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On D-stability and structured singular values

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Abstract

It is shown in this paper that there are close connections between the notion of D-stability of a real square matrix and several quantities related to the structured singular value. As a main result, we show that a real square matrix is D-stable if and only if the real structured singular value of some complex matrix is less than one. This condition implies that checking D-stability may in general be an NP-hard problem. Since the exact verification of D-stability is difficult, we provide several additional conditions that are either necessary or sufficient, and these conditions are also connected to the real or complex structured singular values. These results are further extended to the notion of strong D-stability.

Keywords: D-stability; Strong D-stability; Structured singular values

1. Introduction and preliminaries

The notion of D-stability of a real square matrix was introduced by Arrow and McManus [4] and Enthoven and Arrow [9] in studying stability of equilibria in competitive market dynamic models. Recently, this notion has also found its utility in problems ranging from multiparameter singular perturbation to stability of large-scale systems (cf. [1, 3, 14, 21]), and has been further examined in the context of robust stability analysis [2]. A real square matrix A is said to be D-stable if for any positive diagonal matrix D, DA is stable, i.e., all the eigenvalues of DA have positive[1] real parts. D-stability is in general difficult to verify except for matrices of a size 3 by 3 or less [13, 22].

Many sufficient conditions exist for D-stability (see, e.g., [13]). We mention below those which are of interest to us in this paper. We shall say that a matrix $A \in \mathbb{R}^{n\times n}$ has the property P1 if there exists an $L \in \mathscr{D}$ such that $A^{\mathrm{T}}L + LA$ is positive definite, where $\mathscr{D}$ denotes the set of $n \times n$ positive diagonal matrices and the superscript T denotes transpose. Also, we say that a matrix $A \in \mathbb{R}^{n\times n}$ has the property P2 if, for any $0 \neq x \in \mathbb{C}^n$, there exists an $L \in \mathscr{D}$ such that $x^{\mathrm{H}}(A^{\mathrm{T}}L + LA)x > 0$, where x^{H} denotes the complex conjugate transpose of x. Clearly, property P1 implies the property P2; however, the converse need not be true. The following result states that both the properties P1 and P2 imply D-stability.

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[1] To be consistent with literature on D-stability, we define stability in the positive sense.

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Fact 1 (see, e.g., Johnson [13]). *Let $A \in \mathbb{R}^{n\times n}$. If A has property P1 or P2, then A is D-stable.*

Our main goal in this paper is to show that the D-stability and the properties P1 and P2 can be characterized using a notion called the structured singular value and related quantities. The concept of structured singular value was introduced in [8, 19] as a tool for analysis and synthesis of linear control systems with block diagonal complex matrix perturbations. Later, this notion was extended by de Gaston and Safonov [12] to cases with real matrix perturbations, and by Fan [11] to cases with both real and complex perturbations, and the extended notions may be conveniently referred to as *real* and *mixed* structured singular values. We shall need to introduce some additional notations to define these quantities. First, the symbol I denotes the identity matrix of an appropriate dimension and j denotes the imaginary unit $\sqrt{-1}$. For a square complex matrix M, we denote by $\bar{\sigma}(M)$ its largest singular value, by $|M|$ its determinant and by M^{H} its complex conjugate transpose. Furthermore, we denote by $\rho(M)$ its spectral radius and by $\rho_{\mathbb{R}}(M)$ the maximum modulus of all its real eigenvalues, i.e.,

$$\rho_{\mathbb{R}}(M) = \max\{|\lambda|: |\lambda I - M| = 0, \lambda \in \mathbb{R}\}.$$

If M is a Hermitian matrix, we denote by $\bar{\lambda}(M)$ its largest eigenvalue, and we write $M > 0$ (resp. $M < 0$) if it is positive definite (resp. negative definite). Given a complex vector x, we denote by $\|x\|$ its Euclidean norm. Finally, $\mathscr{D}$ represents the set of positive diagonal matrices in $\mathbb{R}^{n\times n}$ and $\bar{\mathscr{D}}$ is the closure of $\mathscr{D}$.

Consider a matrix $M \in \mathbb{C}^{n\times n}$ and the set $\mathscr{X}(\gamma)$ defined by

$$\mathscr{X}(\gamma) = \{\mathrm{diag}(\delta_1,\dots,\delta_n): \delta_i \in \mathbb{C}, |\delta_i| \leqslant \gamma\}.$$

The (complex) structured singular value[2] $\mu_{\mathscr{X}}(M)$ of M is given by $\mu_{\mathscr{X}}(M) = 0$ if there is no $\Delta \in \mathscr{X}(\infty)$ such that $|I - M\Delta| = 0$; otherwise,

$$\mu_{\mathscr{X}}(M) = \left(\min_{\Delta \in \mathscr{X}(\infty)} \{\bar{\sigma}(\Delta): |I - M\Delta| = 0\}\right)^{-1}.$$

The real structured singular value[3] $\mu_{\mathscr{Y}}(M)$ of M is defined analogously except that $\mathscr{X}(\gamma)$ is replaced by a subset $\mathscr{Y}(\gamma) = \mathscr{X}(\gamma) \cap \mathbb{R}^{n\times n}$, i.e., $\mathscr{Y}(\gamma) = \{\mathrm{diag}(\delta_1,\dots,\delta_n): \delta_i \in \mathbb{R}, |\delta_i| \leqslant \gamma\}$. The following results are collected from [8, 11, 7, 15], and they provide useful relations between these structured singular values and the related quantities.

Fact 2. *Let $M = \{m_{ij}\} \in \mathbb{C}^{n\times n}$. Define $\bar{M} = \{|m_{ij}|\} \in \mathbb{R}^{n\times n}$. Furthermore, define*

$$\hat{\mu}(M) = \inf_{D \in \mathscr{D}} \bar{\sigma}(DMD^{-1}),$$

$$\nu(M) = \max_{\Delta \in \mathscr{X}(1)} \rho(M\Delta),$$

$$\eta(M) = \max_{\Delta \in \mathscr{Y}(1)} \rho_{\mathbb{R}}(M\Delta).$$

Then, the following inequalities hold:

$$\eta(M) = \mu_{\mathscr{Y}}(M) \leqslant \nu(M) = \mu_{\mathscr{X}}(M) \leqslant \hat{\mu}(M) \leqslant \min\{\bar{\sigma}(M), \rho(\bar{M})\}.$$

Moreover, $\mu_{\mathscr{X}}(M) = \hat{\mu}(M)$ if $n \leqslant 3$, and $\eta(M) = \max_{\Delta \in \mathscr{Q}} \rho_{\mathbb{R}}(M\Delta)$ if $M \in \mathbb{R}^{n\times n}$, where $\mathscr{Q} = \{\mathrm{diag}\,(\delta_1,\dots,\delta_n) \in \mathscr{Y}(1): \delta_i = \pm 1\}$.

[2] This corresponds to a special block structure of the original definition of SSV; see [8] for details.

[3] This also corresponds to a special block structure of the original definition of real SSV; see [11] for details.

Our main results in this paper may be summarized as follows. First, we show that verifying D-stability of a real matrix is equivalent to computing the real structured singular value of a complex matrix. Secondly, we show that the property P2 is equivalent to a condition on the complex structured singular value of a real matrix, and the property P1 is equivalent to a condition on a bound of the complex structured singular value. Finally, we give a necessary condition stated in terms of the real structured singular value for a real matrix. These results are then extended to the notion of strong D-stability.

2. Main results

We shall now present the main results of this paper. Our first result provides a characterization of D-stability in terms of the real structured singular value for a complex matrix.

Theorem 1. *Let $A \in \mathbb{R}^{n\times n}$. Then A is D-stable if and only if A is stable and*

$$\mu_{\mathscr{Y}}((\mathrm{j}I + A)^{-1}(\mathrm{j}I - A)) < 1.$$

Proof. From [13], it follows that A is D-stable if and only if A is stable and $|A + \mathrm{j}D| \neq 0$ for all $D \in \mathscr{D}$, or equivalently, for all $D \in \bar{\mathscr{D}}$. Let $R = (I - D)(I + D)^{-1}$. Then, $D \in \bar{\mathscr{D}}$ if and only if $R \in \mathscr{Y}(1)$. Note that $|A + \mathrm{j}D| \neq 0$ for all $D \in \bar{\mathscr{D}}$ if and only if $|A + \mathrm{j}(I + R)^{-1}(I - R)| \neq 0$ for all $R \in \mathscr{Y}(1)$, which is equivalent to that $|(\mathrm{j}I + A) - (\mathrm{j}I - A)R| \neq 0$ for all $R \in \mathscr{Y}(1)$. The latter condition is further equivalent to that $|I - (\mathrm{j}I + A)^{-1}(\mathrm{j}I - A)R| \neq 0$ for all $R \in \mathscr{Y}(1)$. This, however, is precisely the condition $\mu_{\mathscr{Y}}((\mathrm{j}I + A)^{-1}(\mathrm{j}I - A)) < 1$. Hence, the proof is completed. □

Remark 1. A recent result of [6] shows that the exact computation of the real structured singular value is in general an NP-hard problem. Theorem 1 establishes the equivalence between the D-stability of a real matrix and a condition on the real structured singular value of a complex matrix with a somewhat *special* structure. This result thus indicates a strong possibility, though by no means a conclusive statement, that the exact verification of D-stability may also be an NP-hard problem, and that the problem of checking D-stability may be generally intractable when n is large.

Our second result shows that the property P2 is equivalent to a condition on the complex structured singular value of a certain real matrix. Since the exact computation of the complex structured singular value is also difficult, the property P2 is difficult to verify as well.

Theorem 2. *Let $A \in \mathbb{R}^{n\times n}$. Then A has the property P2 if and only if A is stable and*

$$\mu_{\mathscr{X}}((I + A)^{-1}(I - A)) < 1.$$

Remark 2. A typical method [8] for computing the complex structured singular value $\mu_{\mathscr{X}}(M)$ seeks to compute $\hat{\mu}(M)$ and $\nu(M)$. For a general matrix M with a size greater than 3×3, $\hat{\mu}(M)$ is known to be strictly greater than $\mu_{\mathscr{X}}(M)$. However, empirical results [8, 10] show that it is often within a factor of 15% of $\mu_{\mathscr{X}}(M)$. On the other hand, the quantity $\nu(M)$ is equal to $\mu_{\mathscr{X}}(M)$; however, the computation of $\nu(M)$ is difficult [8, 10].

To prove Theorem 2, we will employ the following lemma, which can be easily derived from a result in [11].

Lemma 1. *For any matrix $M \in \mathbb{C}^{n\times n}$, the following equality holds:*

$$\mu_{\mathscr{X}}(M) = \max_{\substack{0 \neq x \in \mathbb{C}^n \\ \gamma \geq 0}} \{\gamma\colon \|DMx\| \geq \gamma \|Dx\| \text{ for all } D \in \mathscr{D}\}.$$

Proof of Theorem 2. Let $M = (I + A)^{-1}(I - A)$. Then, in view of Lemma 1, $\mu_{\mathscr{X}}(M) < 1$ if and only if for each $0 \neq x \in \mathbb{C}^n$, there exists $D \in \mathscr{D}$ such that $\|DMx\| < \|Dx\|$, or, equivalently, there exists a $D \in \mathscr{D}$ such that $\|DM(I + A)x\| < \|D(I + A)x\|$. However, $\|DM(I + A)x\| < \|D(I + A)x\|$ if and only if $\|DM(I + A)x\|^2 < \|D(I + A)x\|^2$, which, alternatively, can be stated as $x^H((I - A)^T D^2 (I - A) - (I + A)^T D^2 (I + A))x < 0$. It is easy to verify that the latter condition is precisely $x^H(A^T D^2 + D^2 A)x > 0$. This completes the proof. □

The verification of the property P1, however, is tractable. The following result shows that this property is equivalent to a condition on the diagonally scaled upper bound for the complex structured singular value, and that this condition may be checked by solving a convex optimization problem.

Theorem 3. *Let $A \in \mathbb{R}^{n \times n}$. Then A has the property P1 if and only if A is stable and*

$$\hat{\mu}((I + A)^{-1}(I - A)) < 1.$$

Furthermore, this condition is equivalent to

$$\min_{D \in \mathscr{D} \cap \mathscr{Y}(1)} \bar{\lambda}(-A^T D - DA) < 0.$$

Proof. Again, let $M = (I + A)^{-1}(I - A)$. It follows that $\hat{\mu}(M) < 1$ if and only if

$$\bar{\sigma}(D(I + A)^{-1}(I - A)D^{-1}) < 1$$

for some $D \in \mathscr{D}$. This condition can be alternatively stated as

$$\bar{\lambda}(D^{-1}(I - A)^T (I + A)^{-T} D^2 (I + A)^{-1}(I - A)D^{-1}) < 1$$

and therefore is equivalent to

$$D^{-1}(I - A)^T (I + A)^{-T} D^2 (I + A)^{-1}(I - A)D^{-1} - I < 0.$$

A simple manipulation of this matrix inequality shows further that it is equivalent to $A^T D^2 + D^2 A > 0$. Hence, we have proved the first part of the theorem. The second part follows directly by definition. □

Remark 3. Given any $M \in \mathbb{C}^{n \times n}$, it was shown in e.g., [20, 17] that $\hat{\mu}(M)$ can be computed by solving a convex optimization problem. Hence, the first condition in Theorem 3 shows that the property P1 can be verified by solving a convex optimization problem as well. This assertion is also demonstrated by the second condition in Theorem 3. Indeed, the latter is one of the standard eigenvalue minimization problems, which, as shown in [16, 5], can be solved using several available methods.

As an immediate consequence of Fact 2 and Theorems 2 and 3, the following corollaries are clear.

Corollary 1. *Suppose that $n \leqslant 3$. Then A has the property P1 if and only if it has the property P2.*

Corollary 2. *Suppose that A is stable. Let $M = (I + A)^{-1}(I - A)$. Then A has the property P1 and P2 if $\bar{\sigma}(M) < 1$ or $\rho(\bar{M}) < 1$.*

Finally, we give the following necessary condition for D-stability.

Theorem 4. *Let $A \in \mathbb{R}^{n \times n}$. A necessary condition for A to be D-stable is that A is stable and*

$$\mu_{\mathscr{Y}}((I + A)^{-1}(I - A)) < 1.$$

Proof. By definition, A is D-stable if and only if AD is stable for all $D \in \mathscr{D}$. Thus, a necessary condition for A to be D-stable is that $|I + AD| \neq 0$ for all $D \in \bar{\mathscr{D}}$. Let $D = (I + R)^{-1}(I - R)$. Then, the necessary condition is equivalent to that $|I + A(I + R)^{-1}(I - R)| \neq 0$, or $|(I + A) + (I - A)R| \neq 0$ for all $R \in \mathscr{Y}(1)$. Since

$|I + A| \neq 0$, this is further equivalent to $|(I + A) + (I - A)R| \neq 0$ for all $R \in \mathscr{Y}(1)$. The result then follows from the definition of $\mu_{\mathscr{Y}}(\cdot)$. □

Remark 4. The difference between the condition in Theorem 1 and that in Theorem 4 is that the latter amounts to computing the real structured singular value of a *real* matrix, and hence it can be checked via a finite number of steps. Specifically, it follows from Fact 2 that in this case the quantity $\eta((I + A)^{-1}(I - A))$ can be computed via a finite number of evaluations.

3. Extensions to strong D-stability

The notion of strong D-stability was introduced by Abed [1] in attempts to study *robustness* of D-stability properties. A real square matrix A is said to be *strongly D-stable* if it is D-stable and remains D-stable for sufficiently small perturbations of A. Specifically, A is strongly D-stable if A is D-stable and there exists a $\varepsilon > 0$, such that for any $\Delta A \in \mathbb{R}^{n \times n}$ with $\bar{\sigma}(\Delta A) < \varepsilon$, $A + \Delta A$ is D-stable. Clearly, strong D-stability implies D-stability; however, a simple example may demonstrate that the converse need not be true (see, e.g., [1]).

It follows, nevertheless, from the continuity properties of the matrix $A^T L + LA$, where $L \in \mathscr{D}$, that both the properties P1 and P2 imply strong D-stability. Hence, both Theorems 2 and 3 in the preceding section may be used to test strong D-stability. Since strong D-stability implies D-stability, Theorem 4 is a necessary condition for strong D-stability as well. It turns out that strong D-stability may also be characterized using the real structured singular value with respect to a modified block structure. Specifically, we may consider the block structure

$$\mathscr{X}(\gamma) = \{\mathrm{diag}(\Delta_1 \;\; \Delta_2)\colon \Delta_1 \in \mathscr{Y}(\gamma), \Delta_2 \in \mathbb{R}^{n \times n}, \bar{\sigma}(\Delta_2) \leqslant \gamma\}.$$

Furthermore, we may define analogously the real structured singular value $\mu_{\mathscr{X}}(\cdot)$ with respect to $\mathscr{X}(\gamma)$, which has exactly the same form as $\mu_{\mathscr{X}}(\cdot)$ but is defined with respect to a different structure.

Theorem 5. *Let $A \in \mathbb{R}^{n \times n}$. Suppose that $\mu_{\mathscr{X}}(\cdot)$ is defined as above. Then A is strongly D-stable if and only if A is stable and there exists some $\varepsilon > 0$ such that $\mu_{\mathscr{X}}(M) < 1$, where*

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = \begin{bmatrix} (\mathrm{j}I + A)^{-1}(\mathrm{j}I - A) & 2\mathrm{j}(\mathrm{j}I + A)^{-1} \\ \varepsilon(\mathrm{j}I + A)^{-1} & -\varepsilon(\mathrm{j}I + A)^{-1} \end{bmatrix}.$$

Proof. According to Theorem 1, A is strongly D-stable if and only if A is stable and there exists some $\varepsilon > 0$ such that for all $\Delta A \in \mathbb{R}^{n \times n}$ with $\bar{\sigma}(\Delta A) < \varepsilon$, the inequality

$$\mu_{\mathscr{Y}}((\mathrm{j}I + A + \Delta A)^{-1}(\mathrm{j}I - A - \Delta A)) < 1$$

holds. Let $M(\Delta A) = (\mathrm{j}I + A + \Delta A)^{-1}(\mathrm{j}I - A - \Delta A)$. Then

$$\begin{aligned} M(\Delta A) &= (\mathrm{j}I + A + \Delta A)^{-1}(\mathrm{j}I - A - \Delta A) \\ &= 2\mathrm{j}(\mathrm{j}I + A + \Delta A)^{-1} - I \\ &= 2\mathrm{j}(I + (\mathrm{j}I + A)^{-1}\Delta A)^{-1}(\mathrm{j}I + A)^{-1} - I \\ &= 2\mathrm{j}(\mathrm{j}I + A)^{-1} - 2\mathrm{j}(\mathrm{j}I + A)^{-1}\Delta A(I + (\mathrm{j}I + A)^{-1}\Delta A)^{-1}(\mathrm{j}I + A)^{-1} - I \\ &= (\mathrm{j}I + A)^{-1}(\mathrm{j}I - A) - 2\mathrm{j}(\mathrm{j}I + A)^{-1}\Delta A(I + (\mathrm{j}I + A)^{-1}\Delta A)^{-1}(\mathrm{j}I + A)^{-1} \\ &= M_{11} - M_{12}\frac{\Delta A}{\varepsilon}\left(I - M_{22}\frac{\Delta A}{\varepsilon}\right)^{-1} M_{21}. \end{aligned}$$

This shows that $M(\Delta A)$ is a *linear fractional transform* of $\Delta A/\varepsilon$. Hence, it follows from [17] that $\mu_{\mathscr{Y}}(M(\Delta A)) < 1$ for all $\bar{\sigma}(\Delta A) < \varepsilon$ if and only if $\mu_{\mathscr{Y}}(M_{11}) < 1$ and $\mu_{\mathscr{X}}(M) < 1$. This completes the proof. □

Remark 5. Following an argument based on continuity properties, we may infer strong D-stability from D-stability for a specific matrix A by examining the continuity property of $\mu_{\mathscr{Y}}(\mathrm{j}I + A)^{-1}(\mathrm{j}I - A)$ with respect to A. It follows that a D-stable matrix A is strongly D-stable if and only if $\mu_{\mathscr{Y}}((\mathrm{j}I + A)^{-1}(\mathrm{j}I - A))$ is continuous with respect to the elements of A. Similarly, one can also establish the strong D-stability of A by establishing the continuity of $\mu_{\mathscr{X}}(M)$ at $\varepsilon = 0$, where M is defined in Theorem 5. It follows that A is strongly D-stable if and only if A is D-stable and in addition

$$\lim_{\varepsilon \to 0} \mu_{\mathscr{X}}(M) = \mu_{\mathscr{Y}}((\mathrm{j}I + A)^{-1}(\mathrm{j}I - A)).$$

Unfortunately, the verification of continuity properties for both $\mu_{\mathscr{Y}}((\mathrm{j}I + A)^{-1}(\mathrm{j}I - A))$ and $\mu_{\mathscr{X}}(M)$ may be complicated by the fact that the real structured singular value may not be a continuous function [18]. This is in sharp contrast to the fact that both the complex structured singular value $\mu_{\mathscr{X}}((I + A)^{-1}(I - A))$ and its upper bound $\hat{\mu}((I + A)^{-1}(I - A))$ are continuous functions of A (see, e.g., [18]). The continuity properties of these two quantities explain from an alternative perspective why the properties P1 and P2 are invariant of perturbations.

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