Dynamic Analysis of a Propagating Antiplane Interface Crack

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Abstract: In this study, the transient problem of a propagating interface crack between two different media is analyzed. For time $t < 0$, the crack is stress free and at rest. At $t = 0$, a pair of concentrated antiplane dynamic point loadings are applied at the stationary crack faces. It is assumed that the stationary crack will begin to propagate along the interface with a subsonic speed as the incident wave generated by the point loading in the upper crack face or in the lower crack face arrives at the crack tip. A new fundamental solution is proposed in this study and the transient solution is determined by superposition of the fundamental solution in the Laplace transform domain. The proposed fundamental problem is the problem of applying an exponentially distributed traction (in the Laplace transform domain) on the propagating crack faces. The Cagniard-de Hoop method of Laplace inversion is used to obtain the transient solution in time domain. Theoretical results indicate that the shear stress along the interface of stationary crack in a bimaterial will jump to the corresponding static value in a homogeneous medium after the lower shear wave reaches the observation point. Moreover, the dynamic stress intensity factor of a propagating interface crack has an interesting form of the product of a universal function and the corresponding static value of a homogeneous crack.

INTRODUCTION

Many structures are composed of different materials formed in layers for both man-made and natural origin. The layers are bonded together along interfaces. For the last two decades, the importance of composite materials has increased very rapidly in engineering applications because of their high strength and light weight. However, flaws contained at the interfaces of composite bodies due to improper adhesion may lead to serious danger, and a better understanding of interface fracture mechanics is needed. Since the inherent time dependence of a dynamic fracture process results in mathematical models that are more complex than equivalent quasistatic models, most of the analyses done regarding cracked composite bodies are quasistatic. However, because of increasing applications of multilayered materials in modern engineering structures, there is still substantial interest in the dynamic interface fracture problem and many efforts should be added in this field.

The asymptotic elastic fields of a semiinfinite crack lying along a interface between dissimilar isotropic materials subjected to static loading was first considered by Williams (1959) for plane strain condition. A number of solutions for the stress and the displacement field near the crack tip are obtained by England (1965), Erdogan (1965) and Rice and Sih (1965). Extensions to anisotropic elasticity for the near tip field have been made by Gotoh (1967), Bogy (1972), and Kuo and Bogy (1974), and recently by Ting (1986, 1990) and Qu and Bassani (1989). The exact full field solutions of interface cracks in anisotropic dissimilar media is obtained by Ma and Luo (1996). In the field of propagating interface cracks, Willis (1971) investigated the energy release rate of a steadily extending interface crack by means of the local form of the Griffith virtual work argument. He also derived an explicit fracture criterion, which involves a suitable defined "stress concentration vector." In the recent years, Wu (1991) treated the similar but anisotropic problem and derived the crack-tip fields and energy release rate successfully by employing the Stroh formalism for anisotropic elasticity. Deng (1992) analyzed the near-tip fields for steadily growing interface cracks in dissimilar isotropic materials. Yang et al. (1991) have analyzed the problems of steadily propagating interface cracks in dissimilar isotropic and orthotropic bimaterials. They solved the crack-tip fields by the method of Stroh formulation and discussed the singularities for antiplane and in-plane deformations carefully. The stress singularities and the angular stress distributions near a propagating interface crack in different transonic regimes for both antiplane and in-plane cases are determined by Yu and Yang (1994, 1995). Zhu and Kuang (1995) have solved the antiplane strain problem of a straight interface propagating between two elastic half-spaces under arbitrary variable loading. They obtained the stresses along the interface and stress intensity factor of a propagating crack.

The extension of an interface crack under the influence of transient horizontally polarized shear wave was analyzed by Brock and Achenbach (1973). It is assumed that the adhesive behaves as a perfectly plastic material, so that the stress in the zone of interface yielding is uniform and equal to the yield stress. Analytic solutions for the time of rupture and for the interface stress ahead of yield zone are obtained by applying integral transform methods. Recently, Chung and Robinson (1992) solved the transient problem of a mode-III crack propagating along the interface between two different media. In their study, the compound body is loaded by a constant shear traction at infinity such that the problem becomes self-similar. This self-similar problem can be solved effectively by the method of self-similar potentials (SSP). In a series of papers, Freund (1972, 1973, 1974) developed important analytical methods for evaluation of the transient stress field of a propagating crack in a homogeneous material under a quite general dynamic loading situation. These particular cases analyzed by Freund are also self-similar, but they are solved by means of integral transform methods rather than by direct application to similarity arguments. An indirect analytical approach was proposed by Freund based on superposition over a fundamental solution. Based on the superposition method proposed by Freund, a series of problems for nonplanar crack propagation in an infinite domain was solved by Ma and Burgers (1986, 1987, 1988) and Ma (1988, 1990). For the aforementioned problems (except for the SSP method), either the direct application of the well-known Wiener-Hopf technique (Noble 1958) is used or the superposition method proposed by Freund is performed to solve the problem. However,

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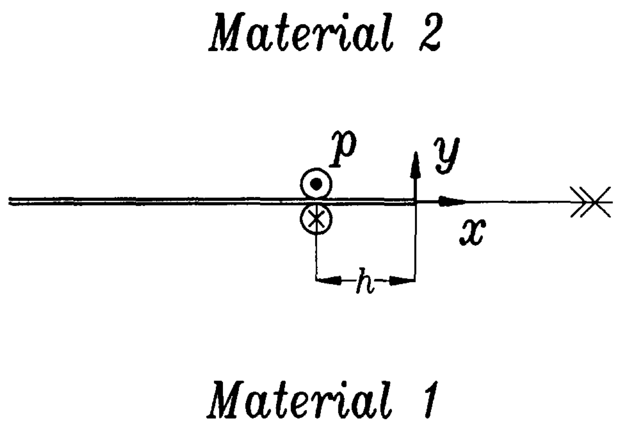

FIG. 1. Configuration and Coordinate System of a Stationary Interface Crack in a Bimaterial Medium

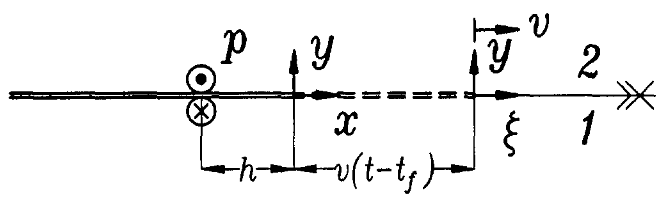

FIG. 2. Configuration and Coordinate System of a Propagating Interface Crack in a Bimaterial Medium

if a crack is subjected to incident nonplanar waves, none of the known methods can be used directly to obtain the transient solutions.

In this study, the transient problem of an interface crack subjected to dynamic loadings and propagating with a subsonic speed along the interface is considered. At time $t = 0$, the crack is at rest and a pair of antiplane concentrated dynamic loadings are applied at stationary crack faces as shown in Fig. 1. After some delay time t_f, the crack begins to run along the interface with a constant velocity v as shown in Fig. 2. Two propagating cases are considered in this study. The first one is that the crack starts to propagate immediately after the faster incident cylindrical wave passes the stationary crack tip (i.e., $t_f = b_2 h$). The second one is that the crack begins to propagate after the incident cylindrical wave with smaller shear wave speed arrives at crack tip (i.e., $t_f = b_1 h$). A new fundamental solution is proposed in this study and it is successfully applied towards solving the problem. The fundamental problem is the problem of applying an exponentially distributed traction on the propagating crack faces in the Laplace transform domain. This alternative superposition scheme has been used to solve many transient problems for the interaction of incident cylindrical waves with cracks in a homogeneous medium successfully, e.g., Tsai and Ma (1992) for a stationary crack and Ma and Ing (1995) for a propagating crack. The transient full-field solutions of the stationary interface crack and the dynamic stress intensity factor of the propagating interface crack are obtained and are expressed in a closed form. Numerical results of interfacial stresses and dynamic stress intensity factors for the problem are evaluated and discussed in detail.

FUNDAMENTAL PROBLEM

Consider a fundamental problem of antiplane deformation for an extending interface crack in dissimilar materials. The interface crack propagates with a constant velocity v, which is less than the lower shear wave speed of these two materials. Fig. 1 shows the interface crack geometry and the coordinate systems. Materials 1 and 2 occupy the two half-spaces. The coordinate ξ defined by $\xi = x - vt$ is fixed with respect to the moving crack tip. In analyzing this problem, it is convenient to express the governing equations of wave motions in the moving coordinates $\xi - y$ as follows:

$$(1 - b_j^2 v^2)\frac{\partial^2 w_j}{\partial \xi^2} + \frac{\partial^2 w_j}{\partial y^2} + 2b_j^2 v \frac{\partial^2 w_j}{\partial \xi \partial t} - b_j^2 \frac{\partial^2 w_j}{\partial t^2} = 0; \quad j = 1, 2 \qquad (1a,b)$$

where the subscript j ($j = 1, 2$) refers to the lower and upper media, respectively; w_j = out-of-plane displacements; and b_j = slownesses of the shear waves given by

$$b_j = \frac{1}{c_{sj}} = \sqrt{\frac{\rho_j}{\mu_j}}$$

in which c_{sj} = shear wave speeds, μ_j and ρ_j = shear moduli and the mass densities of two materials, respectively. Without loss of generality, we assume $b_1 > b_2$; that is, the shear wave speed in the lower material is less than in the upper material. The nonvanishing shear stresses are

$$\tau_{yzj} = \mu_j \frac{\partial w_j}{\partial y}; \quad \tau_{xzj} = \mu_j \frac{\partial w_j}{\partial x} \qquad (2a,b)$$

The solution for an exponentially distributed loading applied at the crack faces in the Laplace transform domain will be referred to as the fundamental solution. Then the boundary conditions on the crack surfaces expressed in the Laplace transform domain can be described as follows:

$$\bar{\tau}_{yz1}(\xi, 0, s) = \bar{\tau}_{yz2}(\xi, 0, s) = e^{s\eta\xi}; \quad -\infty < \xi < 0 \qquad (3a,b)$$

where s = Laplace transform parameter; and η = constant. The overbar symbol is used for denoting the transform on time t. The one-sided Laplace transform with respect to time and the two-sided Laplace transform with respect to ξ are defined by

$$\bar{w}(\xi, y, s) = \int_0^\infty w(\xi, y, t)e^{-st}\, dt$$

$$\bar{w}^*(\lambda, y, s) = \int_{-\infty}^\infty \bar{w}(\xi, y, s)e^{-s\lambda\xi}\, d\xi$$

The displacements and shear stresses must be continuous on the interface, which gives the following conditions on the interface:

$$\bar{\tau}_{yz1}(\xi, 0, s) = \bar{\tau}_{yz2}(\xi, 0, s); \quad 0 < \xi < \infty \qquad (4a,b)$$

$$\bar{w}_1(\xi, 0, s) = \bar{w}_2(\xi, 0, s); \quad 0 < \xi < \infty \qquad (5a,b)$$

The solution of the proposed fundmental problem can be obtained in the usual way by making use of integral transform methods. Apply a one-sided Laplace transform with respect to t and a two-sided Laplace transform with respect to ξ on (1). General solutions in the transform domain, which are bounded as $y \to -\infty$ (and $+\infty$, respectively), can be expressed as

$$\bar{w}_1^*(\lambda, y, s) = A_1(s, \lambda)e^{s\alpha_1^*(\lambda)y} \qquad (6)$$

$$\bar{w}_2^*(\lambda, y, s) = A_2(s, \lambda)e^{s\alpha_2^*(\lambda)y} \qquad (7)$$

where

$$\alpha_j^*(\lambda) = \sqrt{b_j + \lambda(1 - b_j v)}\,\sqrt{b_j - \lambda(1 + b_j v)}$$

$$= \alpha_{j+}^*(\lambda)\alpha_{j-}^*(\lambda); \quad j = 1, 2 \qquad (8)$$

and A_1, A_2 = unknown functions. We define $b_{j,1} = b_j/(1 + b_j v)$ and $b_{j,2} = b_j/(1 - b_j v)$. The branch cuts of α_j^* are introduced to ensure $\text{Re}(\alpha_j^*) \geq 0$ in the entire cut complex λ-plane, where Re = real part.

Application of the Laplace transforms to the boundary conditions (3)–(5) and aid with (6) and (7), the transformed displacements and shear stresses along the crack line $y = 0$ are obtained as

$$\bar{w}_1^*(\lambda, 0, s) = A_1 = A_+ + A_{1-} \quad (9)$$

$$\bar{w}_2^*(\lambda, 0, s) = A_2 = A_+ + A_{2-} \quad (10)$$

$$\mu_1 s\alpha_1^*(\lambda)A_1 = -\mu_2 s\alpha_2^*(\lambda)A_2 = \frac{1}{s(\eta - \lambda)} + \bar{\tau}_{yz+}^* \quad (11)$$

where $\bar{\tau}_{yz+}^*$, A_{1-}, A_{2-}, and A_+ = unknown functions. Eliminating A_+ through (9)–(11), we have

$$A_- = \frac{\mu_1\alpha_1^*(\lambda) + \mu_2\alpha_2^*(\lambda)}{s\mu_1\mu_2\alpha_1^*(\lambda)\alpha_2^*(\lambda)}\left[\frac{1}{s(\eta - \lambda)} + \bar{\tau}_{yz+}^*\right] \quad (12)$$

where $A_- \equiv A_{1-} - A_{2-}$ = transformed crack-opening displacement. At this point it is convenient to introduce a new function $Q^*(\lambda)$ by defining

$$Q^*(\lambda) = \frac{\mu_1\alpha_1^*(\lambda) + \mu_2\alpha_2^*(\lambda)}{\mu_1\mu_2 k\alpha_1^*(\lambda)} \quad (13)$$

where $k = (\mu_1\sqrt{1 - b_1^2v^2} + \mu_2\sqrt{1 - b_2^2v^2})/(\mu_1\mu_2\sqrt{1 - b_1^2v^2})$. From the general product factorization method, $Q^*(\lambda)$ can be written as the product of two regular functions $Q_+^*(\lambda)$ and $Q_-^*(\lambda)$, where

$$Q_+^*(\lambda) = \exp\left\{\frac{-1}{\pi}\int_{b_{2,2}}^{b_{1,2}} \tan^{-1}\left[\frac{\mu_2|\alpha_2^*(-z)|}{\mu_1\alpha_1^*(-z)}\right]\frac{dz}{z + \lambda}\right\} \quad (14)$$

and

$$Q_-^*(\lambda) = \exp\left\{\frac{-1}{\pi}\int_{b_{2,1}}^{b_{1,1}} \tan^{-1}\left[\frac{\mu_2|\alpha_2^*(z)|}{\mu_1\alpha_1^*(z)}\right]\frac{dz}{z - \lambda}\right\} \quad (15)$$

In view of the previous discussion (12) may be rewritten as

$$\frac{s\alpha_{2-}^*(\lambda)A_-}{Q_-^*(\lambda)} - \frac{kQ_+^*(\eta)}{s(\eta - \lambda)\alpha_{2+}^*(\eta)} = \frac{k}{s(\eta - \lambda)}\left[\frac{Q_+^*(\lambda)}{\alpha_{2+}^*(\lambda)} - \frac{Q_+^*(\eta)}{\alpha_{2+}^*(\eta)}\right] + \frac{kQ_+^*(\lambda)}{\alpha_{2+}^*(\lambda)}\bar{\tau}_{yz+}^* \quad (16)$$

The left-hand side of this equation is regular for $\mathrm{Re}(\lambda) < b_{2,1}$, while the right-hand side is regular for $\mathrm{Re}(\lambda) > -b_{2,2}$. The usual reasoning now leads to the solution

$$A_- \equiv A_{1-} - A_{2-} = \frac{kQ_+^*(\eta)Q_-^*(\lambda)}{s^2(\eta - \lambda)\alpha_{2+}^*(\eta)\alpha_{2-}^*(\lambda)} \quad (17)$$

Making use of (9)–(11) and eliminating $\bar{\tau}_{yz+}^*$, we can obtain the transformed displacement A_+. Then substituting A_{2-} from (17) into the expression of A_+, the amplitude of $\bar{w}_1^*$ in the transform domain can be found as

$$A_1 = \frac{Q_+^*(\eta)\alpha_{2+}^*(\lambda)}{s^2\mu_1\alpha_{2+}^*(\eta)(\eta - \lambda)\alpha_1^*(\lambda)Q_+^*(\lambda)} \quad (18)$$

Similarly substituting A_{1-} from (17) into the expression of A_+, we have

$$A_2 = \frac{-Q_+^*(\eta)}{s^2\mu_2\alpha_{2+}^*(\eta)(\eta - \lambda)\alpha_{2-}^*(\lambda)Q_+^*(\lambda)} \quad (19)$$

In view of (18), (19), (6), and (7) inverting the two-sided Laplace transform, we obtain the solutions of stresses and displacements for the fundamental problem in the Laplace transform domain as follows:

$$\bar{\tau}_{yz1}(\xi, y, s) = \frac{1}{2\pi i}\int \frac{Q_+^*(\eta)\alpha_{2+}^*(\lambda)}{\alpha_{2+}^*(\eta)(\eta - \lambda)Q_+^*(\lambda)} e^{s\alpha_1^*(\lambda)y + s\lambda\xi}\, d\lambda \quad (20)$$

$$\bar{\tau}_{xz1}(\xi, y, s) = \frac{1}{2\pi i}\int \frac{\lambda Q_+^*(\eta)\alpha_{2+}^*(\lambda)}{\alpha_{2+}^*(\eta)(\eta - \lambda)\alpha_1^*(\lambda)Q_+^*(\lambda)} e^{s\alpha_1^*(\lambda)y + s\lambda\xi}\, d\lambda \quad (21)$$

$$\bar{w}_1(\xi, y, s) = \frac{1}{2\pi i}\int \frac{Q_+^*(\eta)\alpha_{2+}^*(\lambda)}{s\mu_1\alpha_{2+}^*(\eta)(\eta - \lambda)\alpha_1^*(\lambda)Q_+^*(\lambda)} e^{s\alpha_1^*(\lambda)y + s\lambda\xi}\, d\lambda \quad (22)$$

$$\bar{\tau}_{yz2}(\xi, y, s) = \frac{1}{2\pi i}\int \frac{Q_+^*(\eta)\alpha_{2+}^*(\lambda)}{\alpha_{2+}^*(\eta)(\eta - \lambda)Q_+^*(\lambda)} e^{-s\alpha_2^*(\lambda)y + s\lambda\xi}\, d\lambda \quad (23)$$

$$\bar{\tau}_{xz2}(\xi, y, s) = \frac{1}{2\pi i}\int \frac{-\lambda Q_+^*(\eta)}{\alpha_{2+}^*(\eta)(\eta - \lambda)\alpha_{2-}^*(\lambda)Q_+^*(\lambda)} e^{-s\alpha_2^*(\lambda)y + s\lambda\xi}\, d\lambda \quad (24)$$

$$\bar{w}_2(\xi, y, s) = \frac{1}{2\pi i}\int \frac{-Q_+^*(\eta)}{s\mu_2\alpha_{2+}^*(\eta)(\eta - \lambda)\alpha_{2-}^*(\lambda)Q_+^*(\lambda)} e^{-s\alpha_2^*(\lambda)y + s\lambda\xi}\, d\lambda \quad (25)$$

The corresponding result of the dynamic stress intensity factor expressed in the Laplace transform domain is

$$\bar{K}(s) = \lim_{\xi\to 0}\sqrt{2\pi\xi}\,\bar{\tau}_{yz1}(\xi, 0, s) = \lim_{\xi\to 0}\sqrt{2\pi\xi}\,\bar{\tau}_{yz2}(\xi, 0, s) = \frac{-\sqrt{2(1 - b_2v)}\,Q_+^*(\eta)}{\sqrt{s}\,\alpha_{2+}^*(\eta)} \quad (26)$$

TRANSIENT FULL FIELD RESPONSE FOR STATIONARY CRACK

As shown in Fig. 1, a bimaterial medium is composed of two homogeneous, isotropic, and linearly elastic solids. Materials 1 and 2 occupy the lower and upper half-planes, respectively. Without loss of generality, we assume $b_1 > b_2$. A semiinfinite crack lying along the interface of the bimaterial is initially stress free and at rest. At time $t = 0$, a pair of equal and opposite concentrated antiplane dynamic loadings with magnitude p are applied at the crack faces with a distance h from the tip. The time dependence of the concentrated loading is represented by the Heaviside step function $H(t)$. Dynamic stress intensity factor will be induced as the incident cylindrical waves generated from point loadings arrive at the crack tip. In this section, it is assumed that the dynamic stress intensity factor is always less than the corresponding fracture toughness, i.e., the stationary crack will not propagate under this loading condition.

The incident field of the cylindrical wave generated by the concentrated loading expressed in the Laplace transform domain can be represented as follows:

$$\bar{\tau}_{yz}^i(x, 0, s) = \frac{1}{2\pi i}\int -pe^{s\lambda(x+h)}\, d\lambda \quad (27)$$

The applied traction on the crack faces as indicated in (27) has the functional form $e^{s\lambda x}$. Since the solutions of applying traction $e^{s\eta x}$ on crack faces have been solved in the previous section (by setting $v = 0$), the diffracted field generated from the stationary semiinfinite crack can be constructed by superimposing the incident wave traction that is equal to (27). When we combine (20), (21), (23), and (24) by setting $v = 0$ and (27), the stress fields for lower and upper planes expressed in the Laplace transform domain can be obtained as follows:

$$\bar{\tau}_{yz1}^d(x, y, s) = \frac{1}{2\pi i}\int_{\Gamma_{\eta 1}} -pe^{s\eta_1 h} \cdot \left\{\frac{1}{2\pi i}\int_{\Gamma_{\eta 2}} \frac{Q_+(\eta_1)\alpha_{2+}(\eta_2)}{\alpha_{2+}(\eta_1)(\eta_1 - \eta_2)Q_+(\eta_2)} e^{s\alpha_1(\eta_2)y + s\eta_2 x}\, d\eta_2\right\} d\eta_1$$
$$= \frac{p}{4\pi^2}\int_{\Gamma_{\eta 1}}\int_{\Gamma_{\eta 2}} T(\eta_1, \eta_2)e^{s\eta_1 h}e^{s\alpha_1(\eta_2)y + s\eta_2 x}\, d\eta_2\, d\eta_1 \quad (28)$$

$$\bar{\tau}^d_{xz1}(x, y, s) = \frac{p}{4\pi^2}\int_{\Gamma_{\eta 1}}\int_{\Gamma_{\eta 2}} \frac{\eta_2}{\alpha_1(\eta_2)} T(\eta_1, \eta_2)e^{s\eta_1 h}e^{s\alpha_1(\eta_2)y+s\eta_2 x}\, d\eta_2\, d\eta_1 \quad (29)$$

$$\bar{\tau}^d_{yz2}(x, y, s) = \frac{p}{4\pi^2}\int_{\Gamma_{\eta 1}}\int_{\Gamma_{\eta 2}} T(\eta_1, \eta_2)e^{s\eta_1 h}e^{-s\alpha_2(\eta_2)y+x\eta_2 x}\, d\eta_2\, d\eta_1 \quad (30)$$

$$\bar{\tau}^d_{xz2}(x, y, s) = \frac{-p}{4\pi^2}\int_{\Gamma_{\eta 1}}\int_{\Gamma_{\eta 2}} \frac{\eta_2}{\alpha_2(\eta_2)} T(\eta_1, \eta_2)e^{s\eta_1 h}e^{-s\alpha_2(\eta_2)y+s\eta_2 x}\, d\eta_2\, d\eta_1 \quad (31)$$

in which

$$T(\eta_1, \eta_2) = \frac{Q_+(\eta_1)\alpha_{2+}(\eta_2)}{\alpha_{2+}(\eta_1)(\eta_1 - \eta_2)Q_+(\eta_2)}$$

$$Q(\lambda) = Q_+(\lambda)Q_-(\lambda) = Q^*_+(\lambda)|_{v=0}Q^*_-(\lambda)|_{v=0}$$

$$\alpha_j(\lambda) = \alpha_{j+}(\lambda)\alpha_{j-}(\lambda) = \alpha^*_{j+}(\lambda)|_{v=0}\alpha^*_{j-}(\lambda)|_{v=0} = \sqrt{b_j + \lambda}\sqrt{b_j - \lambda}$$

Applying the Cagniard-de Hoop method of Laplace inversion, the solutions of stress field for the stationary crack in time domain are obtained as follows:

$$\tau^d_{yz1}(x, y, t) = \frac{-pt \sin\theta H(t - b_1 r)}{\pi r(t^2 - b_1^2 r^2)^{1/2}} + \frac{p}{\pi^2}\int_{b_2 h}^{t-b_1 R} \mathrm{Re}\left[T(\eta_1^+, \eta_{d1}^+)\frac{\partial\eta_1^+}{\partial t_1}\frac{\partial\eta_{d1}^+}{\partial t_{d1}}\right] dt_1 + \frac{p}{\pi^2}\int_{b_2 h}^{t} \mathrm{Re}\left[T(\eta_1^+, \eta_H^+)\frac{\partial\eta_1^+}{\partial t_1}\frac{\partial\eta_H^+}{\partial t_2}\right][H(t - t_1 - t_H) - H(t - t_1 - b_1 R)]H\left(\cos\varphi - \frac{b_2}{b_1}\right) \quad (32)$$

$$\tau^d_{xz1}(x, y, t) = \frac{-pt \cos\theta H(t - b_1 r)}{\pi r(t^2 - b_1^2 r^2)^{1/2}} + \frac{p}{\pi^2}\int_{b_2 h}^{t-b_1 R} \mathrm{Re}\left[\frac{\eta_{d1}^+}{\alpha_1(\eta_{d1}^+)}T(\eta_1^+, \eta_{d1}^+)\frac{\partial\eta_1^+}{\partial t_1}\frac{\partial\eta_{d1}^+}{\partial t_{d1}}\right] dt_1 + \frac{p}{\pi^2}\int_{b_2 h}^{t} \mathrm{Re}\left[\frac{\eta_H^+}{\alpha_1(\eta_H^+)}T(\eta_1^+, \eta_H^+)\frac{\partial\eta_1^+}{\partial t_1}\frac{\partial\eta_H^+}{\partial t_2}\right] dt_1[H(t - t_1 - t_H) - H(t - t_1 - b_1 R)]H\left(\cos\varphi - \frac{b_2}{b_1}\right) \quad (33)$$

$$\tau^d_{yz2}(x, y, t) = \frac{-pt \sin\theta H(t - b_2 r)}{\pi r(t^2 - b_2^2 r^2)^{1/2}} + \frac{p}{\pi^2}\int_{b_2 h}^{t-b_2 R} \mathrm{Re}\left[T(\eta_1, \eta_2)\frac{\partial\eta_1^+}{\partial t_1}\frac{\partial\eta_{d2}^+}{\partial t_{d2}}\right] dt_1 \quad (34)$$

$$\tau^d_{yz2}(x, y, t) = \frac{-pt \cos\theta H(t - b_2 r)}{\pi r(t^2 - b_2^2 r^2)^{1/2}} - \frac{p}{\pi^2}\int_{b_2 h}^{t-b_2 R} \mathrm{Re}\left[\frac{\eta_{d2}^+}{\alpha_2(\eta_{d2}^+)}T(\eta_1, \eta_2)\frac{\partial\eta_1^+}{\partial t_1}\frac{\partial\eta_{d2}^+}{\partial t_{d2}}\right] dt_1 \quad (35)$$

where

$$\eta_1^+ = \frac{t_1}{h} + i\varepsilon$$

$$\eta_{dj}^+ = \frac{-t_{dj}}{R}\cos\varphi + \frac{i\sin\varphi}{R}(t_{dj}^2 - b_j^2 R^2)^{1/2}; \quad j = 1, 2$$

$$\eta_H^+ = \frac{-t}{R}\cos\varphi + \frac{\sin\varphi}{R}(b_1^2 R^2 - t^2)^{1/2} + i\varepsilon$$

$$t_H = b_2 R|\cos\varphi| - R\sin\varphi\sqrt{b_1^2 - b_2^2}$$

$$r = [(x + h)^2 + y^2]^{1/2}; \quad \theta = \cos^{-1}\left(\frac{x + h}{r}\right)$$

$$R = (x^2 + y^2)^{1/2}; \quad \varphi = \cos^{-1}\left(\frac{x}{R}\right)$$

$$t_1 + t_{dj} = t, j = 1, 2$$

The first terms in (32)–(35) represent the incident cylindrical waves due to the applied loadings in two half-planes. The second terms in (32)–(35) represent the diffracted cylindrical waves, which are radiated from the stationary crack tip after the incident waves arrive at the tip. The last terms shown in (32) and (33) describe head waves generated by the mismatch bimaterial. In view of the integrals in (32) and (33), we can find that there is a singularity in the denominator through the material-dependent function Q_+. The corresponding physical meaning is that every integral in (32) and (33) includes contribution of two elastic waves. The first one is induced by the incident wave in the upper plane, and the second one is caused by the incident wave in the lower plane. The shear stress along the interface can be obtained by letting $y = 0$ in (34) and the result is

$$\tau^d_{yz2}(x, 0, t) = \frac{-p}{\pi}\int_{b_2 h}^{t-b_2 x} \frac{\sqrt{h(t - t_1 - b_2 x)}Q_+(-t_1/h)}{\sqrt{x(t_1 - b_2 h)}[(x + h)t_1 - th]Q_+\left(\dfrac{-t + t_1}{x}\right)} dt_1 \quad (36)$$

After the slower shear wave passes the observation point, i.e., $t > b_1(h + x)$, the integral in (36) can be evaluated analytically by contour integration in the complex t_1-plane. We choose a branch cut along $b_2 h < t_1 < t - b_2 x$ and take a close contour of infinitely large radius by using Cauchy's theorem. Finally, we can obtain the transient shear stress along the interface for $t > b_1(h + x)$ as follows:

$$\tau^d_{yz2}(x, 0, t) = \tau^s_{yz2} = \frac{p}{\pi(x + h)}\sqrt{\frac{h}{x}} \quad (37)$$

It is worth noting that the solution in (37) is independent of material constants. In addition, it indicates that the dynamic shear stress along the interface for a pair of concentrated forces applied at crack faces in a bimaterial is the same as the corresponding static value in a homogeneous medium after the slower shear wave passes the material point.

Making use of (27) and (26) (by setting $v = 0$), we can obtain the corresponding stress intensity factor expressed in the Laplace transform domain as follows:

$$\bar{K}^d(s) = \frac{1}{2\pi i}\int - pe^{s\lambda h}\left\{\frac{-\sqrt{2}Q_+(\lambda)}{\sqrt{s}\alpha_{2+}(\lambda)}\right\} d\lambda = \frac{1}{2\pi i}\int \frac{\sqrt{2}pQ_+(\lambda)}{\sqrt{s}\alpha_{2+}(\lambda)} e^{s\lambda h}\, d\lambda \quad (38)$$

The dynamic stress intensity factor of the stationary interface crack expressed in time domain will be

$$K^d(t) = \int_{b_2 h}^{t} \frac{\sqrt{2}p}{\pi^{3/2}\sqrt{t - \tau}} \mathrm{Im}\left\{\frac{Q_+(\eta_1^+)\dfrac{\partial\eta_1^+}{\partial t}}{\alpha_{2+}(\eta_1^+)}\right\}_{t=\tau} d\tau$$

$$= \sqrt{\frac{2}{h}}\frac{p}{\pi^{3/2}}\int_{b_2}^{t/h}\frac{\mathrm{Re}[Q_-(\eta)]}{\sqrt{t/h-\eta}\sqrt{\eta-b_2}}\,d\eta \quad (39)$$

After the slower shear wave generated from point loading arrives at the crack tip, i.e., $t > b_1 h$, the integral in (39) can be evaluated by using contour integration and yields

$$K^d(t) = K^s H(t - b_1 h) \quad (40)$$

in which

$$K^s = p\sqrt{\frac{2}{\pi h}} \quad (41)$$

is the corresponding static solution in a homogeneous medium.

As one would expect, the dynamic stress intensity factor in a bimaterial is the same as the corresponding static value K^s in a homogeneous medium after the slower shear wave passed the crack tip. If $b_1 = b_2 = b$, for the homogeneous case, (39) can be evaluated as before by letting $Q_+(\eta) = 1$ and yields

$$K_h^d(t) = p\sqrt{\frac{2}{\pi h}}H(t - bh) \quad (42)$$

The result expressed in (42) is the well-known solution of dynamic stress intensity factor for a semiinfinite crack in a homogeneous medium and subjected to a pair of concentrated forces on its faces. It is interesting to note that the dynamic stress intensity factor jumps from zero to the corresponding static value after the incident cylindrical wave generated from the loading point arrives at the crack tip.

DYNAMIC STRESS INTENSITY FACTOR OF A PROPAGATING CRACK

When the dynamic stress intensity factor of a crack reaches its fracture toughness, the crack will start to propagate and release energy. From the previous section, we know that the dynamic stress intensity factor will reach its corresponding static value immediately after the incident wave with slower shear wave speed passed the stationary crack tip. This means that a stationary interface crack subjected to a pair of concentrated forces on its faces can propagate only at time $b_2 h \le t \le b_1 h$. In this section, we consider the same interface crack as discussed in the previous section. At $t = 0$, a pair of concentrated loadings act on the stationary crack faces and we assume that at time $t = t_f$ the dynamic stress intensity factor reaches its critical value and the crack starts to propagate with a constant subsonic speed v ($v < b_1^{-1} < b_2^{-1}$) along $y = 0$ as shown in Fig. 2. For convenience, we consider two special propagating cases. The first one is that the crack starts to propagate at once when the incident cylindrical wave with higher speed arrives at the stationary crack tip (i.e., $t_f = b_2 h$). The second one is that the crack starts to propagate when the slower incident wave arrives at the crack tip (i.e., $t_f = b_1 h$). Since the stress intensity factor is the key parameter in characterizing dynamic crack growth, we will focus our attention on the determination of the dynamic stress intensity factor.

Case 1: Delay Time $t_f = b_2 h$

In this case, the crack starts to propagate immediately as the incident cylindrical wave generated from dynamic point loading in medium 2 passes the stationary crack tip. The applied concentrated loading on the interfacial crack faces written in the Laplace transform domain for the moving coordinate system will have the following form:

$$\bar{\tau}_{yz}^{l}(\xi_1, 0, s) = \frac{1}{2\pi i}\int\frac{pd}{\lambda - d}e^{sh(1-b_2 v)\lambda + s\lambda\xi_1}\,d\lambda \quad (43)$$

in which $d = 1/v$ = slowness of the crack velocity; and $\xi_1 = x - v(t - b_2 h)$. The applied traction on the crack faces as expressed in (43), has the functional form $e^{s\lambda\xi_1}$. Since the Laplace transform solutions of applying traction $e^{s\eta\xi}$ on crack faces have been solved previously, the dynamic stress intensity factor can be constructed by superimposing the fundamental solutions (26) and the stress distribution in (43). The result of dynamic stress intensity factor expressed in the Laplace transform domain will be

$$\bar{K}^{v,1}(s) = \frac{1}{2\pi i}\int\frac{pd}{\lambda - d}e^{sh(1-b_2 v)\lambda}\left\{\frac{-\sqrt{2(1 - b_2 v)}Q_{\ddagger}^{*}(\lambda)}{\sqrt{s}\alpha_{2+}^{*}(\lambda)}\right\}d\lambda$$

$$= \frac{-1}{2\pi i}\int\frac{pd\sqrt{2(1 - b_2 v)}Q_{\ddagger}^{*}(\lambda)}{\sqrt{s}(\lambda - d)\alpha_{2+}^{*}(\lambda)}e^{sh(1-b_2 v)\lambda}\,d\lambda \quad (44)$$

Inverting the Laplace transform of (44), the dynamic stress intensity factor for a propagating interface crack at an unbounded bimaterial medium in time domain can be obtained as follows:

$$K^{v,1}(t) = \frac{pd\sqrt{2h(1 - b_2 v)}}{\pi^{3/2}}\int_{b_2 h}^{t}\frac{\mathrm{Re}\left[Q_{\ddagger}^{*}\left(\frac{-\tau}{h(1 - b_2 v)}\right)\right]}{\sqrt{t - \tau}\sqrt{\tau - b_2 h}[\tau + h(d - b_2)]}\,d\tau \quad (45)$$

If $t > t_c$, which is the time that the slower incident wave in material 1 catches up with the propagating crack tip, then the integration in (45) can be carried out and the final result is

$$K^{v,1}(t) = p\sqrt{\frac{2}{\pi[v(t - b_2 h) + h]}}Q_{\ddagger}^{*}(d)(1 - b_2 v)^{1/2}H(t - t_c) \quad (46)$$

where

$$t_c = \frac{b_1 h(1 - b_2 v)}{1 - b_1 v}$$

The expression for $K^{v,1}(t)$ in (46) has the interesting form of the product of a function $Q_{\ddagger}^{*}(d)(1 - b_2 v)^{1/2}$ and the corresponding static stress intensity factor K^s in (41) for applying a pair of concentrated loadings at crack faces with a distance $v(t - b_2 h) + h$ from the crack tip. The value $Q_{\ddagger}^{*}(d)(1 - b_2 v)^{1/2}$ and the corresponding static stress intensity factor K^s in (41) for applying a pair of concentrated loadings at crack faces with a distance $v(t - b_2 h) + h$ from the crack tip. The value $Q_{\ddagger}^{*}(d)(1 - b_2 v)^{1/2}$ is an universal function, which depends only on crack speed and material properties. For any combination of material constants, the universal function always decreases monotonically from one at $v = 0$ to zero when the crack speed reaches the smaller shear wave speed. If $b_1 = b_2$ and $\mu_1 = \mu_2$, we have $Q_{\ddagger}^{*}(d) = 1$ and the solution in (46) for the propagating interface crack in a bimaterial can be reduced to that obtained by Ma and Ing (1995) in a homogeneous medium.

Case 2: Delay Time $t_f = b_1 h$

For the second case, the fracture toughness of the bimaterial is assumed to be equal to the corresponding static value in (41). Hence, the interface crack starts to propagate at time $t = b_1 h$ when the slower shear wave arrives at the crack tip. For $b_2 h < t < b_1 h$, the crack is still at rest and the dynamic stress intensity factor for this stationary crack can be calculated by using the formulation in (39). For $t > b_1 h$, the crack begins to propagate along the interface with a constant velocity v, which is less than the smaller shear wave speed of these two materials. To obtain the dynamic stress intensity factor after propagation, we also express the applied concentrated loading in the Laplace transform domain for the moving coordinate system as follows:

$$\bar{\tau}^{i}_{yz}(\xi, 0, s) = \frac{1}{2\pi i}\int \frac{pd}{\lambda - d} e^{sh(1-b_1v)\lambda + s\lambda\xi_2}\, d\lambda \quad (47)$$

in which $\xi_2 = x - v(t - b_1h)$.

Using the fundamental solution in (26) and the stress distribution in (47), the dynamic stress intensity factor for the propagating interface crack expressed in the Laplace transform domain can be constructed as follows:

$$\bar{K}^{v,2}(s) = \frac{1}{2\pi i}\int \frac{pd}{\lambda - d} e^{sh(1-b_1v)\lambda}\left\{\frac{-\sqrt{2(1-b_2v)}Q^{*}_{\ddagger}(\lambda)}{\sqrt{s}\,\alpha^{*}_{2+}(\lambda)}\right\} d\lambda$$

$$= \frac{-1}{2\pi i}\int \frac{pd\sqrt{2(1-b_2v)}Q^{*}_{\ddagger}(\lambda)}{\sqrt{s}(\lambda - d)\alpha^{*}_{2+}(\lambda)} e^{sh(1-b_1v)\lambda}\, d\lambda \quad (48)$$

The dynamic stress intensity factor expressed in time domain can be obtained by inverting the Laplace transform of (48). The result is

$$K^{v,2}(t) = \frac{pd\sqrt{2h(1-b_2v)}}{\pi^{3/2}} \cdot \int_{b_1h}^{t} \frac{\mathrm{Re}\left[Q^{*}_{\ddagger}\left(\frac{-\tau}{h(1-b_1v)}\right)\right]}{\sqrt{t-\tau}\sqrt{(1-b_2v)\tau/(1-b_1v) - b_2h[\tau + h(d-b_1)]}}\, d\tau \quad (49)$$

The integral in (49) can be evaluated by using contour integration in the complex τ-plane and the final result is obtained in an analytical form as follows:

$$K^{v,2}(t) = p\sqrt{\frac{2}{\pi[v(t-b_1h) + h]}}\, Q^{*}_{\ddagger}(d)(1 - b_2v)^{1/2}H(t - b_1h) \quad (50)$$

The expression for $K^{v,2}(t)$ from (50) is similar to $K^{v,1}(t)$ in (46) and has an interesting form of the product of a function $Q^{*}_{\ddagger}(d)(1 - b_2v)^{1/2}$ and the corresponding static stress intensity factor K^s in (41) with a distance $v(t - b_1h) + h$ from the crack tip.

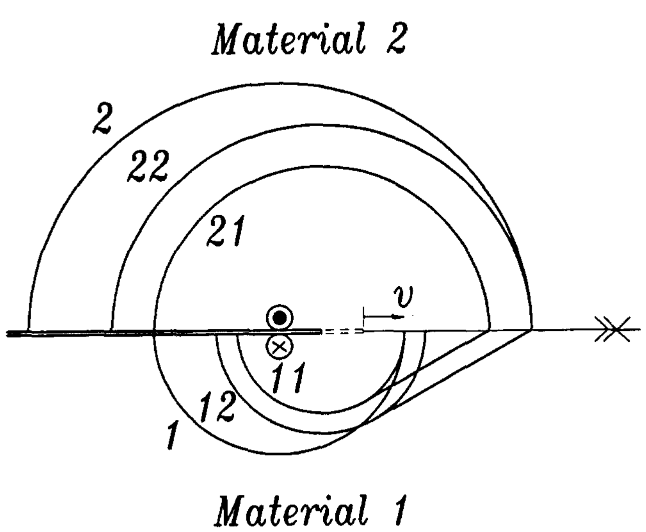

FIG. 3. Wave Fronts of Incident and Diffracted Waves of Case 2 Situation for $t > t_r$

NUMERICAL RESULTS

In the previous sections, we have derived the transient solution of dynamic stress intensity factors for a propagating interface crack subjected to a pair of concentrated loadings on crack faces. The complete wave fronts for the propagating crack are plotted in Fig. 3. In this figure, 1 and 2 indicate the waves produced by the applied concentrated forces in the material 1 and 2, respectively. The diffracted waves ij ($i, j = 1, 2$), denote the waves in medium i resulting from the diffraction of a disturbance induced by the applied loading in medium j. The stress along the interface of a stationary interface crack is calculated and plotted in Fig. 4. It is shown in Fig. 4 that the shear stresses jump to the same static value at $t = 2b_1h$ for all the material combination. Figs. 5 and 6 show the dimensionless stress intensity factors K^d/K^s of the stationary crack versus dimensionless time for various values of μ_1/μ_2 and b_1/b_2, respectively. In these figures, it can be seen that the dynamic stress intensity factors are almost constants for $t < b_1h$ and will

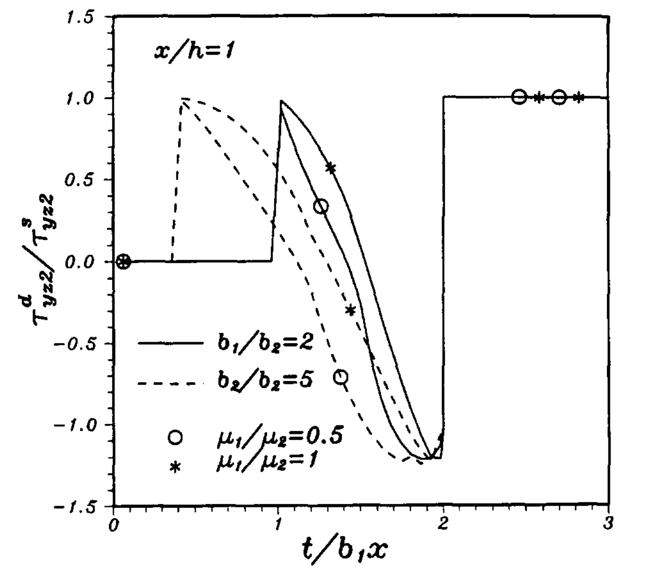

FIG. 4. Transient Stress Field along Interface of a Stationary Crack

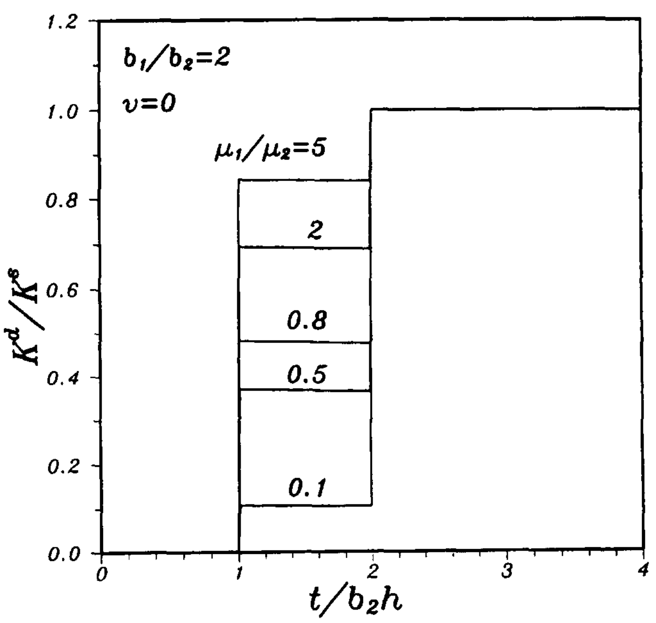

FIG. 5. Stress Intensity Factors of a Stationary Interface Crack for Different Values of μ_1/μ_2

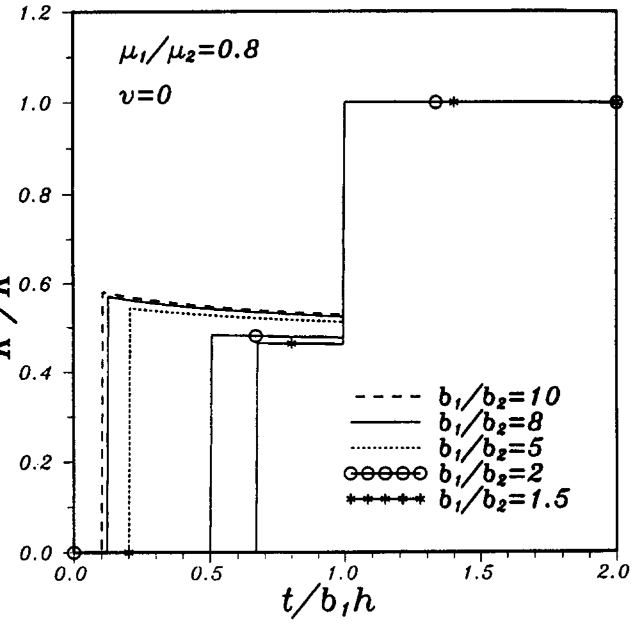

FIG. 6. Stress Intensity Factors of a Stationary Interface Crack for Different Values of b_1/b_2

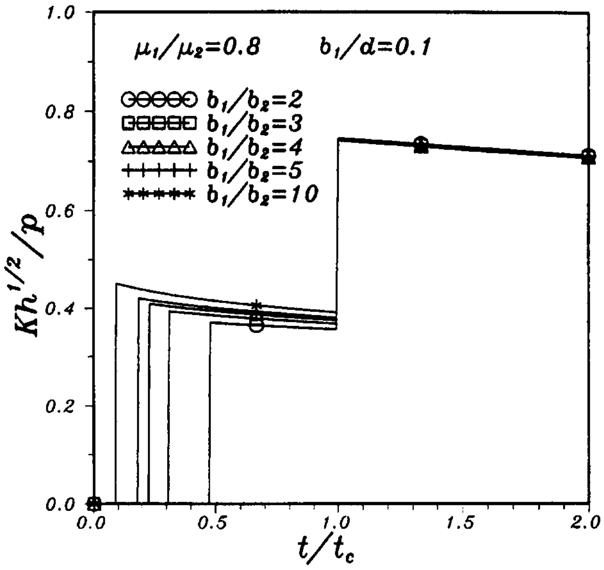

FIG. 8. Stress Intensity Factors of a Propagating Interface Crack in Case 1 for Different Values of b_1/b_2

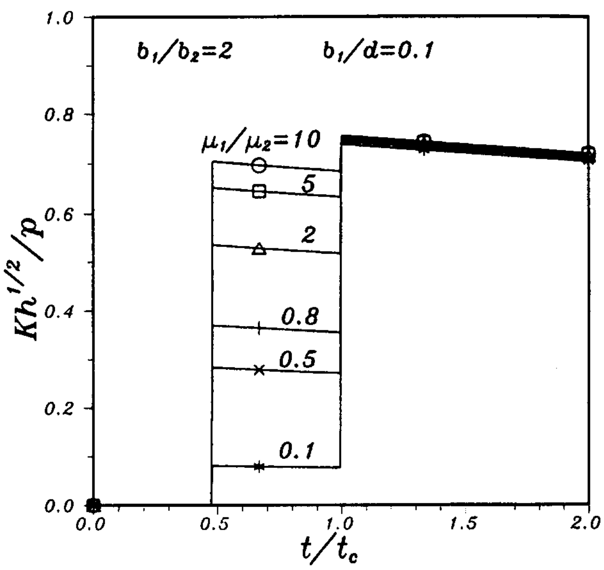

FIG. 7. Stress Intensity Factors of a Propagating Interface Crack in Case 1 for Different Values of μ_1/μ_2

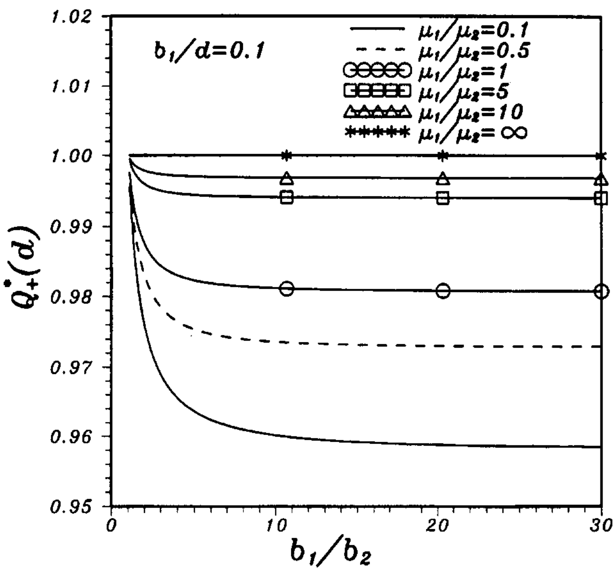

FIG. 9. Values of $Q_+^*(d)$ for $b_1/d = 0.1$ under Diffferent Material Combination

jump to the corresponding static values immediately at $t = b_1h$. It also indicates in Fig. 5 that for $t < b_1h$, the larger μ_1/μ_2 is, the closer the dynamic stress intensity factor to the corresponding static value.

Figs. 7 and 8 shows the dimensionless stress intensity factors $Kh^{1/2}/p$ for the case 1 situation of the propagating interface crack versus dimensionless time t/t_c for various values of μ_1/μ_2 and b_1/b_2, respectively. It is of interest to see that for $t > t_c$ the dynamic stress intensity factors are almost equal for small crack velocity $v = 0.1c_{s1}$ under different material combination. From (46), we know that the dynamic stress intensity factor for $t > t_c$ is equal to the homogeneous static solution times a universal function $Q_+^*(d)(1 - b_2v)^{1/2}$. Figs. 9 and 10 plot the values of $Q_+^*(d)$ for different material combination for $b_1/d = 0.1$ and $b_1/d = 0.2$, respectively. It is interesting to point out that for small crack velocity, the values of $Q_+^*(d)$ are approximately equal to one for all kinds of material combination. Consequently, for lower crack velocity ($b_1/d \leq 0.1$), the dynamic stress intensity factor in (46) can be approximated as follows:

$$K^{u,1}(t) = p\sqrt{\frac{2}{\pi[v(t-b_2h)+h]}}\,Q_+^*(d)(1-b_2v)^{1/2}H(t-t_c)$$
$$\approx p\sqrt{\frac{2}{\pi[v(t-b_2h)+h]}}\,(1-b_2v)^{1/2}H(t-t_c) \tag{51}$$

The dynamic stress intensity factors for different values of b_1/d under constant b_1/b_2 and μ_1/μ_2 are shown in Fig. 11. It can be found that the higher crack velocity, the smaller dynamic stress intensity factor. Hence, the stationary crack has the largest dynamic stress intensity factor among different running cases.

Figs. 12–14 show the dynamic stress intensity factors for the case 2 situation of the propagating interface crack. It also can be seen in Figs. 12 and 13 that dynamic stress intensity factors are almost equal for $t > b_1h$. It means that for lower

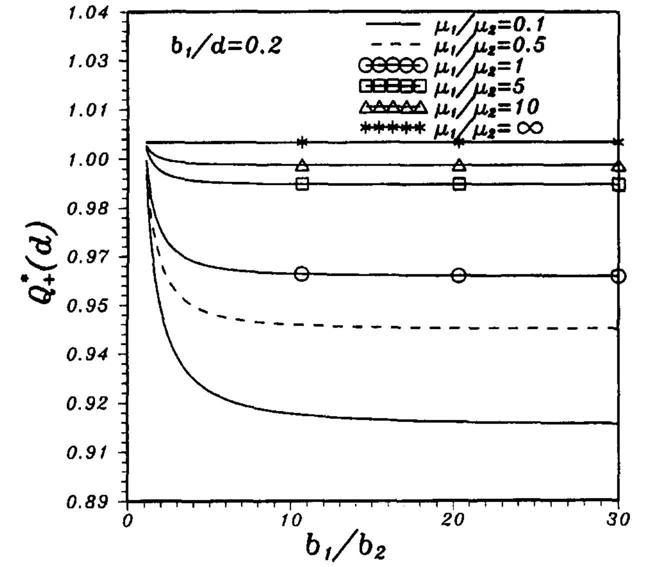

FIG. 10. Values of $Q_+^*(d)$ for $b_1/d = 0.2$ under Diffferent Material Combination

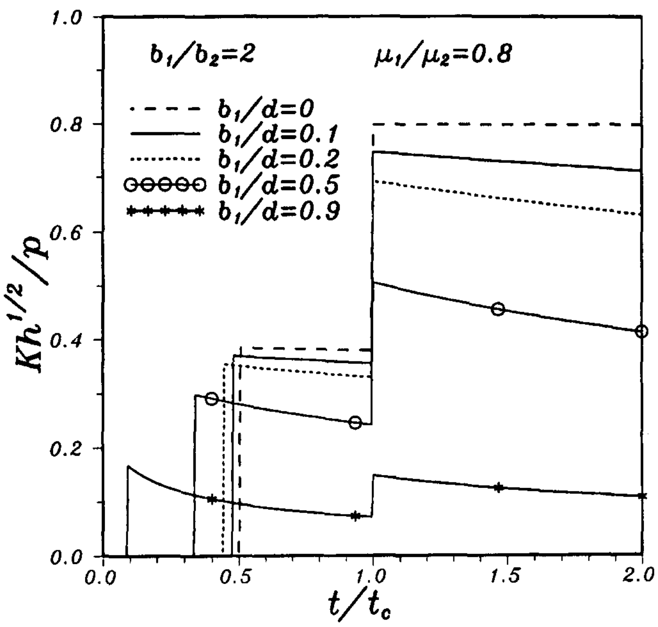

FIG. 11. Stress Intensity Factors of a Propagating Interface Crack in Case 1 for Different Values of Crack Velocity v

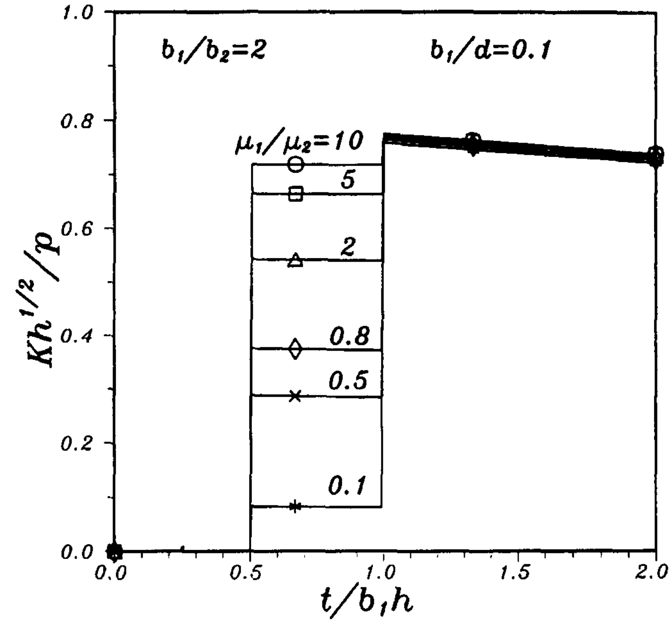

FIG. 12. Stress Intensity Factors of a Propagating Interface Crack in Case 2 for Different Values of μ_1/μ_2

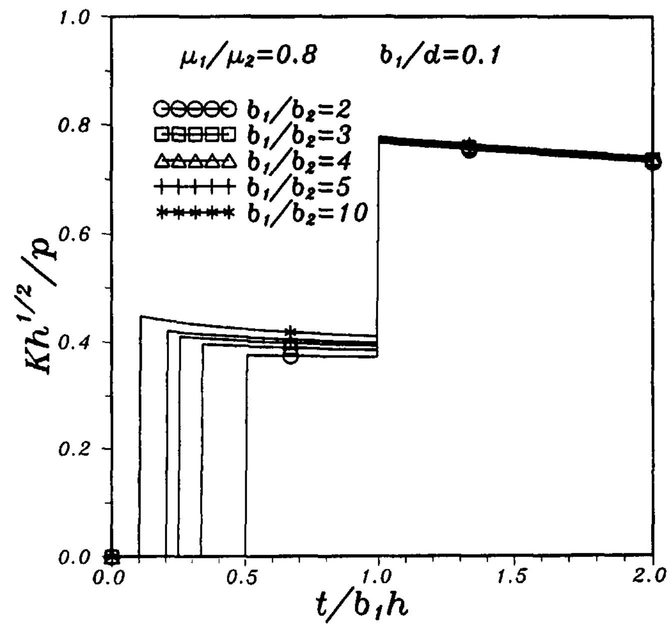

FIG. 13. Stress Intensity Factors of a Propagating Interface Crack in Case 2 for Different Values of b_1/b_2

crack velocity, the dynamic stress intensity factor is nearly independent of material parameters after the slower shear wave passed the propagating crack tip. So we have

$$K^{v,2}(t) = p\sqrt{\frac{2}{\pi[v(t - b_1h) + h]}}\,Q_+^*(d)(1 - b_2v)^{1/2}H(t - b_1h)$$

$$\approx p\sqrt{\frac{2}{\pi[v(t - b_1h) + h]}}\,(1 - b_2v)^{1/2}H(t - b_1h) \tag{52}$$

CONCLUSIONS

The mechanical behavior of many newly developed multiphase materials are mainly controlled by the response of the interface. Many researchers have devoted to investigate the field of dynamic debonding along a bimaterial interface. The transient problem of a propagating interface crack in an infinite bimaterial is considered in this study. The equivalent steady-state problem has been studied by many investigators in the past 20 years, but the transient solution is still very few. In this paper, the transient response of a propagating interface crack in an infinite medium subjected to a pair of concentrated loadings applied on its faces is obtained. These transient solutions are obtained by superposition of a proposed fundamental solution in the Laplace transform domain. The proposed fundamental solution is an exponentially distributed traction applied on the propagating crack faces. This fundamental solution is successfully applied towards solving this transient problem and is demonstrated as an efficient methodology to solve other similar problems.

Some interesting and important results are obtained in this study. We find that the stress along the interface (or the dy-

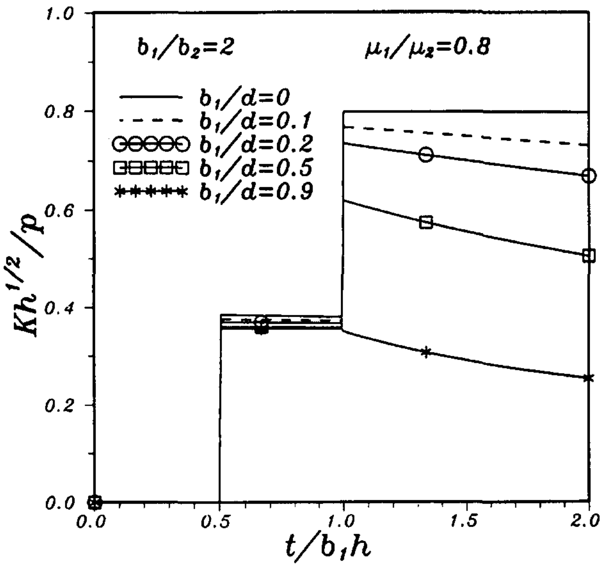

FIG. 14. Stress Intensity Factors of a Propagating Interface Crack in Case 2 for Different Values of Crack Velocity *v*

namic stress intensity factor) of a stationary bimaterial crack is the same as the corresponding static value in a homogeneous medium after all the transient waves passes the material point (or the crack tip). Two propagating cases are considered and the results have many interesting phenomena. It is worth noting that the expression of dynamic stress intensity factor of a propagating interface crack has the form of the product of a universal function and the corresponding static stress intensity factor after the slower incident shear wave arrives at the propagating crack tip. Moreover, the dynamic stress intensity factor of a propagating interface crack is approximately equal to that in a homogeneous medium for low crack propagating velocity ($v \leq 0.1c_{s1}$).

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