PROCESS EXHAUST DUCT SYSTEM DESIGN USING DYNAMIC PROGRAMMING METHODS

Huei-Jiunn Chen, Huan-Ruei Shiu and Sih-Li Chen*

ABSTRACT

This paper explores the design methods for process exhaust duct systems, including velocity method, the equal friction method, the static regain method and the T-method, and produces actual case designs and system simulations to compare the applicability of respective design methods. The study shows that none of the methods are able to meet the demand of cost optimization under the constraints of duct size and velocity, and the requirement of pressure balance. Dynamic programming is therefore proposed. The results show that dynamic programming is better than the other methods. It is able to achieve the optimization of life-cycle cost while satisfying all constraints (flow rate, duct size, velocity and pressure balancing). The differences between the design and simulation results using dynamic programming are much lower than those of other methods. Thus a process exhaust duct system that approximates the actual operation best may be designed using dynamic programming.

Key Words: exhaust duct system, dynamic programming method.

I. INTRODUCTION

The main purpose of a process exhaust duct system is to discharge waste gases generated in a process outside the plant. Exhaust volume is the first factor to be considered. In addition, the air velocity in each duct section must be constrained within an acceptable range. Other problems, such as noise, vibration, pressure balance, costs and space limitations during construction have to be factored in as well. These factors are mostly inter-containing. For example, increasing design velocity or reducing duct size can lower initial costs, but fan total pressure needs to be increased which might require bigger fan, resulting in higher operating cost or bringing about noise or vibration problems. Thus it is always a challenge for designers to come up with an optimum exhaust duct system while satisfying individual constraints.

Conventional design methods for exhaust duct systems, such as the velocity method, the equal friction method, and the static regain method (Carrier, 1960) are unable to obtain pressure balancing or control the air velocity within a certain range. The resulting designs are usually not highly accurate and totally leave out the issue of cost optimization. The T-method (Tsal *et al.*, 1988, 1990) is able to obtain system pressure balance and minimal life-cycle cost. But the system is designed in the absence of any constraint. If constraints (e.g. duct size or velocity) are included, the total pressure balance will be sacrificed. In light of the deficiencies of the conventional design methods that cannot totally satisfy the needs of the designers, this study explores the design of a process exhaust duct system using dynamic programming methods (DPM) in the hope of helping designers attain cost optimization while satisfying the constraints of duct size and flow rate, and achieving pressure balance.

Dynamic programming is a technique in operation research applied mostly in business or management problems. In recent years, it has been applied

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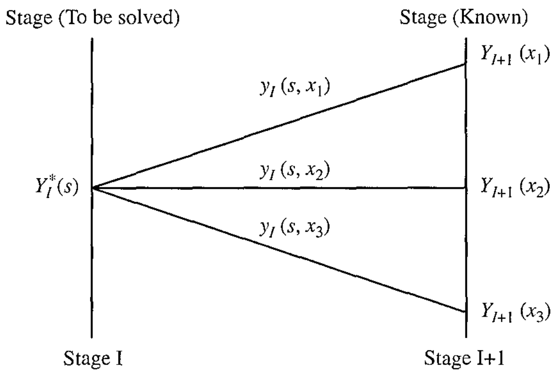

Fig. 1 Model of backward dynamic programming

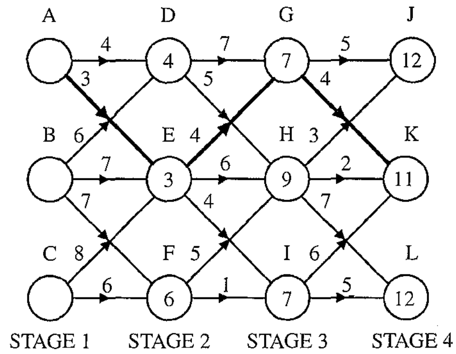

Fig. 2 Four-stage policy of optimality network

to the planning of reservoirs and scheduling of production units. There are few mentions in the literature with regard to the use of this method in the design of exhaust duct systems. DPM is a mathematical programming approach first developed by Bellman in 1957. It is suitable for solving complicated and multi-stage decision problems by finding the optimal strategy. Bellman reckons that "an optimal policy has the property that whatever the initial state and the initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision." In other words, if the current state and the planned decision are known, an optimal policy formed in the future will be independent of the past policy already formed. In the initial stage when the problem is relatively simple, an optimum decision is made based on the known conditions. In subsequent stages the conditions become more complicated, besides the known conditions for the said stage, the optimal decision made in the last stage needs to be factored in to obtain the optimum decision for the current stage. The process goes on in sequence until the optimum decision for the last stage is obtained. Based on such iterative characteristics, DPM solves a problem by finding a chain-like decision and maximizing (or minimizing) the anticipated objective function. The chain-like decision that turns out maximal total benefit (or minimal total expenses) is called the policy of optimality. The purpose of dynamic programming is to apply numerical methods to multi-stage decision problems and find the optimum policy. Thus dynamic programming is most suitable for multi-stage, sequential problems, particularly for optimization problems where the objective functions are undifferentialable as in the case of optimal duct system design.

II. DYNAMIC PROGRAMMING METHOD

Dynamic programming method (DPM) has two approaches according to the direction of operation - forward and backward. Forward DPM performs calculation from the first stage to the last stage and then goes back to decide the optimum solution. Backward DPM calculates from the last stage backward to the first stage and then going from the first stage in sequence to find the optimum solution. Both approaches purport to search for an optimum path to bring the costs to a minimum. This study employs backward dynamic programming in calculation. As shown in Fig. 1, the optimum path from stage I+1 to stage I is decided by means of recurrence relations as shown below:

$$Y_I^*(s) = \underset{x_I}{Min}\left[y_I(s, x_i) + y_{I+1}^*(x_i)\right] \tag{1}$$

where $Y_I^*(s)$ represents the minimum cost under state S in stage I, i.e., the minimum total cost from the final stage to this state; $y_{I+1}^*(x_i)$ represents the minimum cost under state x_i in stage I+1, i.e. the minimum cost from the last stage to this state. $y_I(s, x_i)$ represents the cost required from Stage I+1 State x_i to Stage I State S. If each stage has S number of conditions, there are S^{N-1} probable paths after $(N-1)$ stages. Using the backward dynamic programming algorithm, only $(N-2)S^2+S$ calculations need to be performed, making the process of problem solving markedly efficient.

The problem-solving algorithm may be illustrated with Fig. 2. The figure shows a network with three starting points available - A, B and C, and three probable destinations - J, K and L after four decision stages. The numerals on each arrow line represent the cost of each operating activity. The three optimum paths from the first decision stage to the nodes of the second stage are 4, 3, and 6, i.e., the respective cost of two paths to node D is 4 and 6, of which 4 is more optimal. Thus the smaller value 4 is recorded in node D. The cost of three paths to node E is respectively 3, 7 and 8, of which, 3 is the more optimal. Thus smaller value 3 is recorded in node E. Similarly the cost of two paths to node F is respectively

7 and 6, of which 6 is more optimal, thus 6 is recorded in node F. This way, the policies of optimality for two stages are obtained. Similarly, using the optimum results 7, 9 and 7 in stage 3 as starting points, select the smaller sum and record them in the respective node in stage 4. This way, the multi-stage decision path with the smallest sum is obtained, which is 11 as shown by thick line in the figure. The operating activities linked up in this path result in the smallest total cost.

Next, objective functions are to be decided. The optimal design for an exhaust duct system is to find a duct assembly having minimum system pressure drop with known air volume at each outlet to achieve optimization of total cost. Thus the objective function of system optimization is life-cycle cost through the selection of optimal duct size and optimal total pressure of the fan. The life-cycle cost of the duct system includes initial cost and energy cost (in operation). The former consists of the costs of duct fabrication and installation, while the latter comprises basic rates and variable rates of electricity. The method of optimization is to find the fan total pressure that can keep the cost at minimum. Given that some of the costs mentioned are constants which are not correlated with optimization, only initial cost and energy cost are taken as objective functions, which may be expressed in Eq. (2) below:

$$E=E_p(PWEF)+E_s \tag{2}$$

where

E	life-cycle cost ($)
E_P	energy cost in first-year operation
E_S	initial cost
$PWEF$	present worth escalation factor

Given that a system usually lasts for several years, future annual escalation rate and interest rate variation should be factored in. Thus in Eq. (2), E_P is multiplied by $PWEF$ which may be obtained in Eq. (3):

$$PWEF=\frac{\left(\frac{1+AER}{1+AIR}\right)^a-1}{1-\left(\frac{1+AIR}{1+AER}\right)} \tag{3}$$

where

AER	annual escalation rate
AIR	annual interest rate
a	amortization period

The greater the annual escalation rate or amortization period, the greater the $PWEF$; while the greater the annual interest rate, the smaller the $PWEF$. The energy cost of the system is in fact the power consumption cost, while fan is the major component in the system that consumes electricity. Thus

$$E_p=\frac{Q_{fan}P_{fan}}{\eta_f\eta_c}\cdot E_c\cdot Y \tag{4}$$

where

Q_{fan}	flow rate of fan (m^3/s)
P_{fan}	total pressure of fan (Pa)
η_f	fan efficiency (%)
η_c	efficiency of motor (%)
E_c	unit price of electricity ($/kW-h)
Y	annual operation hours of system (hr/year)

The initial cost of the system is the unit area cost of the duct multiplied by the total surface area of duct section. The unit area cost should include material cost, handling cost and labor cost for installation. The costs of fittings should be included as well. For duct section with round cross section:

$$E_s=S_d(\pi DL) \tag{5a}$$

for duct section with rectangular cross section:

$$E_s=2S_d(H+W)L \tag{5b}$$

where

S_d	duct unit area cost ($/$m^2$)
L	section length (m)
D	diameter of round section (m)
H	height of rectangular section (m)
W	height of rectangular section (m)

Substitute Eqs. (3) to (5) into Eq. (2) and obtain the life-cycle cost of round and rectangular ducts as:

Round duct

$$E=\left(\frac{Q_{fan}P_{fan}}{\eta_f\eta_c}\cdot E_cY\right)PWEF+S_d(\pi DL) \tag{6a}$$

Rectangular duct:

$$E=\left(\frac{Q_{fan}P_{fan}}{\eta_f\eta_c}\cdot E_cY\right)PWEF+2S_d(H+W)L \tag{6b}$$

This analysis assumes the use of round ducts. Thus Eq. (6a) will be employed in the computation of objective functions. In the setting of cost-related parameters, fan efficiency is 80% and motor efficiency is 80%; the unit area cost of duct is base price $500/$m^2$ plus 30% for miscellaneous items, such as flanges, supports, gaskets, labor and transportation,

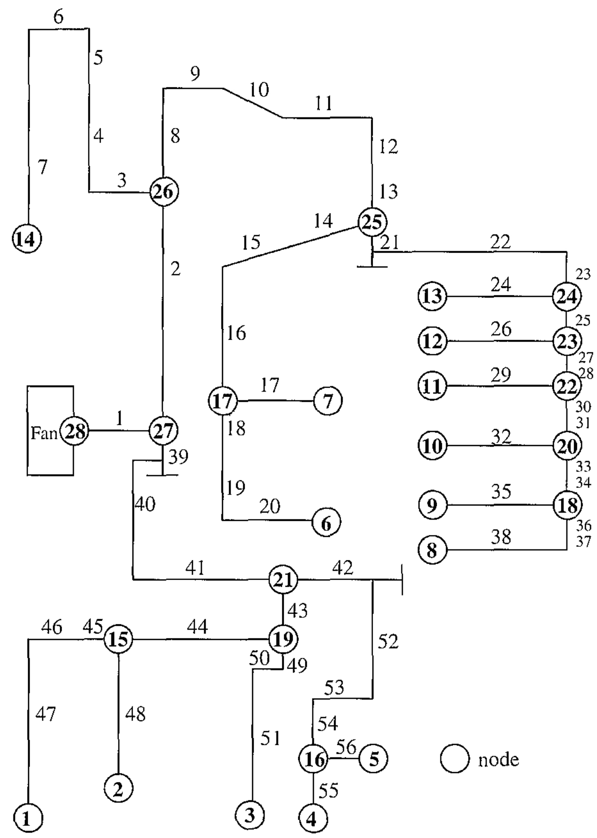

Fig. 3 Nodes configuration using dynamic programming

for a total of \$650/m^2. In the calculation of present worth escalation factor, assuming the system has a lifespan of 10 years, and the annual interest rate and annual escalation rate are 6% and 3% respectively, the present worth escalation factor obtained is 8.568.

III. CASE ANALYSIS

DPM is used in the design and analysis of an alkaline gas exhaust system at a semiconductor plant. Fig. 3 depicts the exhaust system configuration. The exhaust volume is determined by the volume of waste gases generated in the process. The length of each duct section in the system depends on equipment location and architectural structure. In Fig. 3, duct sections are numbered by different flow rates, duct sizes and shapes of cross sections, and divided into 56 sections and 14 paths. Section number represents the straight duct location in the system layout. Path number indicates the air flow route from upstream inlet to fan. The flow rate and section length are known conditions, as shown in Table 1. In consideration of control, connectivity and safety, the duct system is installed with relevant fittings (ASHRAE Handbook 1997). The design points are jointly determined by constraints, plant requirements, and exhaust flow rate,

Table 1 Sectional length and flow rate

Duct No	Flow rate (m^3/s)	Section length (m)	Duct No	Flow rate (m^3/s)	Section length (m)
1	10.620	1.000	29	1.720	21.400
2	8.280	38.000	30	3.050	1.000
3	0.370	3.800	31	3.050	13.400
4	0.370	3.000	32	1.720	21.400
5	0.370	1.500	33	1.330	1.000
6	0.370	9.600	34	1.330	6.000
7	0.370	12.600	35	0.720	18.000
8	7.910	0.500	36	0.610	1.000
9	7.910	2.500	37	0.610	14.400
10	7.910	4.500	38	0.610	22.400
11	7.910	10.500	39	2.340	1.000
12	7.910	14.700	40	2.340	3.200
13	7.910	4.000	41	2.340	2.000
14	2.110	0.500	42	0.820	2.000
15	2.110	6.400	43	1.520	15.000
16	2.110	20.400	44	1.170	3.000
17	1.060	23.000	45	0.700	1.000
18	1.050	2.000	46	0.700	7.000
19	1.050	47.600	47	0.700	12.000
20	1.050	20.200	48	0.470	12.000
21	5.800	0.500	49	0.330	17.500
22	5.800	57.000	50	0.330	8.600
23	5.800	6.000	51	0.330	10.400
24	0.530	22.000	52	0.820	21.500
25	5.270	16.000	53	0.820	15.000
26	0.500	22.000	54	0.820	24.000
27	4.770	1.000	55	0.560	5.000
28	4.770	5.400	56	0.260	10.200

Table 2 Design point of alkaline gas exhaust system

Constraint	Parameter value
Min. safety velocity	5 m/sec
Max. safety velocity	15 m/sec
Design velocity	7.5 m/sec
Total air volume required	10.70 m^3/sec

the paramount considerations are safe velocity and total exhaust volume. As shown in Table 2, the maximum and minimum safe velocities are determined by parameters, including type of exhaust, humidity, vibration and noise as suggested in manufacturer or reference handbook (ASHRAE, 1995; Howell *et al.*, 1998).

The first step in the calculation is to set nodes based on the principle where "nodes are required at duct divergence (at confluence or distribution point) and air outlet (or return)." Fig. 3 presents the configuration of system nodes. What follows is to stratify the duct system; set the total pressure on path terminal at 0, then calculate from level 1 toward the last level, and calculate all acceptable total pressure, duct sizes and air velocities on every node from level 1 to the last level. Substitute the total pressure and total sectional area into objective function to obtain optimum value after comparison. In level layout, level 1 must be related to air outlet (or air return), while the last level is the fan side. The whole calculation process is carried out backward from the air outlet toward the fan. The principle of stratification is: "required data for nodes in level N+1 must include known results of level N." Nine levels are obtained from the stratification: Level 1 contains nodes 15, 16, 17 and 18; level 2 contains nodes 19 and 20; level 3 contains nodes 21 and 22; level 4 contains nodes 23; Level 5 contains node 24; level 6 contains 25; level 7 contains node 26; Level 8 contains node 27, and level 9 contains node 28. Given that nodes 1 through 14 are connected with the atmosphere, their total pressure values are 0, i.e. $Pt_1=Pt_2=Pt_3=\ldots\ldots Pt_{14}=0$.

The design for level 1 with nodes 15, 16, 17 and 18 is carried out first. Since each node is independent of every other, calculation may start with any of the nodes. This study first considers the total pressure on node 15 which has a range of 1Pa to 600Pa with an increment interval of 1Pa and the maximum total pressure value can be adjusted arbitrarily based on the size of duct work. The maximum total pressure for this duct system is set at 600Pa. When the pressure of node 15 is $P_{15}=P_{15}^{(1)}=1$Pa, the total pressure drop from node 15 to node 1 and node 2 is respectively

$$\Delta P_{15\sim1}^{(1)} = P_{15}^{(1)} - P_1 \tag{7a}$$

$$\Delta P_{15\sim2}^{(1)} = P_{15}^{(1)} - P_2 \tag{7b}$$

Given that $P_1=P_2=0$, the equations above may be rewritten as

$$\Delta P_{15\sim1}^{(1)} = P_{15}^{(1)} \tag{7c}$$

$$\Delta P_{15\sim2}^{(1)} = P_{15}^{(1)} \tag{7d}$$

The duct diameter required may be obtained by the total pressure drop above using the following equations:

$$\Delta P_l = \left(\frac{fL}{D} + C\right)\frac{\rho V^2}{2} \tag{8a}$$

$$f = 0.11\left(\frac{\varepsilon}{D} + \frac{68}{\mathrm{Re}}\right)^{0.25} \tag{8b}$$

$$V^2 = \frac{16Q^2}{\pi^2 D^4} \tag{8c}$$

$$\mathrm{Re} = \frac{VD}{V} \tag{8d}$$

Substitute Eqs. (8b), (8c) and (8d) into (8a) to obtain (8e),

$$\Delta P_l = \left[0.11\left(\frac{\varepsilon}{D} + \frac{17\pi D v}{Q}\right)^{0.25}\frac{L}{D} + C\right]\cdot\left(\frac{\rho}{2}\frac{16Q^2}{\pi^2 D^4}\right) \tag{8e}$$

From Eq. (8e), solutions for node 15 at level 1 may be expressed as follows:

$$\Delta P_{15-1} = \left[0.11\left(\frac{\varepsilon}{D_{15-1}} + \frac{17\pi D_{15-1} v}{Q_{15-1}}\right)^{0.25}\frac{L_{15-1}}{D_{15-1}} + C_{15-1}\right] \times\left(\frac{\rho}{2}\frac{16Q_{15-1}^2}{\pi^2 D_{15-1}^4}\right) \tag{9a}$$

$$\Delta P_{15-2} = \left[0.11\left(\frac{\varepsilon}{D_{15-2}} + \frac{17\pi D_{15-2} v}{Q_{15-2}}\right)^{0.25}\frac{L_{15-2}}{D_{15-2}} + C_{15-2}\right] \times\left(\frac{\rho}{2}\frac{16Q_{15-2}^2}{\pi^2 D_{15-2}^4}\right) \tag{9b}$$

$$D_{\min} \le D_{15-1} \le D_{\max} \tag{10a}$$

$$D_{\min} \le D_{15-2} \le D_{\max} \tag{10b}$$

$$V_{\min} \le V_{15-1} \le V_{\max} \tag{10c}$$

$$V_{\min} \le V_{15-2} \le V_{\max} \tag{10d}$$

From Eq. (9), the corresponding duct size may be derived if total pressure drop is known. But said equation does not produce duct diameter directly. A numerical method needs to be applied and the Newton method is used in this paper. If the resulting diameter or velocity cannot satisfy the constraints set in Eqs. (10a)~(10d), the pressure value will be rejected. The so-called unacceptable duct sizes are sizes that exceed the space allowed, which is defined at 1.5 m in this paper.

After obtaining $D_{15-1}^{(1)}$ and $D_{15-2}^{(1)}$, the initial costs of these two duct sections $Es_{15-1}^{(1)}$ and $Es_{15-2}^{(1)}$ may be computed using Eq. (5a). Then calculate the corresponding duct size, velocity and initial cost for the rest of total pressure by the increment of 1Pa. These acceptable values are stored in the computer in arrays as shown in Table 3. $Es_{15}^{(1)}$ in the farthest right column of Table 3 represents the total sectional area (the sum of surface area of duct sections 45, 46, 47 and 48) connected to node 15 under $P_{15}^{(1)}$, and so on. Given that node 15 is connected to the outlet at the end of the ductwork, $Es_{15-1}^{(1)}$ may also be taken as the minimum total sectional area under $P_{15}^{(1)}$ after node 15, and so on. The same operation may be applied to the calculations for node 16, 17 and 18.

The same design method just described is applied to the next level, up to level 9 where node 28 is the fan side. Thus the total pressure on node 28 is the total pressure the fan should supply. The equation for the solutions on this level is expressed below:

$$\Delta P_{28-27}$$

$$=\left[0.11\left(\frac{\varepsilon}{D_{28-27}}+\frac{17\pi D_{28-27}\nu}{Q_{28-27}}\right)^{0.25}\frac{L_{28-27}}{D_{28-27}}+C_{28-27}\right]$$

$$\times\left(\frac{\rho}{2}\frac{16Q_{28-27}^2}{\pi^2 D_{28-27}^4}\right) \tag{11}$$

$$D_{\min}\leq D_{28-27}\leq D_{\max} \tag{12a}$$

$$V_{\min}\leq V_{28-27}\leq V_{\max} \tag{12b}$$

As described, there might be a number of options of P_{27} for P_{28}. Thus the options must be compared to obtain P_{27} with the smallest sectional area. The resulting parameters are listed in Table 3, in which $P_{28,27}^{(1)}$ represents the total pressure of node 27 corresponding to $P_{28}^{(1)}$, whereas $E_{S28}^{(1)}$ in the farthest right column represents the minimum sectional area of the entire ductwork. That is when the total pressure on node 28 is $P_{28}^{(1)}$ and the total pressure on node 27 is $P_{28,27}^{(1)}$, the minimum area of the entire ductwork is $E_{S28}^{(1)}$.

After calculating all acceptable total pressure values, P_{28}, on node 28 and corresponding total initial costs, E_{S28}, we will proceed with the comparison of minimum life-cycle cost. Given that P_{28} represents the fan total pressure required which is related to initial cost, the minimum objective function value may be selected by substituting E_{S28} and P_{28} into objective function and by comparison. The minimum operating cost and initial cost in the life cycle are the optimum design for the duct system.

Table 3 Pressure value and initial cost of designed nodes at Level 1

Node 15					
$P_{15}^{(1)}$	$D_{15-1}^{(1)}$	$V_{15-1}^{(1)}$	$D_{15-2}^{(1)}$	$V_{15-2}^{(1)}$	$Es_{15-1}^{(1)}+Es_{15-2}^{(1)}=Es_{15}^{(1)}$
$P_{15}^{(2)}$	$D_{15-1}^{(2)}$	$V_{15-1}^{(2)}$	$D_{15-2}^{(2)}$	$V_{15-2}^{(2)}$	$Es_{15-1}^{(2)}+Es_{15-2}^{(2)}=Es_{15}^{(2)}$
…	…	…	…	…	… …
…	…	…	…	…	… …
$P_{15}^{(S)}$	$D_{15-1}^{(S)}$	$V_{15-1}^{(S)}$	$D_{15-2}^{(S)}$	$V_{15-2}^{(S)}$	$Es_{15-1}^{(S)}+Es_{15-2}^{(S)}=Es_{15}^{(S)}$
Node 16					
$P_{16}^{(1)}$	$D_{16-4}^{(1)}$	$V_{16-4}^{(1)}$	$D_{16-5}^{(1)}$	$V_{16-5}^{(1)}$	$Es_{16-4}^{(1)}+Es_{16-5}^{(1)}=Es_{16}^{(1)}$
$P_{16}^{(2)}$	$D_{16-4}^{(2)}$	$V_{16-4}^{(2)}$	$D_{16-5}^{(2)}$	$V_{16-5}^{(2)}$	$Es_{16-4}^{(2)}+Es_{16-5}^{(2)}=Es_{16}^{(2)}$
…	…	…	…	…	… …
…	…	…	…	…	… …
$P_{16}^{(S)}$	$D_{16-4}^{(S)}$	$V_{16-4}^{(S)}$	$D_{16-5}^{(S)}$	$V_{16-5}^{(S)}$	$Es_{16-4}^{(S)}+Es_{16-5}^{(S)}=Es_{16}^{(S)}$
Node 17					
$P_{17}^{(1)}$	$D_{17-7}^{(1)}$	$V_{17-7}^{(1)}$	$D_{17-7}^{(1)}$	$V_{17-6}^{(1)}$	$Es_{17-7}^{(1)}+Es_{17-6}^{(1)}=Es_{17}^{(1)}$
$P_{17}^{(2)}$	$D_{17-7}^{(2)}$	$V_{17-7}^{(2)}$	$D_{17-6}^{(2)}$	$V_{17-6}^{(2)}$	$Es_{17-7}^{(2)}+Es_{17-6}^{(2)}=Es_{17}^{(2)}$
…	…	…	…	…	… …
…	…	…	…	…	… …
$P_{17}^{(S)}$	$D_{17-7}^{(S)}$	$V_{17-7}^{(S)}$	$D_{17-6}^{(S)}$	$V_{17-6}^{(S)}$	$Es_{17-7}^{(S)}+Es_{17-6}^{(S)}=Es_{17}^{(S)}$
Node 18					
$P_{18}^{(1)}$	$D_{18-8}^{(1)}$	$V_{18-8}^{(1)}$	$D_{18-9}^{(1)}$	$V_{18-9}^{(1)}$	$Es_{18-8}^{(1)}+Es_{18-9}^{(1)}=Es_{18}^{(1)}$
$P_{18}^{(2)}$	$D_{18-8}^{(2)}$	$V_{18-8}^{(2)}$	$D_{18-9}^{(2)}$	$V_{18-9}^{(2)}$	$Es_{18-8}^{(2)}+Es_{18-9}^{(2)}=Es_{18}^{(2)}$
…	…	…	…	…	… …
…	…	…	…	…	… …
$P_{18}^{(S)}$	$D_{18-8}^{(S)}$	$V_{18-8}^{(S)}$	$D_{18-9}^{(S)}$	$V_{18-9}^{(S)}$	$Es_{18-8}^{(S)}+Es_{18-9}^{(S)}=Es_{18}^{(S)}$

IV. RESULTS AND DISCUSSION

1. Comparison of Velocity

The main purpose of an exhaust system is to discharge waste gases generated in the process outside the plant through the ductwork. If the air velocity is too low, solid granules may not be discharged, but instead, build up inside the ductwork and pose potential hazards. Thus the air velocity inside the ductwork must be maintained above an allowable lower limit. On the other hand, air velocity that is too high might cause noise or vibration problems, thus the designed velocity may not exceed an allowable upper limit. From Fig. 4 one can see that the velocity method set the velocity in each section as equal, mainly because velocity was kept at a constant through known design points. Under the static regain method, the designed system velocity was relatively low and some sections had velocities below the minimum allowed. This is because under the static regain method, "At the nodes of the duct system, the dynamic pressure is lowered in exchange of rising static pressure and the static regain is used to offset the loss of static pressure drop from friction in subsequent section after the node." Thus although of dynamic pressure is exchanged for the rise of static

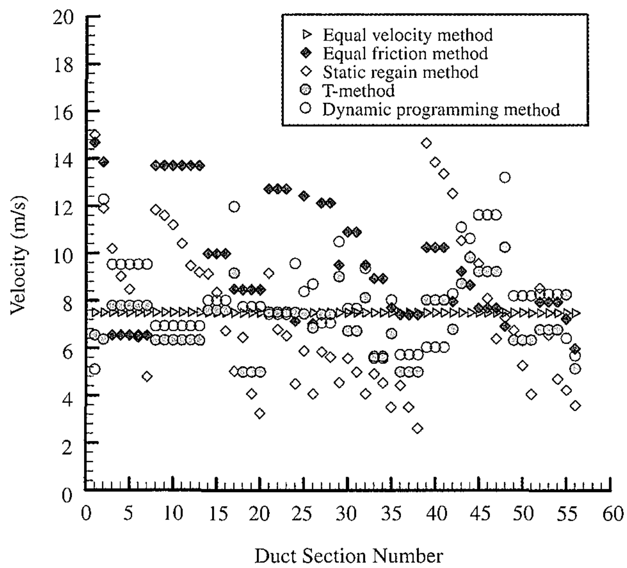

Fig. 4 Comparison of design point velocity utilizing different methods

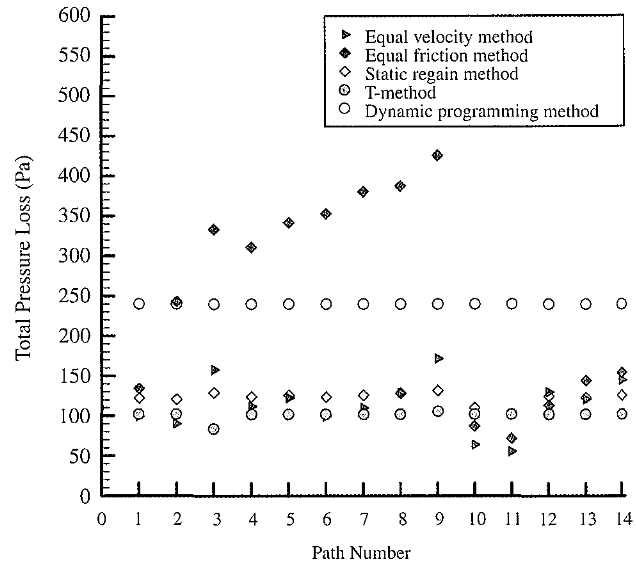

Fig. 5 Comparison of total pressure loss

pressure, the air velocity is reduced as a result. Under the equal friction method, the average velocity is relatively high. The system velocity designed by dynamic programming also tended to be high, but within the allowable range. This is because its total pressure loss tended to be high. The tendency of rising velocity is also observed in Eq. (8a), mainly because total pressure loss is directly proportional to the square of velocity.

2. Comparison of Pressure Loss

In actual operation, the total pressure of the exhaust duct system will adjust automatically to attain a balance, that is, the total pressure drop in each path becomes equal. If pressure balance is not achieved in the design stage, the velocity and flow in each duct section won't meet the design needs. Then more time and money will need to be spent on air flow adjustment after the system is constructed. In other words, the closer the designed pressure loss of paths to each other, the more the system will approximate the real scenario. From Fig. 5 one can see that the system pressure values designed by dynamic programming are more equal. This is because pressure balance was a basic constraint under this method. *T*-method also has the constraint of pressure balance. But it will be sacrificed when other constraints, such as duct size or velocity are included. In this case, pressure imbalance has resulted in the design using *T*-method due to the constraint of velocity range. In the design using the static regain method, the static regain factor was set as 1.0, meaning the static pressures at upstream and downstream of the distributing duct are equal. Thus a fairly balanced pressure system was derived. It is also shown in Fig. 5 that the pressure imbalance rate under the velocity method was rather high, exceeded only by the equal friction method. The equal friction method produces a design with the worst pressure balance. This is because the basic assumption of the method is "all sections of the system have equal unit length pressure loss." Thus it is more suitable for the design of a symmetrical system. But for an asymmetrical system as is the case here, a system with disparate system pressure results using the equal friction method.

3. Comparison of Total Surface Area

The total surface area of the exhaust duct system represents its initial cost. To cut down initial cost, duct size needs to be reduced. But reduced duct size will result in elevation of velocity and pressures, and high velocity will produce noise and vibration problems. If the pressure drop is too high, larger fans will be required. Duct size is also related to total surface area of the entire ductwork. Fig. 6 shows that the design utilizing dynamic programming produced the smallest total surface area, meaning its initial cost would be the smallest in comparison with other methods, followed by the equal friction method. The static regain method resulted in the highest initial cost, because it had bigger duct sizes and required lower fan pressure.

4. Results of Velocity Simulation

Testing the performance of the designed system

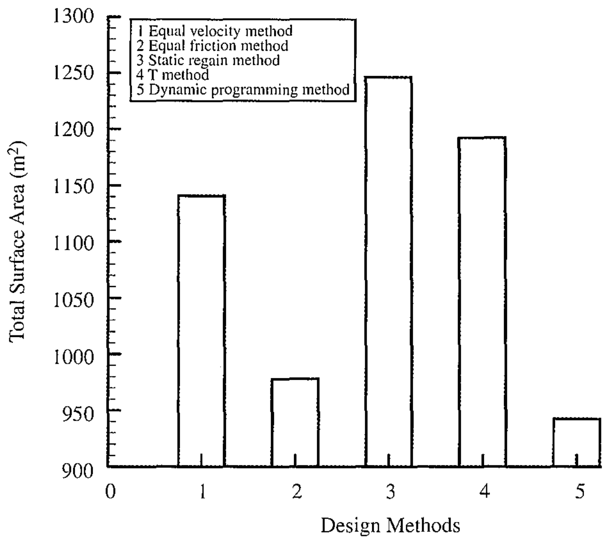

Fig. 6 Comparison of total surface area using different methods

in actual operation may be carried out by means of experiment or computer simulation. To save cost and facilitate changes in design, computer-aided simulation is employed for testing. System simulation is modeled after a method proposed by Tsal *et al.* in 1990. Said method may apply to simulation of a designed duct system where the flow rate and velocity in each duct section during system operation are computed with known duct size conditions and data of fittings.

Figure 7 shows that velocities derived from dynamic programming and *T*-method mostly fall within the originally designed limit. This is because both methods factored in velocity limitations. Hence their simulation results are consistent with the design values. Other conventional design methods (velocity, equal friction and static regain) also set velocity limits, but the limitation is unidirectional (set the velocity under a certain value), instead of being a specific range. Thus Fig. 7 shows that the distribution of velocity values obtained from these methods was not within the set range. The static regain method produced the widest distribution range and relatively low system velocity.

5. Air Flow Rate at Each Return Outlet

Table 5 shows that flow errors under three conventional methods were relatively big; the velocity method had errors up to 68.49%, and the equal friction method had errors up to 124.86%, followed by 104.04%. The biggest error under the static regain method was 58.92%, and under the *T*-method was 81.92%. Dynamic programming produced the smallest errors overall with the biggest being 25.71%. It is also found that the three conventional design

Table 4 Pressure value and initial cost of designed nodes at Level 9

Node 28				
$P_{28}^{(1)}$	$D_{28-27}^{(1)}$	$V_{28-27}^{(1)}$	$P_{28,27}^{(1)}$	$Es_{28-27}^{(1)}+Es_{27}^{(\min)}=Es_{28}^{(1)}$
$P_{28}^{(2)}$	$D_{28-27}^{(2)}$	$V_{28-27}^{(2)}$	$P_{28,27}^{(2)}$	$Es_{28-27}^{(2)}+Es_{27}^{(\min)}=Es_{28}^{(2)}$
…	…	…	…	… …
…	…	…	…	… …
$P_{28}^{(S)}$	$D_{28-27}^{(S)}$	$V_{28-27}^{(S)}$	$P_{28,27}^{(S)}$	$Es_{28-27}^{(3)}+Es_{27}^{(\min)}=Es_{28}^{(3)}$

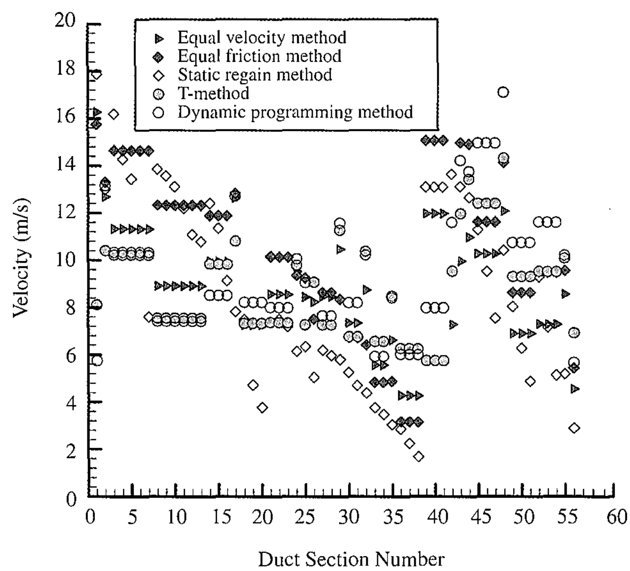

Fig. 7 Comparison of simulated sectional velocity yielded by different methods

methods had similar error tendencies, meaning that when one of the methods had a bigger error at a certain return outlet, the same situation occurred with the other two methods. This is because these three methods did not set strict limits on parameters such as duct size, velocity and air flow since optimal life-cycle cost was not taken into consideration. Thus their flow control is far inferior to that of *T*-method and dynamic programming, which showed less flow error. Dynamic programming produced the smallest error of all, because it can find the optimal design while satisfying all constraints. *T*-method has the same objective functions as dynamic programming. But the total pressure drop obtained from this method will not be balanced with the inclusion of other constraints (such as duct size or air velocity) and cannot meet the requirement of convergence, meaning the design is not a real optimal design. Dynamic programming showed small errors in paths other than path 13. This is because the pressure loss coefficient of some sections on path 13 did not have the correct

Table 5 Comparison of flow rate at each return outlet

Duct	Design Value	Velocity Method		Equal Friction Method		Static Regain Method		T Method		Dynamic Programming Method	
	Flow rate (m^3/s)	Flow rate (m^3/s)	Error (%)	Flow rate (m^3/s)	Error (%)	Flow rate (m^3/s)	Error (%)	Flow rate (m^3/s)	Error (%)	Flow rate (m^3/s)	Error (%)
7	0.37	0.56	+51.35	0.832	+124.86	0.588	+58.92	0.49	+32.43	0.345	-6.76
17	1.06	1.786	+68.49	1.604	+51.32	1.647	+55.38	1.259	+18.77	0.977	-7.83
20	1.05	1.014	-3.423	0.909	-13.43	1.218	+16.00	1.539	+46.57	0.969	-7.71
24	0.53	0.672	+26.79	0.692	+30.57	0.726	+36.98	0.651	+22.83	0.489	-7.74
26	0.50	0.548	+9.6	0.534	+6.80	0.622	+24.40	0.604	+20.80	0.457	-8.60
29	1.72	2.399	+39.48	1.512	-12.09	2.194	+27.56	1.932	+12.33	1.639	-4.71
32	1.72	2.002	+16.4	1.161	-32.50	1.852	+7.67	1.879	+9.24	1.628	-5.35
35	0.72	0.637	-11.53	0.455	-36.81	0.624	-13.33	0.775	+7.64	0.657	-8.75
38	0.61	0.349	-42.79	0.262	-57.05	0.394	-35.41	0.762	+24.92	0.554	-9.18
47	0.70	0.96	+37.14	1.054	+50.57	0.824	+17.71	0.948	+35.43	0.717	-2.43
48	0.47	0.754	+60.43	0.959	+104.04	0.669	+42.34	0.664	+41.28	0.483	+2.77
51	0.33	0.304	-7.88	0.448	+35.76	0.394	+19.39	0.49	+48.48	0.344	+4.24
55	0.56	0.637	+13.75	0.742	+32.50	0.684	+22.14	0.689	+23.04	0.704	+25.71
56	0.26	0.158	-39.23	0.236	-9.23	0.21	-19.23	0.473	+81.92	0.209	-19.62

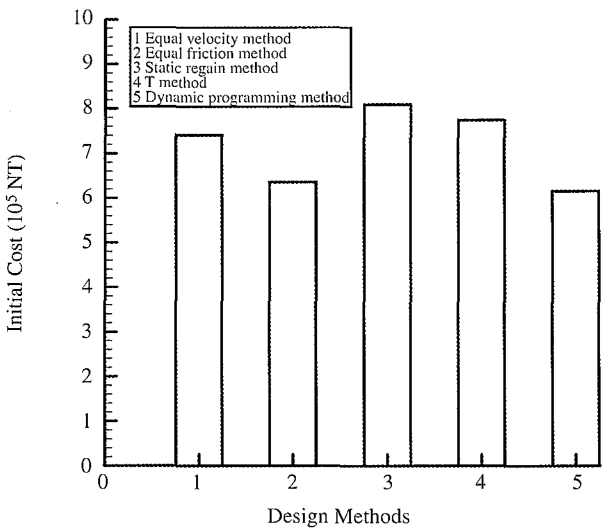

Fig. 8 Comparison of initial cost

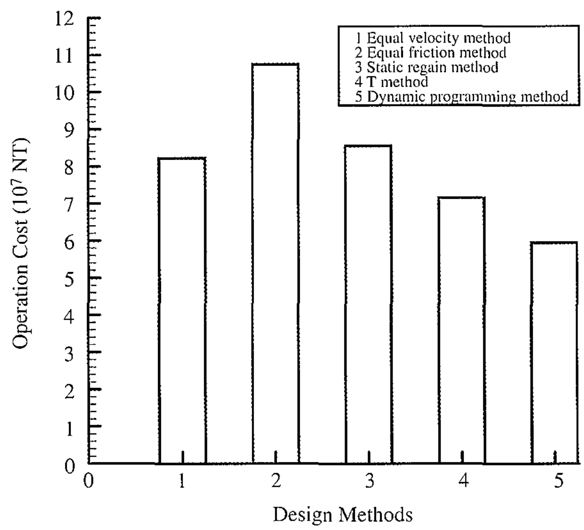

Fig. 9 Comparison of operating cost

setting. Overall, dynamic programming produced the smallest air flow error, while system designed under equal friction showed the largest error.

6. Life-cycle Cost

Figures 8 to10 compare the initial cost, energy cost and life-cycle cost of the system. Fig. 8 shows that initial cost was the lowest with dynamic programming and highest with static regain, because the latter designed the largest duct size and the former produced the smallest duct size. Fig. 9 shows that the energy cost was the lowest with dynamic programming and highest with equal friction, because the former had the smallest value from operating point pressure multiplied by air flow and the latter had the largest such value. Life-cycle cost is the sum of initial cost and operating cost. Fig. 10 shows that both dynamic programming and *T*-method could achieve minimization of life-cycle cost with the former producing the smallest life-cycle cost. Aside from the life-cycle cost, some systems with pressure imbalance would require damper or other balancing devices for pressure adjustment, which also adds to the cost. In

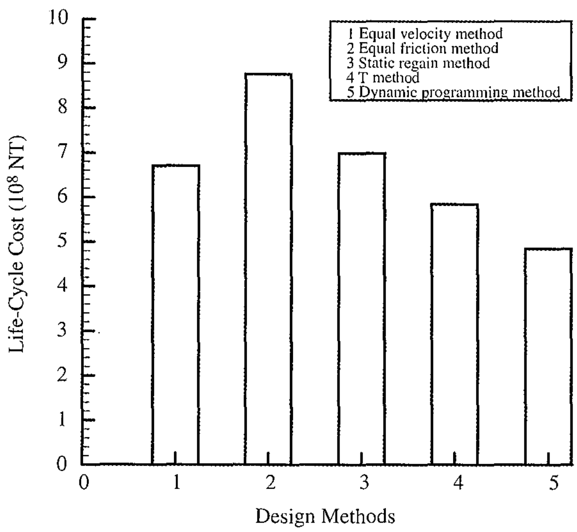

Fig. 10 Comparison of life-cycle cost

fact, except for dynamic programming, designs of other methods would face the problem of pressure imbalance under the constraints and incur additional cost for flow rate adjustment.

V. CONCLUSIONS

Dynamic programming with criterion of minimum cost yields a design with a lower lower-cycle cost than other methods when applied to the design of an exhaust duct system. Designs by conventional me-thods, such as velocity method, equal friction method and static regain method, do not obtain pressure balance. The same problem occurs with *T*-method if other constraints, such as duct size or velocity, are included in the design. Dynamic programming can find the optimum design while satisfying all constraints and pressure balance. The differences between the design and simulation results using dynamic programming are much lower than those of other methods. Thus a process exhaust duct system that approximates the actual operation best may be designed using dynamic programming method.

NOMENCLATURE

a	amortization period (year)
A	cross section area (m^2)
AER	annual escalation rate
AIR	annual interest rate
C	local loss coefficient
D	diameter of round section (m)
D_h	hydraulic diameter (m)
E	life-cycle cost ($)
E_c	unit price of electricity ($/W-h)
E_p	energy cost in first-year operation ($)
E_s	initial cost ($)
H	height of rectangular section (m)
L	section length (m)
P_t	total pressure (Pa)
P_{fan}	total pressure of fan (Pa)
ΔP_l	total pressure drop (Pa)
$PWEF$	present worth escalation factor
Re	Reynolds number
S_d	duct unit area cost ($/m^2)
Q	flow rate (m^3/s)
Q_{fan}	flow rate of fan (m^3/s)
V	mean velocity (m/s)
W	height of rectangular section (m)
Y	annual operation hours of system (h/year)
Y_i^*	minimum cost ($)

Greek Symbol

η_f	fan efficiency (%)
η_c	efficiency of motor (%)
ε	absolute roughness (m)
ν	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)

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