Minimax Design of Two-Channel Nonuniform-Division FIR Filter Banks with −1, 0, and +1 Coefficients

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***Abstract*—Utilizing an approximation scheme and a WLS algorithm, we present a method to design two-channel nonuniform-division filter (NDF) banks with continuous coefficients under the minimax criterion. It is shown that the optimal filter coefficients can be obtained by solving only linear equations. In conjunction with a proposed filter structure, a method is then presented to obtain the desired design result with filter coefficients taking on -1, 0, and $+1$ only. The effectiveness of the proposed technique is demonstrated by a simulation example.**

***Index Terms*—Algorithms, filters and filtering, optimization.**

I. Introduction

FOR SOME applications, such as the subband coding of speech and audio signals, the most appropriate subband-decomposition of a signal must consider the critical bands of the ear. It has been shown in [1] that these critical bands have nonuniform bandwidths and cannot be easily constructed by a conventional tree structure based on two-channel quadrature mirror filter (QMF) banks. Several structures and design methods with continuous coefficients for nonuniform-division filter (NDF) banks have been presented in [1]–[6]. Recently, one of the authors considered a structure for two-channel NDF banks and proposed design methods for optimally designing NDF banks based on the L_1 error criteria in [7]. Designs with powers-of-two coefficients have been reported in [8] and [9] only for QMF banks. In [12] and [13], approaches have been presented for designing charge transfer device (CTD) filters and QMF banks with −1, 0, and +1 coefficients. However, there are practically no papers concerning the minimax design of NDF banks with coefficients −1, 0, and +1 in the literature. In this paper, we develop a technique to achieve the desired design with coefficients −1, 0, and +1.

II. Two-Channel Nonuniform-Division FIR Filter Banks

Consider the two-channel NDF bank with the architecture given by [7] that is shown in Fig. 1. The desired magnitude responses for the analysis filters $H_0(z)$ and $H_1(z)$ with passband widths equal to $L_0\pi/L$ and $L_1\pi/L$, respectively, are shown in Fig. 2, where $L = L_0 + L_1$. ω_p and ω_s denote the related band-edge frequencies. Assume that the associated magnitude responses are set to $B_0(\omega) = 1$ for $\omega \in [(0, \omega_s)/L_0]$ and $= 0$ for $\omega \in [(2\pi - \omega_s)/L_0, \pi]$, and $B_1(\omega) = 1$ for $\omega \in [0, (\pi - \omega_p)/L_1]$ and $= 0$ for $\omega \in [(\pi+\omega_p)/L_1, \pi]$, respectively. Let $H_0(z)$ and $H_1(z)$ have zero stopband response, $F_0(e^{j\omega}) = H_0(e^{j\omega})$ and $F_1(e^{j\omega}) = -H_1(e^{j\omega})$. By using the linear-phase property, the magnitude response $T(\omega)$ of the NDF bank becomes [7]

$$T(\omega) = \frac{1}{LL_0} H_0^2(\omega) + \frac{1}{LL_1} H_1^2(\omega). \tag{1}$$

The resulting aliasing distortions are given by [7]

$$\begin{aligned} A_1(\omega) = &\frac{1}{LL_0} H_0(\omega)H_0(\omega - \omega_p - \omega_s) \\ &+ \frac{1}{LL_1} H_1(\omega)H_1(\omega - \omega_p - \omega_s) \end{aligned} \tag{2}$$

and

$$\begin{aligned} A_2(\omega) = &\frac{1}{LL_0} H_0(\omega)H_0(\omega + \omega_p + \omega_s) \\ &+ \frac{1}{LL_1} H_1(\omega)H_1(\omega + \omega_p + \omega_s). \end{aligned} \tag{3}$$

Following (1)–(3), we formulate the conditions for perfect reconstruction as

$$\begin{cases} T(\omega) = 1, & \text{for } 0 \le \omega \le \pi \\ H_0(\omega) = 0, & \text{for } \omega_s \le \omega \le \pi \\ H_1(\omega) = 0, & \text{for } 0 \le \omega \le \omega_p \\ \frac{1}{\sqrt{LL_0}} H_0(\omega) & \\ \quad = \frac{1}{\sqrt{LL_1}} H_1(\omega_p + \omega_s - \omega), & \text{for } \omega_p \le \omega \le \omega_s. \end{cases} \tag{4}$$

III. Minimax Design of Two-Channel NDF Banks

According to (4), the overall error function E to be minimized in the minimax sense can be formulated by using the WLS algorithm of [10] as the weighted sum of four errors

$$\begin{aligned} E = &\int_{\omega=0}^{\pi} W_r(\omega)(T(\omega) - 1)^2 \, d\omega \\ &+ \alpha_1 \int_{\omega=0}^{\omega_p} W_1(\omega)H_1^2(\omega)\, d\omega \\ &+ \alpha_2 \int_{\omega=\omega_s}^{\pi} W_0(\omega)H_0^2(\omega)\, d\omega + \alpha_3 \int_{\omega=\omega_p}^{\omega_s} W_t(\omega) \\ &\cdot \left(\frac{1}{\sqrt{LL_0}} H_0(\omega) - \frac{1}{\sqrt{LL_1}} H_1(\omega_p + \omega_s - \omega)\right)^2 d\omega \end{aligned} \tag{5}$$

where $W_r(\omega)$, $W_1(\omega)$, $W_0(\omega)$, and $W_t(\omega)$ denote the required frequency response weighting functions for obtaining the

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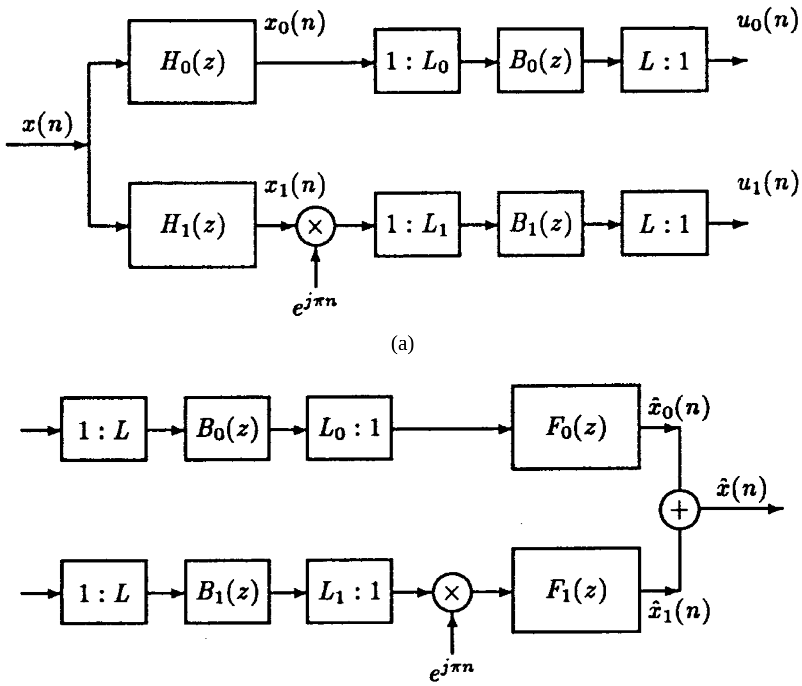

Fig. 1. The two-channel nonuniform-division maximally decimated filter bank system. (a) The analysis system. (b) The synthesis system.

minimax design. α_i, $i = 1, 2, 3$ represent the relative weights. During the optimization process for finding the optimal filter coefficients, let $h_0^l(n)$ and $h_1^l(n)$ be the filter coefficients of $H_0(z)$ and $H_1(z)$, respectively, at the lth iteration. An approximation for $T(\omega)$ is defined by replacing one of $H_i(\omega)$, $i = 0, 1$ in (1) with $H_i^l(\omega)$, respectively, as follows:

$$\hat{T}(\omega) = \frac{1}{LL_0} H_0^l(\omega) H_0(\omega) + \frac{1}{LL_1} H_1^l(\omega) H_1(\omega) \qquad (6)$$

where $H_0^l(\omega)$ and $H_1^l(\omega)$ denote the magnitude responses of $H_0(z)$ and $H_1(z)$ with coefficients $h_0^l(n)$ and $h_1^l(n)$, respectively. Let $\hat{E}$ be the corresponding error function given by (5) for $\hat{T}(\omega)$.

Let $\{\omega_1 = 0, \omega_2, \cdots, \omega_I = \omega_p, \cdots, \omega_J = \omega_s, \cdots, \omega_K = \pi\}$ be a dense grid of frequencies, linearly distributed in $[0, \pi]$.

We define the following matrices

$$\mathbf{U}_0 = [u_0(i, j)] \quad \text{with} \quad u_0(i, j) = 2\cos\left\{\left(\frac{N_0+1}{2} - j\right)\omega_i\right\}, \quad 1 \le i \le K, 1 \le j \le \frac{N_0}{2}$$

$$\mathbf{U}_1 = [u_1(i, j)] \quad \text{with} \quad u_1(i, j) = 2\sin\left\{\left(\frac{N_1+1}{2} - j\right)\omega_i\right\}, \quad 1 \le i \le K, 1 \le j \le \frac{N_1}{2}$$

$$\mathbf{U}_{0p} = [u_{0p}(i, j)] \quad \text{with} \quad u_{0p}(i, j) = 2\cos\left\{\left(\frac{N_0+1}{2} - j\right)\omega_i\right\}, \quad 1 \le i \le I, 1 \le j \le \frac{N_0}{2}$$

$$\mathbf{U}_{1p} = [u_{1p}(i, j)] \quad \text{with} \quad u_{1p}(i, j) = 2\sin\left\{\left(\frac{N_1+1}{2} - j\right)\omega_{i+J-1}\right\}, \quad 1 \le i \le K - J + 1, 1 \le j \le \frac{N_1}{2}$$

$$\mathbf{U}_{0s} = [u_{0s}(i, j)] \quad \text{with} \quad u_{0s}(i, j) = 2\cos\left\{\left(\frac{N_0+1}{2} - j\right)\omega_{i+J-1}\right\}, \quad 1 \le i \le K - J + 1, 1 \le j \le \frac{N_0}{2}$$

$$\mathbf{U}_{1s} = [u_{1s}(i, j)] \quad \text{with} \quad u_{1s}(i, j) = 2\sin\left\{\left(\frac{N_1+1}{2} - j\right)\omega_i\right\}, \quad 1 \le i \le I, 1 \le j \le \frac{N_1}{2}$$

$$\mathbf{U}_{0t} = [u_{0t}(i, j)] \quad \text{with} \quad u_{0t}(i, j) = 2\cos\left\{\left(\frac{N_0+1}{2} - j\right)\omega_{i+I-1}\right\}, \quad 1 \le i \le J - I + 1, 1 \le j \le \frac{N_0}{2}$$

$$\mathbf{U}_{1t} = [u_{1t}(i, j)] \quad \text{with} \quad u_{1t}(i, j) = 2\sin\left\{\left(\frac{N_1+1}{2} - j\right) \cdot (\omega_p + \omega_s - \omega_{i+I-1})\right\}, \quad 1 \le i \le J - I + 1, 1 \le j \le \frac{N_1}{2}.$$

Let two vectors $\mathbf{y}_i^l = [y_i^l(0), y_i^l(1), \cdots, y_i^l((N_i/2) - 1)]^T$, $i = 0, 1$ contain the independent parameters at the lth iteration, where the superscript T denotes the transpose operation.

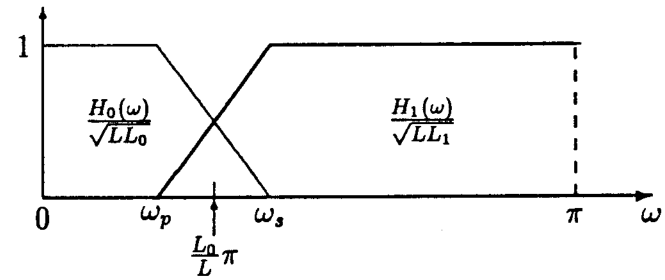

Fig. 2. The desired magnitude specifications for the analysis filters.

Using the above matrix notations, $\hat{E}$ can be approximately reformulated in matrix form as

$$\begin{aligned}\hat{\hat{E}} =& (\mathbf{U}_a\mathbf{y}_0^l + \mathbf{U}_b\mathbf{y}_1^l - \mathbf{1}_K)^T\mathbf{W}_r(\mathbf{U}_a\mathbf{y}_0^l + \mathbf{U}_b\mathbf{y}_1^l - \mathbf{1}_K)\\ &+ \alpha_1(\mathbf{U}_{1s}\mathbf{y}_1^l)^T\mathbf{W}_1(\mathbf{U}_{1s}\mathbf{y}_1^l) + \alpha_2(\mathbf{U}_{0s}\mathbf{y}_0^l)^T\mathbf{W}_0(\mathbf{U}_{0s}\mathbf{y}_0^l)\\ &+ \alpha_3(\mathbf{U}_{0t}\mathbf{y}_0^l - \mathbf{U}_{1t}\mathbf{y}_1^l)^T\mathbf{W}_t(\mathbf{U}_{0t}\mathbf{y}_0^l - \mathbf{U}_{1t}\mathbf{y}_1^l) \qquad (7)\end{aligned}$$

where $\mathbf{U}_a = \mathbf{H}_0^l\mathbf{U}_0$, $\mathbf{U}_b = \mathbf{H}_1^l\mathbf{U}_1$, $\mathbf{H}_0^l = 1/LL_0$ $\mathrm{diag}(\mathrm{H}_0^l(\omega_1), \ H_0^l(\omega_2), \cdots, H_0^l(\omega_K))$, $\mathbf{H}_1^l = 1/LL_1$ $\mathrm{diag}(\mathrm{H}_1^l(\omega_1), \ H_1^l(\omega_2), \cdots, H_1^l(\omega_K))$ and $\mathbf{1}_K$ is a column vector with size K and all entries equal to one. Each of $\mathbf{W}_r$, $\mathbf{W}_1$, $\mathbf{W}_0$, and $\mathbf{W}_t$ represents a diagonal matrix containing the weight values of the corresponding weighting function evaluated on the dense grid. Therefore, finding the optimal $\mathbf{y}_i^{l+1}$, $i = 0, 1$ for minimizing (7) is equivalent to solving the following linear equations:

$$\begin{bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{B}^T & \mathbf{C}\end{bmatrix}\begin{bmatrix}\mathbf{y}_0^{l+1}\\ \mathbf{y}_1^{l+1}\end{bmatrix} = \begin{bmatrix}\mathbf{U}_a^T\mathbf{W}_r\mathbf{1}_k\\ \mathbf{U}_b^T\mathbf{W}_r\mathbf{1}_k\end{bmatrix}$$

or

$$\begin{bmatrix}\mathbf{y}_0^{l+1}\\ \mathbf{y}_1^{l+1}\end{bmatrix} = \mathbf{R}_0^{-1}\begin{bmatrix}\mathbf{U}_a^T\mathbf{W}_r\mathbf{1}_K\\ \mathbf{U}_b^T\mathbf{W}_r\mathbf{1}_K\end{bmatrix} \qquad (8)$$

where

$$\begin{aligned}\mathbf{R}_0 &= \begin{bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{B}^T & \mathbf{C}\end{bmatrix}\\ \mathbf{A} &= \mathbf{U}_a^T\mathbf{W}_r\mathbf{U}_a + \alpha_2\mathbf{U}_{0s}^T\mathbf{W}_0\mathbf{U}_{0s} + \alpha_3\mathbf{U}_{0t}^T\mathbf{W}_t\mathbf{U}_{0t}\\ \mathbf{B} &= \mathbf{U}_a^T\mathbf{W}_r\mathbf{U}_b - \alpha_3\mathbf{U}_{0t}^T\mathbf{W}_t\mathbf{U}_{1t}\\ \mathbf{C} &= \mathbf{U}_b^T\mathbf{W}_r\mathbf{U}_b + \alpha_1\mathbf{U}_{1s}^T\mathbf{W}_1\mathbf{U}_{1s} + \alpha_3\mathbf{U}_{1t}^T\mathbf{W}_t\mathbf{U}_{1t}. \qquad (9)\end{aligned}$$

After getting the parameter vectors $\mathbf{y}_i^{l+1}$ at the $(l+1)$th iteration, we update $h_i(n)$, $i = 0, 1$ as

$$h_i^{l+1}(n) = 0.5(h_i^l(n) + y_i^{l+1}(n)), \quad \text{for } n = 0, 1, \cdots, \frac{N_i}{2} - 1. \qquad (10)$$

To achieve the minimax design, the WLS algorithm of [10] is utilized for adjusting $W_r(\omega)$, $W_0(\omega)$, $W_1(\omega)$, and $W_t(\omega)$ during the design process. Further, the initial vectors $\mathbf{y}_i^0$ are set to a least squares solutions that optimally approximate the desired responses of Fig. 2 as

$$\begin{aligned}\mathbf{y}_0^0 &= \sqrt{LL_0}\left(\mathbf{U}_{0p}^T\mathbf{U}_{0p} + \alpha_2\mathbf{U}_{0s}^T\mathbf{U}_{0s}\right)^{-1}\left(\mathbf{U}_{0p}^T\mathbf{1}_I\right)\\ \mathbf{y}_1^0 &= \sqrt{LL_1}\left(\mathbf{U}_{1p}^T\mathbf{U}_{1p} + \alpha_1\mathbf{U}_{1s}^T\mathbf{U}_{1s}\right)^{-1}\left(\mathbf{U}_{1p}^T\mathbf{1}_{K-J+1}\right).\end{aligned} \qquad (11)$$

Appendix A summarizes the proposed method by showing Design Procedure 1.

IV. Minimax Design using Coefficients −1, 0, and +1

A. The Proposed Filter Structure

Consider the new filter structure shown by Fig. 3 for filter realization. The impulse response $w(m)$ has a finite number of samples with values restricted to -1, 0, and $+1$ only. The reset terminal receives a signal to clear the contents of the delay element of C when $m = nk + 1$. Let the impulse response of the filter structure be $h_d(n)$ with length N. $w(m)$ will have length $N_k = Nk + 1$. Setting $x(n)$ to an impulse sequence, we obtain

$$y_k(0) = \Delta w(0), \qquad y_k(m) = \Delta w(m), \quad \text{for } m = nk + 1, n = 1, 2, \cdots, N \qquad (12)$$

and

$$y_k(m) = 3y_k(m-1) + \Delta w(m), \quad \text{for } m \neq nk + 1, n = 1, 2, \cdots, N. \qquad (13)$$

From (12) and (13), we have

$$\begin{aligned}y_k(0) &= \Delta w(0)\\ y_k(m) &= \Delta \sum_{i=k\langle (m-1)/k\rangle+1}^{m} w(i)3^{m-i}, \quad \text{for } m = 1, 2, \cdots, N_k - 1\end{aligned} \qquad (14)$$

where $\langle x\rangle$ denotes the largest integer not greater than x. Let $w(m) = 0$ for $m < 0$. It is easy to show that the relationship between $y(n) = h_d(n)$ and $y_k(m)$ is given by

$$h_d(n) = y(n) = y_k(nk) = \Delta \sum_{m=(n-1)k+1}^{nk} w(m)3^{nk-m}, \quad \text{for } n = 0, 1, \cdots, N-1. \qquad (15)$$

Since $w(m)$ are restricted to -1, 0, and $+1$ only for $m = 0, 1, \cdots, N_k - 1$, we note from (15) that $h_d(n)$ satisfies the inequalities $-(3^k-1)/2 \leq h_d(n)/\Delta \leq (3^k-1)/2$. For any integer $P \in [-(1/2)(3^k-1), (1/2)(3^k-1)]$, it is easy to show that there exists a unique set of $w((n-1)k+1)$, $w((n-1)k+2), \cdots, w(nk)$ such that P can be expressed as $P = \sum_{m=(n-1)k+1}^{nk} w(m)3^{nk-m}$.

B. Discrete Optimization Procedure

1) Constrained Optimization: Let $\mathbf{h} = [h(0), h(1), \cdots, h((N_0+N_1)/2-1)]^T = [h_0(0), h_0(1), \cdots h_0((N_0/2)-1), h_1(0), h_1(1), \cdots, h_1((N_1/2)-1)]^T$. Suppose that $\mathbf{h}^{(i)}$ represents the continuous coefficient vector excluding the ith coefficient $h(i)$ that is fixed at a discrete value. Let $\mathbf{h}_1^{(i)}$ be the vector that minimizes the overall error function $\hat{\hat{E}}$ of (7) when $h(i)$ is fixed at a discrete value $h_d(i)$. Let the difference between the optimal continuous value $h_c(i)$ and the discrete

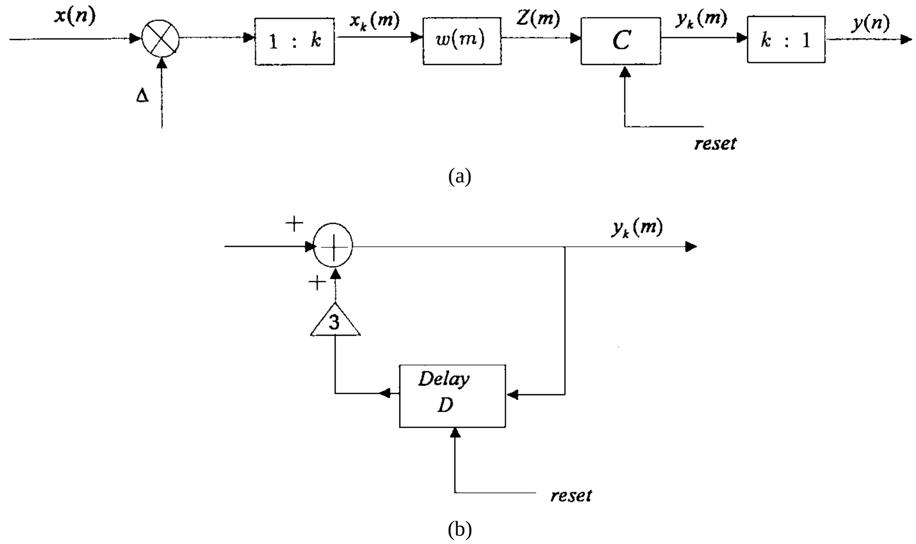

Fig. 3. (a) The proposed new filter structure. (b) The function block C.

value $h_d(i)$ of $h(i)$ be $\delta h(i) = h_d(i) - h_c(i)$. Based on the LMS algorithm of [11], we can obtain $\mathbf{h}_1^{(i)} = \mathbf{h}_c^{(i)} + (\mathbf{G}^{(i)})\delta h(i) = [h_1^{(i)}(0), h_1^{(i)}(1), \cdots, h_1^{(i)}((N_0+N_1)/2-2)]^T$, where $\mathbf{h}_c^{(i)}$ denotes the vector $\mathbf{h}_c = [h_c(0), h_c(1), \cdots, h_c((N_0+N_1)/2-1)]^T$ that is obtained from Design Procedure 1 with $h_c(i)$ omitted. $\mathbf{G}^{(i)}$ is an $((N_0+N_1)/2-1) \times 1$ column vector given by $\mathbf{G}^{(i)} = \mathbf{R}_1^{-1}\mathbf{S}$, where $\mathbf{R}_1$ and $\mathbf{S}$ are obtained from the submatrices of $\mathbf{R}_0$ of (9). Let $\mathbf{R}_0$ be partitioned as

$$\mathbf{R}_0 = \begin{bmatrix} \mathbf{A}_1 & B_1 & \mathbf{A}_2 \\ C_1^T & D & C_2^T \\ \mathbf{A}_3 & B_2 & \mathbf{A}_4 \end{bmatrix}$$

where $[C_1^T \quad D \quad C_2^T]$ and $[B_1^T \quad D \quad B_2^T]^T$ are the ith row and column of $\mathbf{R}_0$, respectively. Then $\mathbf{R}_1 = \left[\begin{smallmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_3 & \mathbf{A}_4 \end{smallmatrix}\right]$ and $\mathbf{S} = \left[\begin{smallmatrix} B_1 \\ B_2 \end{smallmatrix}\right]$. Let $\mathbf{h}_2^{(i,j)}$ be the coefficient vector that minimizes (7) when $h_c(i)$ and $h_1^{(i)}(j)$ are fixed at discrete values. The difference between the optimal continuous value $h_{1c}^{(i)}(j)$ and the discrete value $h_d^{(i)}(j)$ of $h_1^{(i)}(j)$ is $\delta h^{(i)}(j) = h_d^{(i)}(j) - h_{1c}^{(i)}(j)$. Similarly, we obtain $\mathbf{h}_2^{(i,j)} = \mathbf{h}_1^{(i,j)} + \mathbf{G}^{(i,j)}\delta h^{(i)}(j)$, where $\mathbf{h}_1^{(i,j)}$ represents the vector $\mathbf{h}_1^{(i)}$ with $h_1^{(i)}(j)$ omitted. We obtain $\mathbf{G}^{(i,j)}$ by using the same procedure as above with $\mathbf{R}_1$ instead of $\mathbf{R}_0$. This process is repeated until all of the filter coefficients are fixed at discrete values.

2) Tree Search Algorithm: After obtaining the optimal design from (8), we choose a coefficient $h_c(i)$ and fix it at M discrete values in the vicinity of $h_c(i)$. Based on the $\mathbf{h}_1^{(i)}$, M, further optimization problems are produced when the second coefficient $h_1^{(i)}(j)$ is chosen and fixed at M discrete values. Therefore, M^2 optimization problems must be solved when $h_c(i)$ and $h_1^{(i)}(j)$ take on discrete values. We select only M of the M^2 optimization problems for further discretizing the remaining coefficients. The search process continues until all of the filter coefficients are discretized.

Finally, Appendix B summarizes the minimax design with coefficients -1, 0, and $+1$.

3) Filter Coefficient Selection: In general, the grid density decreases as the value of discrete coefficients increases. The effect of discretizing the small coefficients is more easily compensated by the remaining reoptimized coefficients. Hence, we discretize the coefficient with the largest relative sensitivity first, at each tree stage. The relative sensitivity of $h(i)$ is defined as the relative sensitivity of $h(i) = \mathrm{Max}|g^{(i)}(j)|$, where $g^{(i)}(j)$ is the jth element of the vector $\mathbf{G}^{(i)}$.

V. Simulation Example

The parameters K, M, and k are set to $8 \times \max(N_0, N_1)$, 3 and 10, respectively. Let Ω_i denote the stopband of $H_i(\omega)$, $i = 0, 1$. The performance is evaluated in terms of the peak reconstruction error (PRE), the normalized peak stopband ripple (NPSR), and the stopband ripple energies (SRE's) of the designed $H_0(z)$ and $H_1(z)$. They are given by

$$\begin{aligned} \mathrm{PRE} &= \max\{|20\log_{10} T(\omega)|\}, \\ &\quad \text{for } \omega \in [0, \pi] \\ \mathrm{SRE}_i &= \int_{\Omega_i} H_i^2(\omega)\, d\omega \\ \mathrm{NPSR}_i &= \max\left\{20\log_{10}\frac{|H_i(\omega)|}{\sqrt{LL_i}}\right\} \\ &\quad \text{for } \omega \in \Omega_i. \end{aligned} \tag{16}$$

Example: We use the design specifications of Case 1 in Table I and Design Procedure 2. We set $\alpha_1 = \alpha_2 = \alpha_3 = 0.05$, $\kappa_1 = \kappa_2 = \kappa_3 = 0.005$, $\kappa_4 = 0.73$, and $\epsilon = 10^{-3}$. Table I also shows the significant design results for both of continuous and discrete coefficients after 17 iterations. The resulting step size Δ and filter coefficients for the discrete design are listed in Table II. Fig. 4 plots the corresponding $H_0(\omega)/\sqrt{LL_0}$, $H_1(\omega)/\sqrt{LL_1}$, and $T(\omega)$ in decibels of the designed NDF banks. We note that the satisfactory performances of the designed NDF banks with the optimal continuous

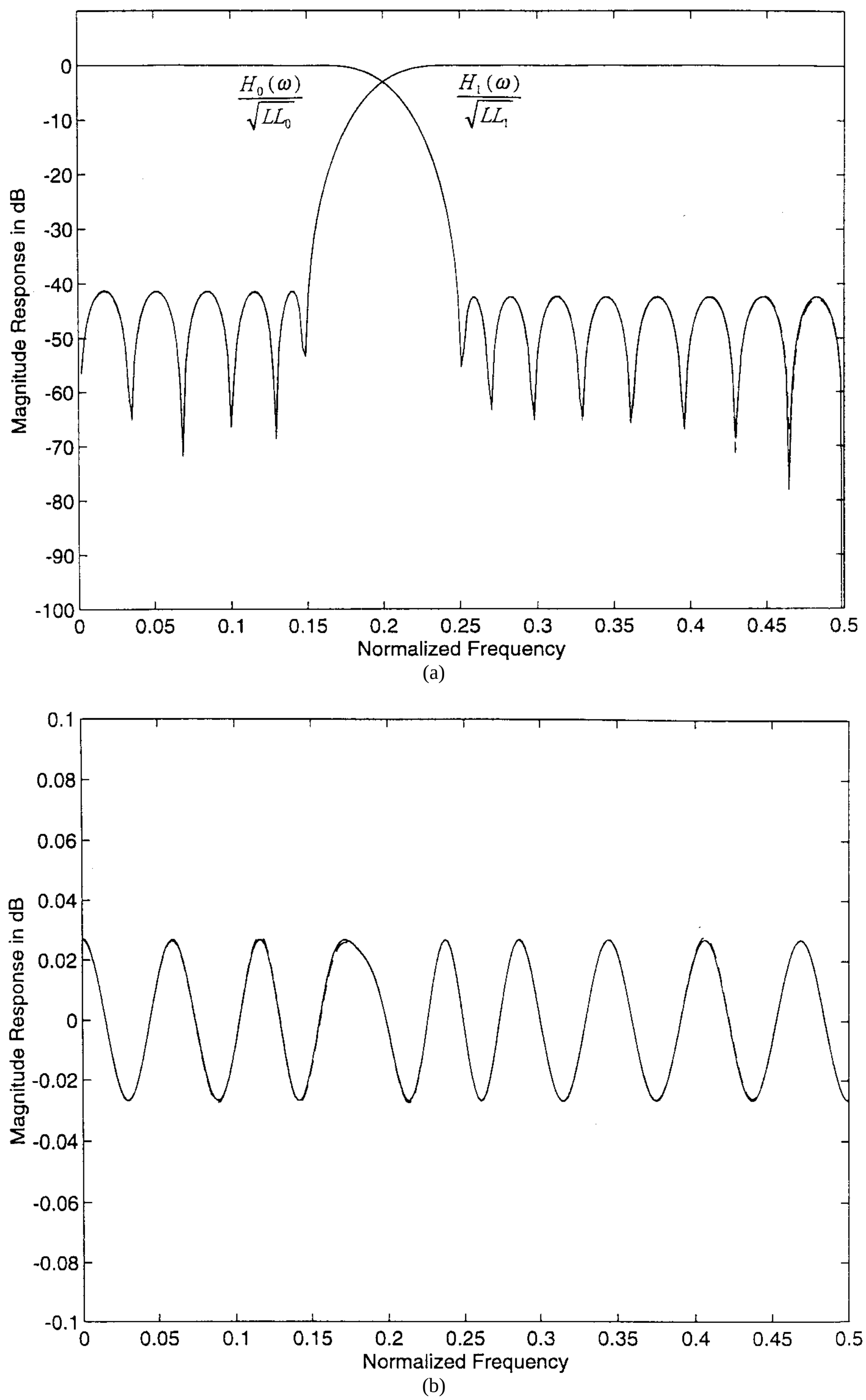

Fig. 4. The magnitude responses for the example. (a) $H_0(\omega)/\sqrt{LL_0}$ and $H_1(\omega)/\sqrt{LL_1}$. (b) $T(\omega)$. With -1, 0, and $+1$ coefficients (dashed-line) and with the optimal continuous coefficients (solid-line).

coefficients and coefficients of -1, 0, and $+1$ only are very close.

Appendix A
Design Procedure 1

Step 1: Specify N_0 and N_1, ω_p and ω_s, the relative weights α_1, α_2, and α_3. Set the iteration number $l = 0$ and each of the weighting matrices to the identity matrix.

Step 2: Obtain $\mathbf{y}_i^0$, $i = 0, 1$, using (11). Compute the magnitude responses $H_0^0(\omega)$ and $H_1^0(\omega)$.

Step 3: Compute the vectors $\mathbf{y}_i^{l+1}$, $i = 0, 1$, at the $(l+1)$th iteration by solving the linear equations of (8).

Step 4: Compute the resulting filter coefficients at the $(l+1)$th iteration using (10).

Step 5: Compute the associated overall error function $\hat{\hat{E}}^{l+1}$. If the stopping criterion, $|(\hat{\hat{E}}^l - \hat{\hat{E}}^{l+1})/\hat{\hat{E}}^l| \le \epsilon$, is satisfied,

TABLE I
(a) THE DESIGN SPECIFICATIONS FOR CASE 1 AND (b) THE SIGNIFICANT DESIGN RESULTS

	Case 1
N_0	32
N_1	32
ω_p	0.3π
ω_s	0.5π
L_0	2
L_1	3

(a)

	Continuous Design	Design with Coefficient -1 , 0 , +1
PRE(dB)	0.0269767970	0.0279254633
NPSR(dB) of $H_0(\omega)$	-42.4243401946	-42.2483936545
NPSR(dB) of $H_1(\omega)$	-41.2303864631	-41.3402438900
SRE of $H_0(\omega)$	0.0004504312	0.0004458339
SRE of $H_1(\omega)$	0.0005199400	0.0005132138

(b)

TABLE II
THE ANALYSIS FILTER COEFFICIENTS

Δ = 1.220703125000000e-004

n	$h_{0d}(n)/\Delta$	$h_{1d}(n)/\Delta$	n
0	10122	16706	31
1	4950	-1393	30
2	-535	-3807	29
3	-2154	-1392	28
4	-498	1145	27
5	1023	1430	26
6	672	161	25
7	-368	-765	24
8	-546	-551	23
9	27	133	22
10	336	404	21
11	99	159	20
12	-154	-139	19
13	-124	-183	18
14	97	-67	17
15	26	24	16

then go to *Step 6,* where ϵ is a preset small positive real number. Otherwise, set $l = l + 1$ and go to *Step 3.*

Step 6: Let $\mathrm{Max}(V_r)$ and $\mathrm{Min}(V_r)$ be the maximum and minimum of $|T(\omega) - 1|$ for $\omega \in [0, \pi]$, $\mathrm{Max}(V_0)$ and $\mathrm{Min}(V_0)$ be the maximum and minimum of $|H_0(\omega)|$ for $\omega \in [\omega_s, \pi]$, $\mathrm{Max}(V_1)$ and $\mathrm{Min}(V_1)$ be the maximum and minimum of $|H_1(\omega)|$ for $\omega \in [0, \omega_p]$, and $\mathrm{Max}(V_t)$ and $\mathrm{Min}(V_t)$ be the maximum and minimum of $|(1/\sqrt{LL_0})H_0(\omega) - (1/\sqrt{LL_1})H_1(\omega_p + \omega_s - \omega)|$ for $\omega \in [\omega_p, \omega_s]$, respectively. If all of the following stopping criteria:

$$\text{a)}\quad \frac{\mathrm{Max}(V_r) - \mathrm{Min}(V_r)}{\mathrm{Max}(V_r)} \le \kappa_1$$

$$\text{b)}\quad \frac{\mathrm{Max}(V_0) - \mathrm{Min}(V_0)}{\mathrm{Max}(V_0)} \le \kappa_2$$

$$\text{c)}\quad \frac{\mathrm{Max}(V_1) - \mathrm{Min}(V_1)}{\mathrm{Max}(V_1)} \le \kappa_3$$

and

$$\text{d)}\quad \frac{\mathrm{Max}(V_t) - \mathrm{Min}(V_t)}{\mathrm{Max}(V_t)} \le \kappa_4$$

are satisfied, then terminate the design process. Otherwise, go to *Step 7.*

Step 7: Adjust $\mathbf{W}_r$, $\mathbf{W}_0$, $\mathbf{W}_1$, and $\mathbf{W}_t$ using the WLS algorithm of [10]. Then, set $l = l + 1$ and go to *Step 3.*

APPENDIX B
DESIGN PROCEDURE 2

Step 1: Use the design method of Section III to find the optimal continuous filter coeffcients $h_i(n)$, $i = 0, 1$.

Step 2: Choose four powers-of-two values in the vicinity of the maximum of $|h_c(n)|/(1/2(3^k - 1))$ as the values for the step size Δ.

Step 3: For a given Δ, perform the discrete optimization procedure of Section IV-B to find the discrete coefficients $h_d(n)$, $n = 0, 1, 2, \cdots, ((N_1 + N_2)/2) - 1$.

Step 4: Compute $\hat{\hat{E}}$ of (7) corresponding to $h_d(n)$, update $\mathbf{U}_a$ and $\mathbf{U}_b$, and adjust $\mathbf{W}_r$, $\mathbf{W}_0$, $\mathbf{W}_1$, and $\mathbf{W}_t$ using the WLS algorithm of [10]. Then, recompute the new optimal $\mathbf{h}_c$ from (8).

Step 5: Repeat *Steps 3* and *4* until $\hat{\hat{E}}$ cannot be further reduced.

Step 6: Select the Δ that makes $\hat{\hat{E}}$ smallest among the four powers-of-two values for Δ. Find the corresponding $w(m)$ by using (15).

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