B_0(z^{1/L_0}W_{L_0}^k W_L^i)B_0(z^{1/L_0}W_{L_0}^k)F_0(z)\\ &+ \frac{1}{LL_1}\sum_{i=0}^{L-1}\sum_{k=0}^{L_1-1} H_1(zW_L^{iL_1})X(zW_L^{iL_1})\\ &\cdot B_1(z^{1/L_1}W_{L_1}^k W_L^i e^{-j\pi/L_1})\\ &\cdot B_1(z^{1/L_1}W_{L_1}^k e^{-j\pi/L_1})F_1(z).\end{aligned} \tag{58}$$

Utilizing the assumption that $B_0(\omega) = 1$, for $\omega \in [0, (\omega_s/L_0)]$ and $= 0$, for $\omega \in [(2\pi - \omega_s)/L_0, \pi]$, and $B_1(\omega) = 1$, for

$\omega \in [0, (\pi - \omega_p)/L_1]$ and $= 0$, for $\omega \in [(\pi + \omega_p)/L_1, \pi]$, respectively, and the assumption that $H_0(z)$ and $H_1(z)$ have zero stopband response, (58) becomes

$$\begin{aligned}\hat{X}(z) = &\frac{1}{LL_0}\, z^{-G_0}[X(z)H_0(z) + X(zW_L^{L_0})H_0(zW_L^{L_0}) \\ &+ X(zW_L^{-L_0})H_0(zW_L^{-L_0})]F_0(z) \\ &+ \frac{1}{LL_1}\, z^{-G_1}[X(z)H_1(z) + X(zW_L^{L_1})H_1(zW_L^{L_1}) \\ &+ X(zW_L^{-L_1})H_1(zW_L^{-L_1})]F_1(z) \qquad (59)\end{aligned}$$

where G_0 and G_1 denote the resulting group delays of the upper and lower channels, respectively. Finally, (1) can be obtained by substituting $z = e^{j\omega}$ into (59).

REFERENCES

[1] R. E. Crochiere, "Digital signal processor: Sub-band coding," *Bell Syst. Tech. J.*, vol. 60, pp. 1633–1653, 1981.

[2] M. G. Bellanger and J. L. Daguet, "TDM-FDM transmultiplexer: Digital polyphase and FFT," *IEEE Trans. Commun.*, vol. COMM-22, pp. 1199–1204, Sept. 1974.

[3] P. Vary and U. Heute, "A short-time spectrum analyzer with polyphase network and DFT," *Signal Process.*, vol. 2, pp. 55–65, 1980.

[4] J. W. Woods and S. D. O'Neil, "Subband coding of images," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-34, pp. 1278–1288, Oct. 1986.

[5] K. Nayebi, T. P. Barnwell, and M. J. T. Smith, "Nonuniform filter banks: A reconstruction and design theory," *IEEE Trans. Signal Processing*, vol. 41, pp. 1114–1127, Mar. 1993.

[6] S. Wada, "Design of nonuniform division multirate FIR filter banks," *IEEE Trans. Circuits Syst. II*, vol. 42, pp. 115–121, Feb. 1995.

[7] J.-H. Lee and S.-C. Huang, "Design of two-channel nonuniform- division maximally decimated filter banks using L_1 criteria," *Proc. Inst. Elect. Eng, Vision, Image Signal Process.*, vol. 143, no. 2, pp. 79–83, Apr. 1996.

[8] Y.-C. Lim, J.-H. Lee, C.-K. Chen, and R. H. Yang, "A weighted least-squares algorithm for quasiequiripple FIR and IIR digital filter design," *IEEE Trans. Signal Processing*, vol. 40, pp. 551–558, Mar. 1992.

[9] L. R. Rabiner and B. Gold, *Theory and Application of Digital Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1975.

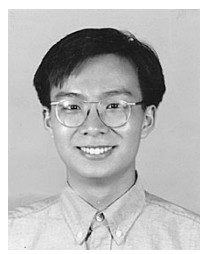
[10] Y. C. Lim and S. R. Parker, "Discrete coefficient FIR digital filter design based on an LMS criteria," *IEEE Trans. Circuits Syst.*, vol. CAS-30, pp. 723–739, Oct. 1983.

[11] B. Liu and L. T. Bruton, "The design of nonuniform-band maximally decimated filter banks," *Proc. IEEE Int. Symp. Circuits Syst.*, Chicago, IL, May 1993, pp. 375–378.

Ju-Hong Lee (M'86) was born in I-Lan, Taiwan, R.O.C., on December 7, 1952. He received the B.S. degree from the National Cheng-Kung University, Tainan, Taiwan, in 1975, the M.S. degree from the National Taiwan University, Taipei, in 1977, and the Ph.D. degree from Rensselaer Polytechnic Institute (RPI), Troy, NY, in 1984, all in electrical engineering.

From September 1980 to July 1984, he was a Research Assistant and was involved in research on multidimensional recursive digital filtering with the Department of Electrical, Computer, and Systems Engineering, RPI. From August 1984 to July 1986, he was a Visiting Associate Professor and, later, in August 1986, became an Associate Professor in the Department of Electrical Engineering, National Taiwan University. Since August 1989, he has been a Professor at the same university. He was appointed Visiting Professor with the Department of Computer Science and Electrical Engineering, University of Maryland, Baltimore, during a sabbatical leave in 1996. His current research interests include multidimensional digital signal processing, image processing, detection and estimation theory, analysis and processing of joint vibration signals for the diagnosis of cartilage pathology, and adaptive signal processing and its applications in communications.

Dr. Lee received Outstanding Research Awards from the National Science Council in 1988, 1989, and 1991 to 1994.



Ding-Chiang Tang was born in Chang-Hua, Taiwan, R.O.C., on August 20, 1974. He received the B.S. and M.S. degrees from the National Taiwan University, Taipei, in 1996 and 1998, respectively, all in electrical engineering.

His current research interests include digital signal processing and its applications.