Moving of a nonhomogeneous, porous floc normal to a rigid plate

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Abstract

The boundary effect on the moving of a porous, nonhomogeneous, spherical floc normal to a rigid plate is analyzed theoretically for the case of low to medium Reynolds number. In particular, the drag force acting on the floc under various conditions is evaluated. A two-layer structure is adopted to simulate the nonhomogeneous nature of a floc. We show that if a floc is away from the plate, the streamlines surrounding the floc are distorted, but the degree of distortion becomes less significant if the floc is near the plate. The modified drag coefficient of a porous floc is orders of magnitude smaller than that of the corresponding rigid particle. For a fixed volume-averaged permeability, the effect of the presence of the plate on the behavior of a nonhomogeneous floc is more significant than that of a homogeneous floc, and this effect depends largely on the structure of a floc. The nonhomogeneous structure of a floc leads to a positive deviation from a Stokes-law-like correlation in the modified drag coefficient, and the smaller the volume-averaged permeability of a floc the greater the deviation. The presence of the plate has the effect of reducing this deviation. The nonhomogeneous structure of a floc on its modified drag coefficient is pronounced when it is close to a boundary.

1. Introduction

The problem that entities moving in a fluid under the influence of the presence of a boundary can be important to processes of practical importance. Typical examples include the settling of flocs formed in water and wastewater treatment and the filtration of particles from dispersions. In the former, the moving of particles will be influenced by the bottom surface of a settling tank, especially when particles are near the surface. In the latter, the interaction between the surface of a filtration medium and particles cannot be overlooked for apparent reasons. In these cases, entities are moving normal to a surface, and it is expected that the behavior of the former depends largely on the entity–surface separation distance. Happel and Brenner [1] analyzed the movement of a rigid particle normal to a rigid plate, along the centerline of a circular tube, and the sedimentation of an ensemble of rigid spheres. For the case where the Reynolds number is in the creeping flow regime, analytical results for the flow field and the drag force acting on a particle were de-

rived. Adopting bipolar coordinates, Brenner [2] was able to derive exact solutions of the Navier–Stokes equations without a convection term for the cases of a rigid sphere moving normal to a rigid plate, away from a free surface, and the combination of these two cases. Cox and Brenner [3] used a singular perturbation approach to calculate the hydrodynamic force acting on a rigid sphere moving perpendicular to a rigid plane. The inertial effect was taken into account, but the result obtained is limited to small Reynolds numbers. As the Reynolds number gets large, the convective motion of the fluid becomes significant, wakes may form to the rear of an entity, and the flow field surrounding it can be complicated. Wham et al. [4] analyzed the movement of a rigid sphere along the centerline of a circular tube for Reynolds number up to 100. It was concluded that both the Reynolds number for the onset of wake formation and its linear size depend largely on the particle–wall distance. Chhabra [5] discussed the terminal velocity of a rigid sphere in a cylindrical tube filled with a Newtonian fluid for Reynolds numbers up to 200. The free settling of particles of various types in a cylinder was examined experimentally by Chhabra [6] for the case where the Reynolds number is smaller than 7. The free settling of a swarm of particles in a cylinder for the case

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where the Reynolds number is in the inertial-flow regime was investigated experimentally by Di Felice et al. [7].

Apart from the presence of a boundary, the structure of a particle can also be important to its behavior in a medium. Alder [8], for example, investigated the aggregation/disaggregation behavior of a permeable spherical floc in both a uniform and a simple shear flow fields, and concluded that the behaviors of floc in these two flow fields are quite different. Based on Darcy's law and a Beaver–Joseph–Saffman slip–flow boundary condition, the movement of a permeable spherical floc toward a rigid wall was investigated by Payatakes and co-workers [9,10] for the case where the Reynolds number is in the creeping flow region. It was concluded that the drag force experienced by a porous particle is much smaller than that experienced by the corresponding rigid particle, especially when a particle is close to a wall. Neale et al. [11] analyzed the movement of a swarm of uniformly distributed permeable spherical flocs. Both Darcy's law and the Darcy–Brinkman model were employed to simulate the flow in a porous medium, and they were able to derive an analytical relationship between the drag force and the porosity. The analysis of Neale et al. [11] was extended by Smith [12] to the case of randomly distributed rigid spherical particles for the case where the Reynolds number is small. Veerapaneni and Wiesner [13] proposed a multilayer floc model to simulate the behavior of a fractal aggregate with radially varying permeability in an infinite medium. Hsu and Hsieh [14,15] investigated the moving of a porous, nonhomogeneous, spherical floc in an infinite medium for the case of low to medium Reynolds number. Wu and Lee studied the moving of both a porous, homogeneous spherical floc [16] and a porous, homogeneous spheroidal floc [17] in an infinite medium. The influence of the presence of a boundary was also examined by considering the moving of a porous, homogeneous spherical floc towards a rigid plate [18], and along the axis of a cylinder [19].

As pointed out by Li and Ganczarczyk [20], the structure of a floc is usually of a complicated nature. Their result implies that using a homogeneous model to simulate floc behavior can be unrealistic for problems of practical significance. This difficulty can be circumvented by assuming that the density or the permeability of a floc is position-dependent. Veerapaneni and Wiesner [13], for example, proposed a multilayer floc model in which a floc is divided into several layers, each having a different permeability. A two-layer model was used by Hsu and Hsieh [14,15] to simulate a porous, nonhomogeneous floc. They showed that, for the same average permeability, the behavior of a nonhomogeneous floc is appreciably different from that of a homogeneous floc both quantitatively and qualitatively. In this study, we consider the movement of a porous, nonhomogeneous spherical floc normal to a rigid plate for the case of low to medium Reynolds numbers. This extends the analysis of Wu and Lee [18] in that the effect of the structure of a floc on its behavior is taken into account. The effects of the key para-
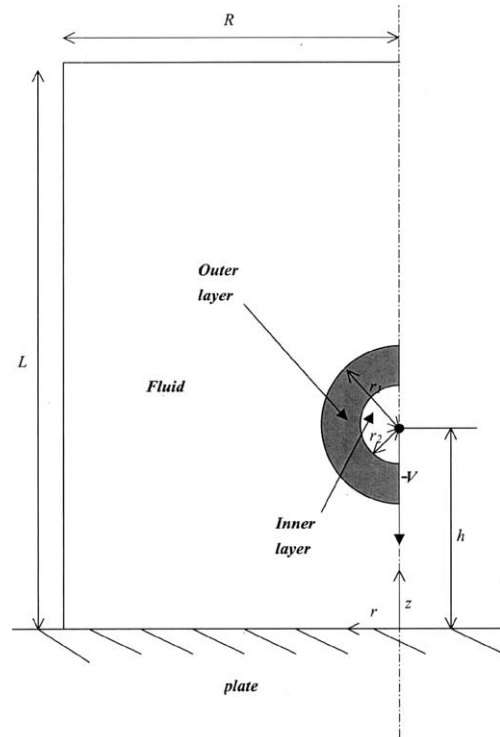

Fig. 1. Schematic representation of the problem considered.

meters of the system under consideration on the drag force acting on a floc are investigated.

2. Theory

Let us consider the problem illustrated in Fig. 1, where a spherical floc of radius r_1 is located above a rigid plate, h is the distance between the center of the floc and the plate. The floc has a two-layer structure with r_2 being the radius of its inner layer. It should be pointed out that although this two-layer model is an idealized one, it is capable of portraying roughly the qualitative nature of a real floc by adjusting the relative magnitudes of the permeabilities of the inner and the outer layers [20,21]. The cylindrical coordinates (r, θ, z) are used with their origin located at the center of the floc; r, θ, and z are the radial, the azimuthal, and the axial coordinates, respectively. Due to the symmetric nature of the present problem only the (r, z) coordinates need to be considered. The floc moves with velocity $-V$ normal to the plate. For convenience, however, we assume that the floc remains fixed and the surrounding fluid flows with a bulk velocity V.

For an incompressible fluid with constant physical properties, the flow field outside the floc is described by the Navier–Stokes equation and the equation of continuity [22]

$$\boldsymbol{u}_f \cdot \nabla \boldsymbol{u}_f = -\nabla P + \frac{2}{Re} \nabla^2 \boldsymbol{u}_f \text{ (outside floc)}, \tag{1}$$

$$\nabla \cdot \boldsymbol{u}_f = 0 \text{ (outside floc)}, \tag{2}$$

where $Re = 2\rho r_1 V/\mu$ is the Reynolds number, ρ, μ, and V being the density of the fluid, the viscosity of the fluid,

and the magnitude of V, respectively. P is the dimensionless pressure scaled by ρV^2, ∇ is the dimensionless gradient operator scaled by $1/r_1$, and $\boldsymbol{u}_f$ is the dimensionless fluid velocity scaled by V. For the flow field inside the floc, we assume that the Darcy–Brinkman model [11] is applicable; that is,

$$\boldsymbol{u}_i + \frac{Re}{2\beta_i^2} \nabla P = \nabla^2 \boldsymbol{u}_i \text{ (inside floc)}, \tag{3}$$

$$\nabla \cdot \boldsymbol{u}_i = 0 \text{ (inside floc)}. \tag{4}$$

Here, the subscript i is a region index ($i = 1$, outer layer, 2, inner layer) and $\beta_i = r_1/\sqrt{k_i}$ is the scaled radius of a floc, k_i and $\boldsymbol{u}_i$ being respectively the permeability and the dimensionless velocity of the fluid in region i scaled by V. We assume that the plate is nonslip, the fluid far away from the plate is stagnant, and both the velocity and its gradient are continuous on the outer surface of the outer layer and on the outer surface of the inner layer. These assumptions lead to the boundary conditions

$$u_z = 1, \quad r \to \infty, \tag{5}$$

$$u_z = 1, \quad z = 0, \tag{6}$$

$$\boldsymbol{u}_f = \boldsymbol{u}_1 \quad \text{and} \quad \nabla \boldsymbol{u}_f = \nabla \boldsymbol{u}_1$$
$$\text{(on the outer surface of outer layer)}, \tag{7}$$

$$\boldsymbol{u}_1 = \boldsymbol{u}_2 \quad \text{and} \quad \nabla \boldsymbol{u}_1 = \nabla \boldsymbol{u}_2$$
$$\text{(on the outer surface of inner layer)}, \tag{8}$$

where $\boldsymbol{u}_z$ represents the dimensionless fluid velocity in the z-direction and u_z is its magnitude. Here, we assume that the viscosity of the liquid in the bulk phase and that inside a floc are the same. If this is not the case, then it can be taken into account of by associating a proportional constant to either side of the second equality of both Eqs. (7) and (8). The symmetric nature of the present problem requires that

$$\frac{\partial \boldsymbol{u}_f}{\partial r} = \frac{\partial \boldsymbol{u}_1}{\partial r} = \frac{\partial \boldsymbol{u}_2}{\partial r} = 0, \quad r = 0. \tag{9}$$

Solving Eqs. (1)–(4) subject to Eqs. (5)–(9) yields the flow field of the problem under consideration, which is then used to evaluate the hydrodynamic drag force acting on the floc, F. Similar to the treatment of Neale et al. [11], we define

$$F = \left(\frac{1}{2}\rho V^2\right)\left(\pi r_1^2\right) C_D \Omega, \tag{10}$$

C_D being the drag coefficient, and Ω is a correction factor. Note that $\Omega \leqslant 1$, and the equality applies when the floc is rigid. It can be shown that [22] for a rigid sphere with $Re \ll 1$,
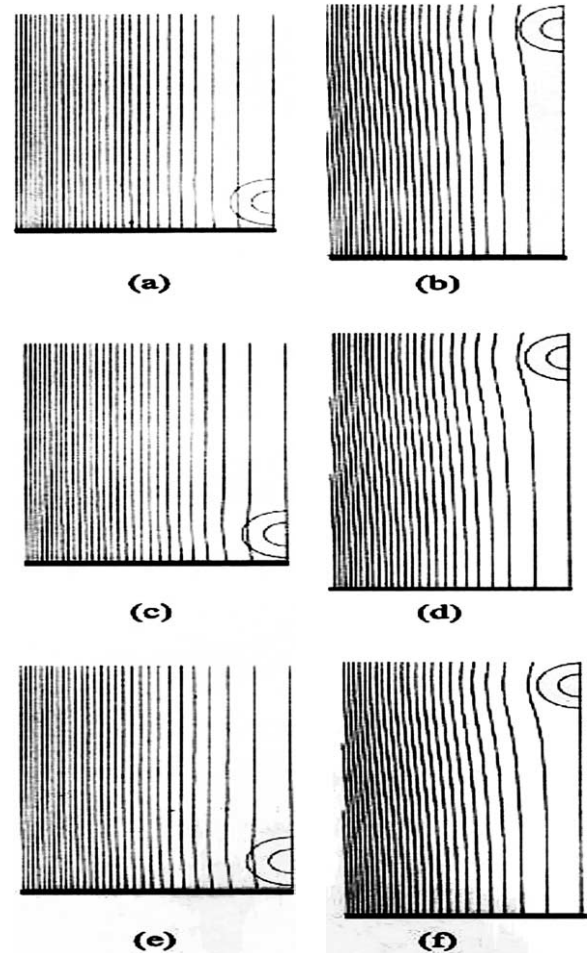
$$C_D = \frac{24}{Re}. \tag{11}$$

Fig. 2. Streamlines at various k_1/k_2 for the case when $Re = 0.1$ and $\bar{\beta} = 2$. (a) $k_1/k_2 = 1$ and $h/r_1 = 1.13$, (b) $k_1/k_2 = 1$ and $h/r_1 = 10.07$, (c) $k_1/k_2 = 10$ and $h/r_1 = 1.13$, (d) $k_1/k_2 = 10$ and $h/r_1 = 10.07$, (e) $k_1/k_2 = 1/10$ and $h/r_1 = 1.13$, (f) $k_1/k_2 = 1/10$ and $h/r_1 = 10.07$. Key: $r_1 = 0.12$ cm, $\rho = 1$ g/cm^3, and $\mu = 0.01$ poise.

3. Results and discussions

The governing equations and the associated boundary conditions are solved numerically by FIDAP 8.6, a commercial program based on a finite element scheme. The computational domain $(R, L - h) = (30 \times r_1, 70 \times r_1)$ is chosen, and the numbers of elements adopted for the fluid domain and the outer and inner layers of a floc are respectively 18,800, 35, and 30. The applicability of the numerical scheme adopted is justified by examining the slow motion of a solid sphere moving toward a wall considered by Happel and Brenner [1], where an analytic result is available. The performance of the numerical scheme is satisfactory, with the maximal percentage deviation in $C_D \Omega$ less than 0.1% for all the cases illustrated by them.

The influence of the relative magnitudes of the permeabilities of the inner and the outer layer of a floc and the floc–plate distance on the flow field at two different levels of Reynolds number are presented in Figs. 2 and 3.
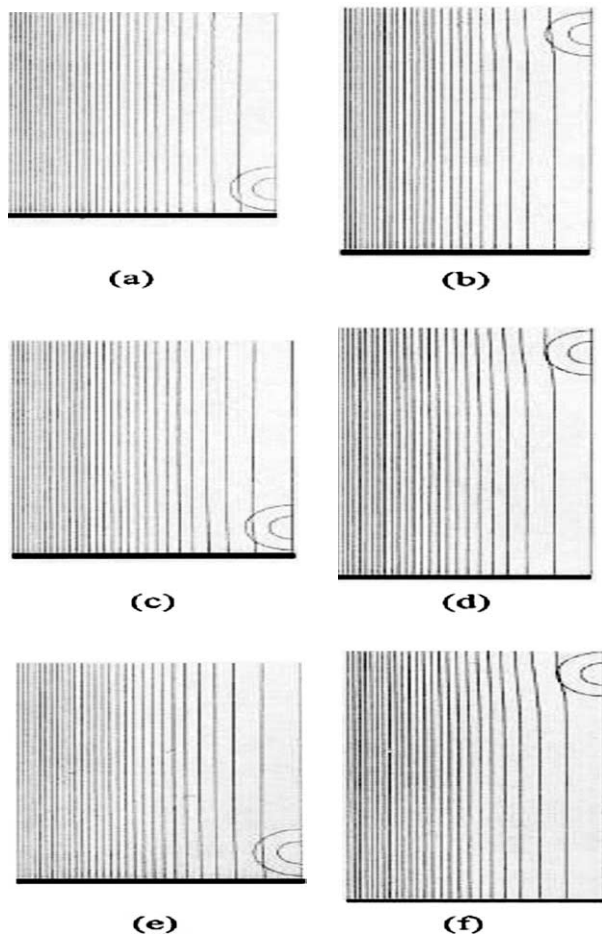
Fig. 3. Streamlines at various k_1/k_2 for the case when $Re = 40$ and $\bar{\beta} = 2$. (a) $k_1/k_2 = 1$ and $h/r_1 = 1.13$, (b) $k_1/k_2 = 1$ and $h/r_1 = 10.07$, (c) $k_1/k_2 = 10$ and $h/r_1 = 1.13$, (d) $k_1/k_2 = 10$ and $h/r_1 = 10.07$, (e) $k_1/k_2 = 1/10$ and $h/r_1 = 1.13$, (f) $k_1/k_2 = 1/10$ and $h/r_1 = 10.07$. Key: same as Fig. 2.

Fig. 4. Variation of $C_D \Omega$ as a function of h/r_1 for various k_1/k_2 at different Re for the case when $\bar{\beta} = 1$. Curve: 1, $k_1/k_2 = 0.1$; 2, $k_1/k_2 = 10$; 3, $k_1/k_2 = 0.2$; 4, $k_1/k_2 = 5$; 5, $k_1/k_2 = 1$. (a) $Re = 0.1$, (b) $Re = 40$. Key: same as Fig. 2.

For convenience, we define $\bar{k} = \sum_{i=1}^{2} V_i k_i / \sum_{i=1}^{2} V_i$ and $\bar{\beta} = r_1/\sqrt{\bar{k}}$. The former represents the volume-averaged permeability of a floc, and the latter is its dimensionless radius. Because both r_1 and $\bar{\beta}$ are fixed in Figs. 2 and 3, so is $\bar{k}$. Figs. 2 and 3 reveal that, due to the presence of a floc, the flow field is distorted, and the degree of distortion is more serious when the floc is away from the plate. This is true regardless of the structure of the floc and the level of Re. A comparison between Figs. 2 and 3 indicates that if a floc is close to a plate, regardless of the level of Re, the streamlines go through the floc and look almost the same. Also, the streamlines in the front region of the floc are symmetric to those in its rear region, which implies that when the plate is close to the floc, the fluid behavior is dominated by the viscous term in the Navier–Stokes equation. On the other hand, if a floc is away from a plate, the streamlines are symmetric about the floc at low Re, but becomes asymmetric at high Re, which implies that when the plate is away from the floc, the convection term in the Navier–Stokes equation become more important.
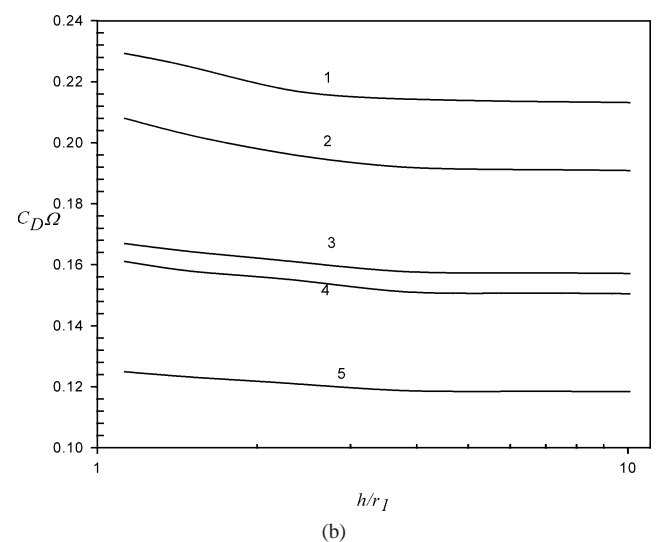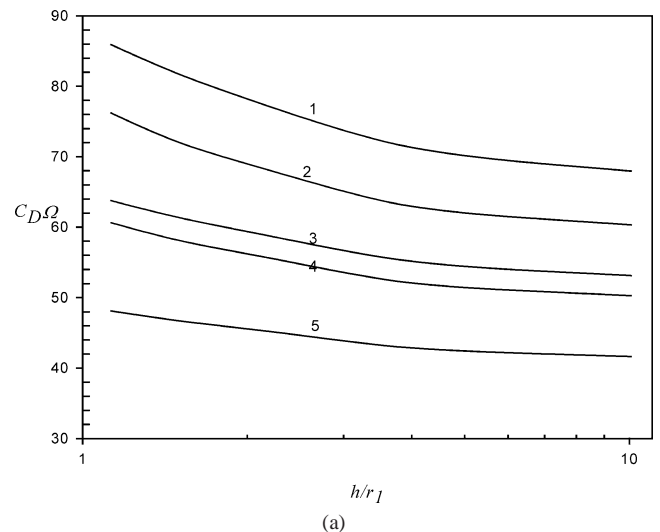
The influence of the structure of a floc on the corrected drag coefficient $C_D \Omega$ is illustrated in Fig. 4 for two different Reynolds numbers.

Note that if $k_1/k_2 = 1$, a floc has a homogeneous structure; if $k_1 < k_2$, the inner layer of a floc is more permeable than its outer layer; and if $k_1 > k_2$, the inner layer of a floc is less permeable than its outer layer. Fig. 4a shows that in general, $C_D \Omega$ decreases with the increase in h/r_1, that is, the farther the floc from the plate the smaller the drag force it experiences. This is because the floc assumes a constant velocity, and the presence of the plate has the effect of retarding its movement; apparently, the closer the floc is to the plate the more significant is this effect. For a fixed volume-averaged permeability, the $C_D \Omega$ for a homogeneous floc is smaller than that for a nonhomogeneous floc, which is true for both $k_1 < k_2$ and $k_1 > k_2$. Regarding the in-

fluence of the floc–plate distance, we have $[C_D\Omega\ (h/r_1 = 1.13)/C_D\Omega\ (h/r_1 = 10.07)] = 1.26, 1.26, 1.2, 1.21$, and 1.16 for $k_1/k_2 = 0.1, 10, 0.2, 5$, and 1, respectively. That is, if the volume-averaged permeability is constant, the effect of the presence of the plate on a homogeneous floc is less significant than that on a nonhomogeneous floc. Note that while $[C_D\Omega\ (h/r_1 = 1.13)/C_D\Omega\ (h/r_1 = 10.07)]$ for $k_1/k_2 = 1/10$ is about the same as that for $k_1/k_2 = 10$, and $[C_D\Omega\ (h/r_1 = 1.13)/C_D\Omega\ (h/r_1 = 10.07)]$ for $k_1/k_2 = 1/5$ is about the same as that for $k_1/k_2 = 5$, $C_D\Omega\ (k_1/k_2 = 1/10) > C_D\Omega\ (k_1/k_2 = 10)$, and $C_D\Omega\ (k_1/k_2 = 1/5) > C_D\Omega\ (k_1/k_2 = 5)$. That is, the boundary effect on a floc with a less permeable outer layer (i.e., $k_1 = k_2/n$, n being an arbitrary constant larger than unity) is more important than that with a less permeable inner layer (i.e., $k_1 = nk_2$). This is because the outer layer of a floc is closer to the plate than its inner layer, and therefore, the effect of the former on $C_D\Omega$ is more important than that of the latter. The qualitative behavior of the curves presented in Fig. 4b is similar to that illustrated in Fig. 4a, but the variation of $[C_D\Omega\ (h/r_1 = 1.13)/C_D\Omega\ (h/r_1 = 10.07)]$ as k_1/k_2 varies is less appreciable. We have $[C_D\Omega\ (h/r_1 = 1.13)/C_D\Omega\ (h/r_1 = 10.07)] = 1.08, 1.09, 1.06, 1.07$, and 1.05 for $k_1/k_2 = 0.1, 10, 0.2, 5$, and 1, respectively. That is, increasing Re has the effect of reducing the influence of the presence of the plate on $C_D\Omega$.

Fig. 5 illustrates the influence of the structure of a floc on the corrected drag coefficient $C_D\Omega$ at two different Reynolds numbers for the case $\bar{\beta}$ is larger than that used in Fig. 4; that is, the floc in Fig. 5 is less permeable than that in Fig. 4. A comparison between Figs. 4a and 5a indicates that the boundary effect in the latter is more serious than that in the former, which is expected. In Fig. 5a, we have $[C_D\Omega\ (h/r_1 = 1.13)/C_D\Omega\ (h/r_1 = 10.07)] = 1.79, 1.70, 1.62, 1.57$, and 1.49 or $k_1/k_2 = 0.1, 10, 0.2, 5$, and 1, respectively. As in the case of Fig. 4, increasing Re has the effect of reducing the ratio $[C_D\Omega\ (h/r_1 = 1.13)/C_D\Omega\ (h/r_1 = 10.07)]$. For example, in Fig. 5b we have $[C_D\Omega\ (h/r_1 = 1.13)/C_D\Omega\ (h/r_1 = 10.07)] = 1.21, 1.23, 1.16, 1.17$, and 1.13 for $k_1/k_2 = 0.1, 10, 0.2, 5$, and 1, respectively.

For the present case, the Stokes-law-like relation, Eq. (11), can be modified as

$$C_D\Omega = \frac{A(\bar{\beta}, k_1/k_2, h/r_1)}{Re} \qquad (12)$$

or

$$\ln(C_D\Omega) = \ln\left[A(\bar{\beta}, k_1/k_2, h/r_1)\right] - \ln(Re), \qquad (13)$$

where A is a function of $\bar{\beta}$, k_1/k_2, and h/r_1. Equation (13) suggests that for a set of $(\bar{\beta}, k_1/k_2, h/r)$, $\ln(C_D\Omega)$ is linearly correlated with $\ln(Re)$. Fig. 6 shows the variation of $\ln(C_D\Omega)$ as a function of $\ln(Re)$ for the case when h/r_1 is large; that for the case when it is small is illustrated in Fig. 7.

Fig. 6 reveals that if Re is small, $\ln(C_D\Omega)$ and $\ln(Re)$ are linearly correlated, but a positive deviation from the linear relation occurs if Re is large. The deviation is more serious when $\bar{\beta}$ becomes larger. In particular, at $Re = 40$ the
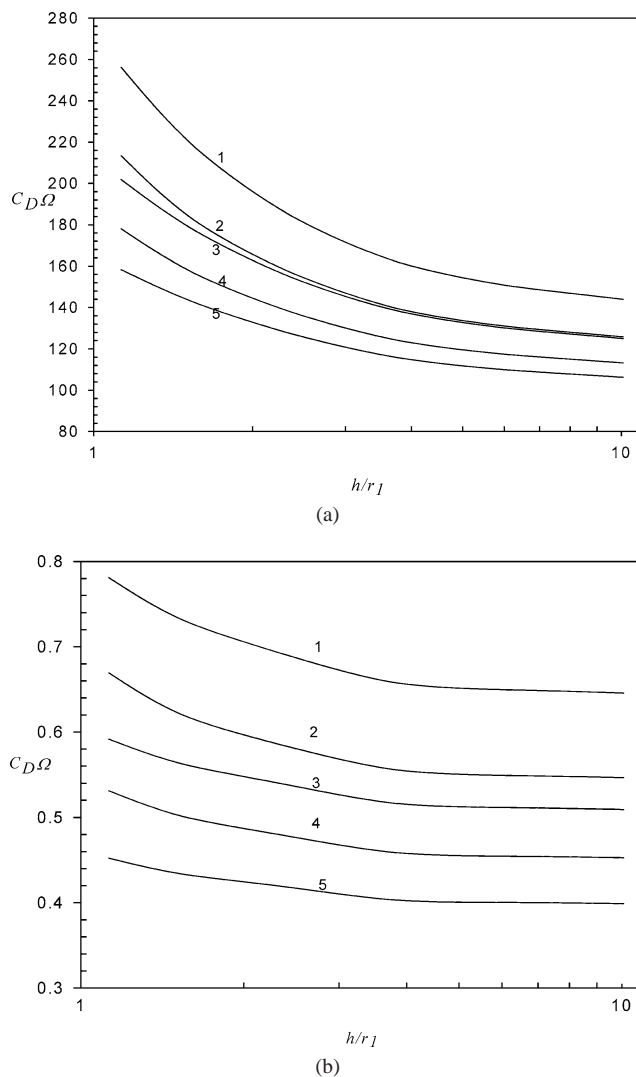

Fig. 5. Variation of $C_D\Omega$ as a function of h/r_1 for various k_1/k_2 at different Re for the case when $\bar{\beta} = 2$. Curve: 1, $k_1/k_2 = 0.1$; 2, $k_1/k_2 = 0.2$; 3, $k_1/k_2 = 10$; 4, $k_1/k_2 = 5$; 5, $k_1/k_2 = 1$. (a) $Re = 0.1$, (b) $Re = 40$. Key: same as Fig. 2.

$C_D\Omega$ of a floc with $k_1/k_2 = 0.1, 10, 0.2, 5$, and 1, for the case when $\bar{\beta} = 1$, deviates from the linear relation, Eq. (13), by 25.5%, 26.6%, 18.3%, 19.7%, and 13.7%, respectively, and by 79.4%, 73.5%, 63%, 60% and 50.2%, respectively, when $\bar{\beta} = 2$. Therefore, if the boundary effect is relatively unimportant, the more nonhomogeneous and/or the less permeable a floc is, the greater is the deviation of $C_D\Omega$ from the modified Stokes-law-like relation, Eq. (13). The deviation is alleviated as the boundary effect becomes significant, as can be seen in Fig. 7.

For instance, the deviations of $C_D\Omega$ from the modified Stokes-law-like relation at $Re = 40$ for $k_1/k_2 = 0.1, 10, 0.2, 5$, and 1 are 6.7%, 9.2%, 4.8%, 6.3%, and 3.8% when $\bar{\beta} = 1$, and are 21.9%, 25.5%, 17.2%, 19.3%, and 14.2%, respectively, when $\bar{\beta} = 2$.

Fig. 8 shows the variation of $C_D\Omega$ as a function of the ratio (radius of inner layer/radius of floc), r_2/r_1, at different
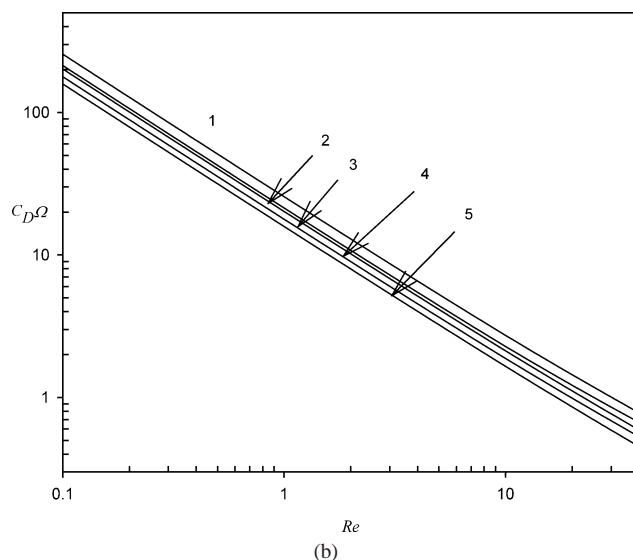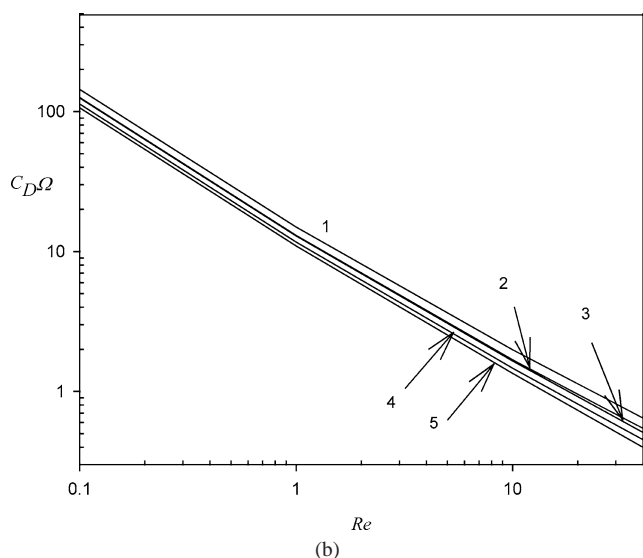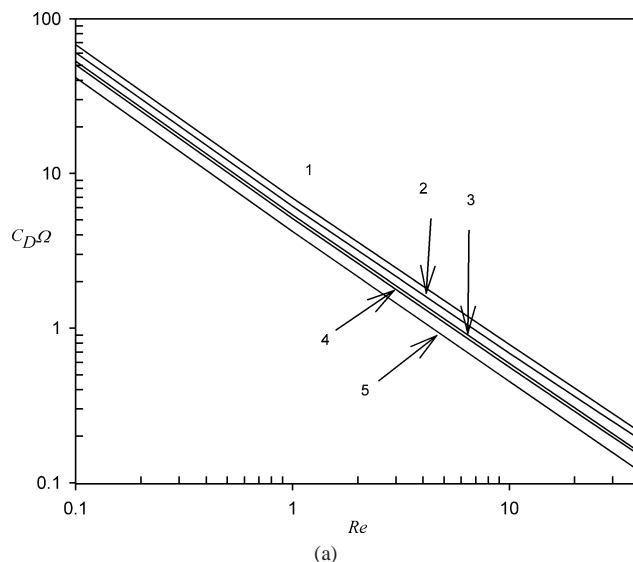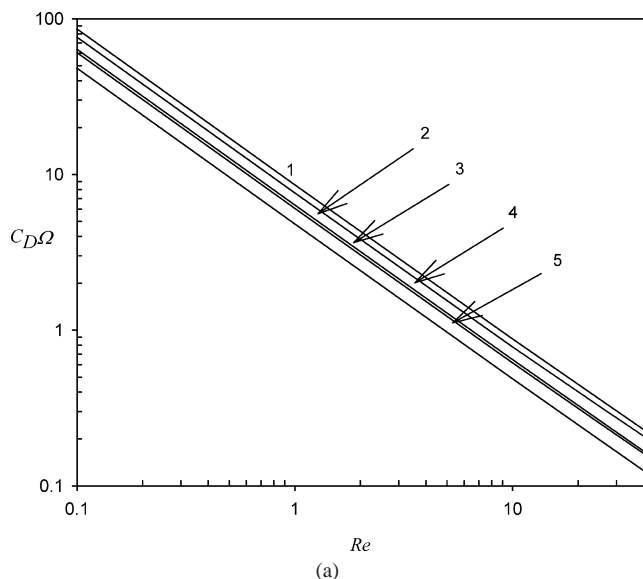
Fig. 6. Variation of $C_D\Omega$ as a function of Re for various k_1/k_2 at different $\bar{\beta}$ for the case when $h/r_1 = 10.07$. Curve: 1, $k_1/k_2 = 0.1$; 2, $k_1/k_2 = 10$; 3, $k_1/k_2 = 0.2$; 4, $k_1/k_2 = 5$; 5, $k_1/k_2 = 1$. (a) $\bar{\beta} = 1$, (b) $\bar{\beta} = 2$. Key: same as Fig. 2.

Fig. 7. Variation of $C_D\Omega$ as a function of Re for various k_1/k_2 at different $\bar{\beta}$ for the case when $h/r_1 = 1.13$. Curve: 1, $k_1/k_2 = 0.1$; 2, $k_1/k_2 = 10$; 3, $k_1/k_2 = 0.2$; 4, $k_1/k_2 = 5$; 5, $k_1/k_2 = 1$. (a) $\bar{\beta} = 1$, (b) $\bar{\beta} = 2$. Key: same as Fig. 2.

h/r_1 for the case when $Re = 0.1$; that for the case when $Re = 40$ is illustrated in Fig. 9.

Curve 5 in these figures represents the result for a homogeneous floc. Fig. 8 reveals that for a nonhomogeneous floc, $C_D\Omega$ increases with the increase in r_2/r_1, regardless of whether its inner layer is less permeable than its outer layer or not. This is because if $k_1/k_2 < 1$, the outer layer of a floc is less permeable than its inner layer. In this case, since the volume-averaged permeability of a floc is fixed, a larger r_2/r_1 implies a less permeable outer layer, and therefore, a larger $C_D\Omega$. On the other hand, if $k_1/k_2 > 1$, the inner layer of a floc is less permeable than its outer layer, and the larger the r_2/r_1 the closer is the inner layer to the plate, which also yields a larger $C_D\Omega$. The qualitative behavior of $C_D\Omega$ pre-

sented in Fig. 8b for the case when the distance between floc and plate is small is similar to that shown in Fig. 8a for the case when it is large. Quantitatively, the closer a floc is to a plate, the larger is the $C_D\Omega$. The behavior of $C_D\Omega$ illustrated in Fig. 9 for the case when Re is increased to 40 is similar to that shown in Fig. 8, except that the magnitude of $C_D\Omega$ decreases dramatically.

4. Conclusion

On the basis of a two-layer model, we have examined the boundary effect on the moving of a nonhomogeneous, spherical floc normal to a rigid plate for the case of low to medium Reynolds number. The results of numerical simula-
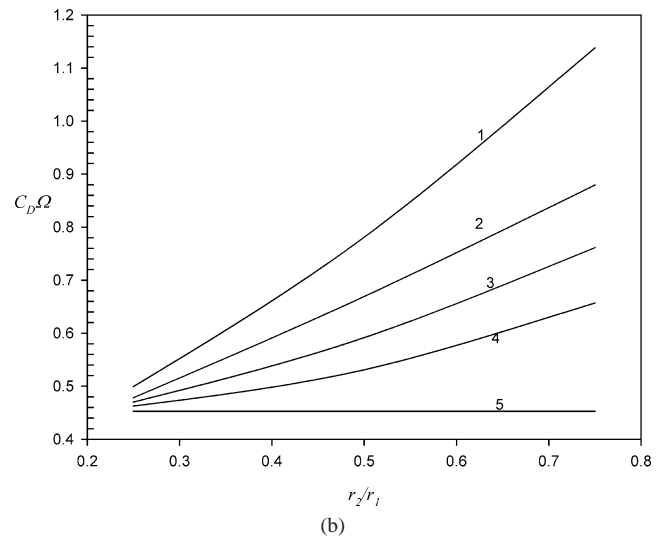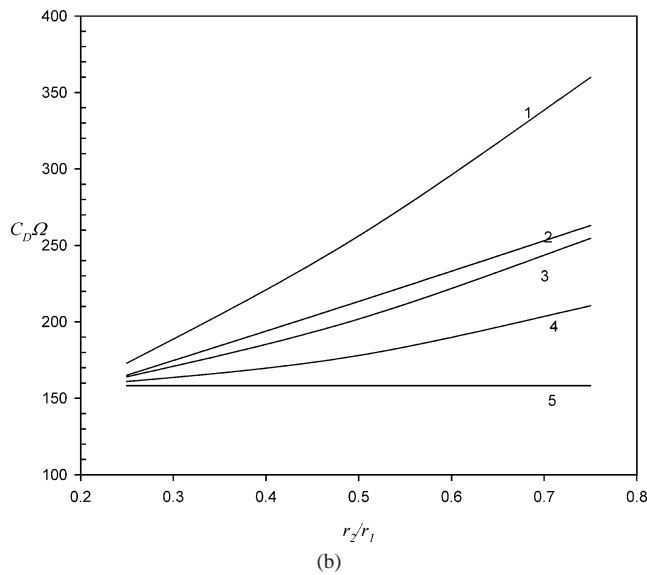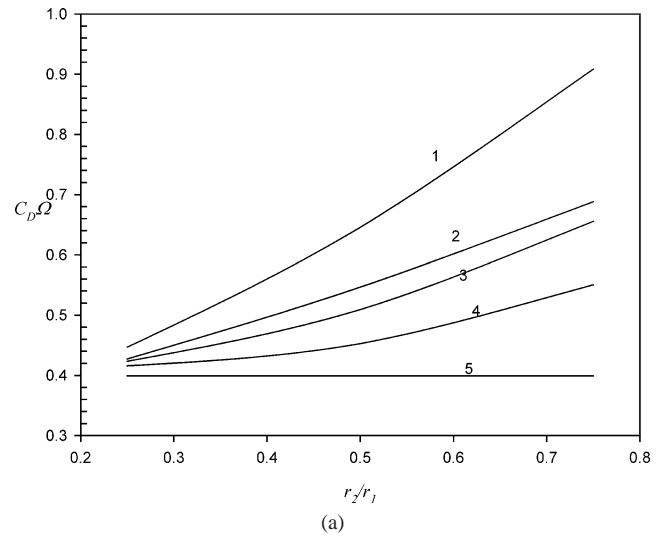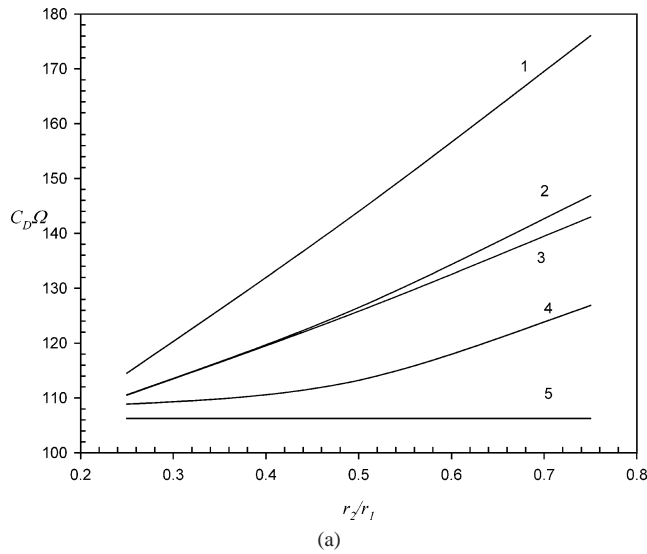
Fig. 8. Variation of $C_D \Omega$ as a function of r_2/r_1 for various k_1/k_2 at different h/r_1 for the case when $\bar{\beta} = 2$ and $Re = 0.1$. Curve: 1, $k_1/k_2 = 0.1$; 2, $k_1/k_2 = 0.2$; 3, $k_1/k_2 = 10$; 4, $k_1/k_2 = 5$; 5, $k_1/k_2 = 1$. (a) $h/r_1 = 10.07$, (b) $h/r_1 = 1.13$. Key: same as Fig. 2.

Fig. 9. Variation of $C_D \Omega$ as a function of r_2/r_1 for various k_1/k_2 at different h/r_1 for the case when $\bar{\beta} = 2$ and $Re = 40$. Curve: 1, $k_1/k_2 = 0.1$; 2, $k_1/k_2 = 10$; 3, $k_1/k_2 = 0.2$; 4, $k_1/k_2 = 5$; 5, $k_1/k_2 = 1$. (a) $h/r_1 = 10.07$, (b) $h/r_1 = 1.13$. Key: same as Fig. 2.

Acknowledgment

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tion can be summarized as followings. (a) The presence of the floc will distort the flow field nearby, and the degree of distortion is more serious when it is away from the plate, which is true regardless of the structure of the floc and the level of Reynolds number. (b) The influence of the plate on the behavior of a nonhomogeneous floc is more significant than that of a homogeneous floc. (c) The nonhomogeneous structure of a floc leads to a positive deviation in the modified drag coefficient from a Stokes-law-like relation. The presence of the plate has the effect of reducing this deviation. (d) Regardless of the magnitude of Reynolds number, the presence of a plate has the effect of increasing the drag force acting on a floc, which arises from the nonhomogeneous structure of a floc.

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