Adaptive fuzzy CMAC control for a class of nonlinear systems with smooth compensation

T.-F. Wu, P.-S. Tsai, F.-R. Chang and L.-S. Wang

Abstract: Adaptive fuzzy cerebellar model articulation controller (CMAC) schemes are proposed to solve the tracking problem for a class of nonlinear systems. The proposed method provides a simple control architecture that merges CMAC and fuzzy logic, so that the complicated structure and the input space dimension in CMAC can be simplified. Adaptive laws are developed to tune all of the control gains online, thereby accommodating the uncertainty of nonlinear systems without any learning phase. In particular, smooth compensation is adopted to overcome the chattering problem associated with conventional switching compensation. By Lyapunov stability analysis, it is guaranteed that all of the closed-loop signals are bounded and the tracking errors converge exponentially to a residual set whose size can be adjusted by changing the design parameters. Simulation results for its applications to three examples are presented to demonstrate the performance of the proposed methodology.

1 Introduction

Owing to its intrinsic difficulty, interest on the control of nonlinear systems has persisted for many years. Various control methodologies have been developed from the perspective of system theory and traditional feedback control theory [1, 2]. Notably, these methods mostly depend on a thorough understanding of the controlled system's dynamics, which makes their application unfavourable for uncertain systems. To deal with uncertainties on dynamical models or disturbances, some techniques in intelligent control have found an application. Examples include neural networks (NNs) using appropriated learning phases [3-5] and fuzzy control by capturing human experiences [6-9].

One subclass of NNs, introduced by Albus [10], called the cerebellar model articulation controller (CMAC), has attracted much attention because of faster learning, better generalisation and simpler computation. In particular, a trained CMAC can approximate nonlinear functions in a generalised lookup-table manner over a domain to any desired accuracy. Numerous researchers have applied it to design the controller of unknown nonlinear systems such as robot manipulators [11] and spacecraft [12]. Recently, various modifications of CMAC have been proposed to enhance the performance. The CMAC with a robust compensation achieves H^{∞} tracking performance [13]. The merging of CMAC and the Hamilton–Jacobin–Bellman (HJB) optimisation theory yields an optimal control design [14]. Combining a fuzzy reasoning mechanism, the resulting fuzzy CMAC (FCMAC) brings about a simple control architecture [3, 12, 15, 16].

Traditionally, the weights in CMAC were trained by an off-line learning phase, so the setting of CMAC may take a long time. The effectiveness of CMAC is limited in treating the problem that requires online tuning. Several studies have suggested the use of the adaptive law to update the CMAC weights online. The tracking performance of CMAC coupled with adaptive laws has been shown by Peng and Woo [3] and Kim and Lewis [14] for the robot manipulators, by Wai et al. [17] for linear piezoelectric ceramic motors and by Lin and Peng [18] for a Chua's chaotic circuit. In these applications, a compensation is required in the adaptive CMAC to attenuate the error of CMAC approximation. This compensation is usually designed to involve a switching function, which gives rise to chattering on the control signals, and an undesirable phenomenon may be excited in turn [1, 2].

To solve the chattering problem without sacrificing the performance, we propose a modified adaptive FCMAC (AFCMAC) scheme on the basis of previous work [16], for dealing with the tracking problem of a class of nonlinear systems. The AFCMAC approximation is adopted as rough tuning, and the smooth compensation is developed as fine tuning, so that (i) the design methodology is easy to realise, (ii) all of the control gains, including the CMAC weights, can be updated online without a prior learning phase, (iii) chattering can be prevented, and (iv) the tracking performance is guaranteed.

2 Structure of fuzzy CMAC

This section introduces the basic structure of CMAC and its modification called the FCMAC that will be used later in this paper. In general, to achieve the desired accuracy with a FCMAC, a complicated structure and a sufficient number of rules may be constructed, so that the dimension of the underlying system becomes higher [19]. However, for the real-time requirement in physical applications, the computational load increases along with the complexity of CMAC and may cause instability owing to the effect of

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time-delay. Some advanced control theories should be integrated with a FCMAC to enhance the system performance if the number of rules is reduced. Consequently, simple controller structures are used in many proposed schemes.

2.1 Basic CMAC design

The physical system to be controlled is assumed to have only one control input and all of the state variables are assumed available. Therefore a single-output CMAC [15] is designed and the output is given by

$$z_{\rm CMAC} = F(s) \tag{1}$$

where $F: \mathbb{R}^L \to \mathbb{R}$ is a nonlinear function of CMAC input variable $s = [s_1, \ldots, s_L]^T \in S \subset \mathbb{R}^L$. To mimic the operation of the cerebellum, the inputs (sensors) are related to the output (response) through an association mechanism with the association memory space A. Any element in A consists of M number of 0s and 1s according to the pattern of the inputs. Mathematically, the relation (1) can be represented by a pair of mappings

$$G: S \longrightarrow A; \quad s \longmapsto G(s) = a(s) \in A$$
 (2)

$$P: A \longrightarrow \mathbf{R}; \quad \mathbf{a} \longmapsto P(\mathbf{a}) \tag{3}$$

In particular, we may choose the function P which generates the output z_{CMAC} as follows

$$z_{\rm CMAC} = P(\boldsymbol{a}) = \boldsymbol{a}^{\rm T} \boldsymbol{w} \tag{4}$$

where the vector *w* denotes the CMAC weight vector.

As an example, we consider the case with two input variables s_1 and s_2 in the range of [-2, 2]. Fig. 1 shows a possible partition of the input variables of the CMAC, in which both s_1 and s_2 are divided into four sub-regions such that 16 blocks (m, n), with m, n = -2, -1, 1, 2, are formed. These sub-regions are further grouped into two regions, (A, B) and (a, b), for s_1 and s_2 , respectively, in the first layer. Their combinations Aa, Ab, Ba and Bb are the hypercubes. By shifting the first layer step-by-step on the sub-regions, we obtain the second and third layers such that an association vector can be expressed as $a^T \equiv [Aa \ Ab \ Ba \ Bb \ Cc \ Cd \ Dc \ Dd \ Ee \ Ef \ Fe \ Ff]$. The map G defined in (2) is then given by associating



Fig. 1 Schematic diagram of fuzzy sets integrated with CMAC

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Fig. 2 Mapping of basic CMAC

2.2 FCMAC design

Owing to possible disturbances on the sensors, the input data, namely s, may not be exact. To accommodate this fuzziness and simplify the input partition, the structure of FCMAC was proposed by Chen *et al.* [15]. For a two-input problem, a fuzzy system with N fuzzy rules may be designed, each of which in the form of

$$\boldsymbol{R}^{(i)}$$
: IF s_1 is F_1^i and s_2 is F_2^i , THEN $\boldsymbol{z}_f^{(i)}$ is $\boldsymbol{a}_i^{\mathrm{T}} \boldsymbol{w}$ (5)

where i = 1, 2, ..., N, and the THEN part is extracted from the CMAC. Given the membership function of fuzzy set F_k^i , k = 1, 2, denoted by $\mu_{F_k^i}$, the following defuzzification process is chosen to compute the output z_{FCMAC}

$$z_{\text{FCMAC}} = \frac{v_1 \boldsymbol{a}_1^{\mathsf{T}} \boldsymbol{w} + v_2 \boldsymbol{a}_2^{\mathsf{T}} \boldsymbol{w} + \dots + v_N \boldsymbol{a}_N^{\mathsf{T}} \boldsymbol{w}}{v_1 + v_2 + \dots + v_N}$$
$$= \frac{\sum_{i=1}^N \boldsymbol{a}_i^{\mathsf{T}} \boldsymbol{w} v_i}{\sum_{i=1}^N v_i}$$
(6)

where $v_i = \prod_{k=1}^{2} \mu_{F_k^i}(s_k)$. The preceding equation may be re-written compactly as

Z

$$\mathbf{w}_{\text{FCMAC}} = \boldsymbol{h}^{\mathrm{T}} \boldsymbol{A} \boldsymbol{w} \tag{7}$$

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where

$$\boldsymbol{h} = \begin{bmatrix} h_1 & h_2 & \cdots & h_N \end{bmatrix}^{\mathrm{T}}, \quad h_i = \frac{v_i}{\sum_{i=1}^N v_i} \text{ and } \boldsymbol{A} = \begin{bmatrix} \boldsymbol{a}_1^* \\ \boldsymbol{a}_2^{\mathrm{T}} \\ \vdots \\ \boldsymbol{a}_N^{\mathrm{T}} \end{bmatrix}$$
(8)

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In (7), the matrix A (determined by CMAC) and the vector h (determined by fuzzy rules) are typically fixed, but the weight vector w is adjustable herein.

For the two-input problem, a set of membership functions may be chosen as shown in Fig. 1, in which P (positive) and N (negative) fuzzy sets are imposed on each variable. Accordingly, there are four fuzzy rules with four association vectors, a_1, a_2, a_3, a_4 , attached to (P, P), (N, P), (N, N),(P, N), respectively. It is seen that, for this example, there are 16 association vectors in CMAC, while only four are used in FCMAC. To determine a_i in FCMAC, the logical operation 'OR' is performed on all possible (in the same region) association vectors in CMAC. For instance, if (s_1, \ldots, s_n) s_2) is in the class (P, P), there are nine blocks in the region and, by performing 'OR' on the corresnine association vectors, we obtain ponding relationship between the fuzzy rules and the CMAC, from which, the matrix A in (7) is given by

Intrinsically, the chosen FCMAC is more like a fuzzy system, with the output function (7) depending on the CMAC structure. Although a fuzzy system has been proven to be a universal approximator [6], it is not easily implemented with a large number of inputs in real-time applications. In this work, we shall adopt the simplest structure of FCMAC comprising two input variables and four fuzzy rules, so that the adaptive control can be integrated easily to achieve basic performance. Moreover, to suppress the approximated error of the FCMAC, a switching compensation is imposed, and a smooth compensation is developed to resolve the arising chattering problem.

3 Adaptive FCMAC controller design

The tracking problem of a class of nonlinear systems is first described. The AFCMAC with different compensation designs is the proposed on the basis of the FCMAC introduced in Section 2.

3.1 Problem description and AFCMAC design

Consider a physical system that can be modelled by the following *n*th-order nonlinear equations [1, 18]

$$x^{(n)} = f(x, \dot{x}, \dots, x^{(n-1)}) + g(x, \dot{x}, \dots, x^{(n-1)})u$$
(10)

$$v = x \tag{11}$$

where $f, g: \mathbb{R}^n \to \mathbb{R}$ are continuous nonlinear bounded functions; $u \in \mathbb{R}$ is the system input and $y \in \mathbb{R}$ is the system output. Define the state vector $x \in \mathbb{R}^n$ as

$$\boldsymbol{x} = [x_1, x_2, \dots, x_n]^{\mathrm{T}} \equiv [x, \dot{x}, \dots, x^{(n-1)}]^{\mathrm{T}}$$
 (12)

For x in a certain controllability region $\Omega_c \in \mathbb{R}^n$, it is necessary that $g(x) \neq 0$. As g(x) is continuous, we may assume that g(x) > 0 for all $x \in \Omega_c$ without loss of generality. Furthermore, to implement the adaptive law proposed in this paper, the function g is assumed to be known. The goal here is to design a controller u such that the system output y follows a desired smooth trajectory y_d . Letting $\tilde{y} \equiv y - y_d$, the aggregate tracking error vector \tilde{y} is defined as

$$\tilde{y} = [\tilde{y}_1, \ \tilde{y}_2, \dots, \tilde{y}_n]^{\mathrm{T}} \equiv [\tilde{y}, \ \dot{\tilde{y}}, \dots, \tilde{y}^{(n-1)}]^{\mathrm{T}}$$
(13)

Now, if the system dynamics f is known, we may apply the ideal control law u^* given by

$$u^* = g^{-1}(\mathbf{x})[-f(\mathbf{x}) + y_d^{(n)} - \mathbf{c}^{\mathrm{T}}\tilde{\mathbf{y}}]$$
(14)

where $\mathbf{c} \equiv [c_n, c_{n-1}, \dots, c_1]^{\mathrm{T}}$ with $c_i, i = 1, 2, \dots, n$ being positive constants such that the polynomial $\Delta(\lambda) = \lambda^n + c_1 \lambda^{n-1} + \dots + c_n$ is Hurwitz. With (14), the error dynamics of the closed-loop system becomes

$$\tilde{y}^{(n)} + c_1 \tilde{y}^{(n-1)} + \dots + c_n \tilde{y} = 0$$
 (15)

and it follows that the tracking error \tilde{y} exponentially approaches zero. However, the ideal control law (14) cannot be directly applied if the function f is unknown. One may need to find a scheme to approximate f, so that an approximated control law can be used. The AFCMAC provides such a scheme.

In applying the two-input FCMAC with simple structure developed in Section 2.2, the input variables are chosen as

$$s_1 = \boldsymbol{d}^{\mathrm{T}} \tilde{\boldsymbol{y}}, \quad s_2 = \dot{s}_1 = \boldsymbol{d}^{\mathrm{T}} \dot{\tilde{\boldsymbol{y}}}$$
 (16)

Table 1 [.]	Relationshin	hetween	fuzzy rules	and CMAC	
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Continuous input (s_1, s_2)		[Aa, Ab, Ba, Bb, Cc, Cd, Dc, Dd,	[Aa, Ab, Ba, Bb, Cc, Cd, Dc, Dd, Ee, Ef, Fe, Ff]			
Class	Blocks	Hypercubes	a _{<i>i</i>} ^T , <i>i</i> = 1, 2, 3, 4			
(P, P)	(-1, -1)	[1 0 0 0 0 0 0 1 1 0 0 0]				
	(-1, 1)	[1 0 0 0 0 0 0 1 0 1 0 0]				
	(-1, 2)	[0 1 0 0 0 0 0 1 0 1 0 0]				
	(1, -1)	[1 0 0 0 0 0 0 1 0 0 1 0]				
	(1, 1)	[1 0 0 0 0 0 0 1 0 0 0 1]	[1 1 1 1 0 0 0 1 1 1 1 1]			
	(1, 2)	[0 1 0 0 0 0 0 1 0 0 0 1]				
	(2, -1)	[0 0 1 0 0 0 0 1 0 0 1 0]				
	(2, 1)	[0 0 1 0 0 0 0 1 0 0 0 1]				
	(2, 2)	[0 0 0 1 0 0 0 1 0 0 0 1]				
(<i>N, P</i>)	÷		[1 1 0 0 0 1 0 1 1 1 1 1]			
(N, N)	:		[1 0 0 0 1 1 1 1 1 1 1]			
(<i>P</i> , <i>N</i>)	÷		[1 0 1 1 0 0 1 1 1 1 1]			

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where $\boldsymbol{d} = [d_1, d_2, \ldots, d_n]^T$ is the coefficient vector. The errors of different orders are thus synthesised in one variable, which is similar to the concept of the sliding surface [1, 15]. With the optimal weight vector \boldsymbol{w}^* that is assumed to exist, it is desired that the output of FCMAC (7), rewritten as $\boldsymbol{u}_{\text{FCMAC}}^* = \boldsymbol{h}^T \boldsymbol{A} \boldsymbol{w}^*$, would be close to the ideal control feedback \boldsymbol{u}^* in (14). Let ε denote their difference

$$\varepsilon = u^* - u^*_{\text{FCMAC}} \tag{17}$$

which is assumed to be bounded with a small positive bound D [17]

$$|\varepsilon| \le D \tag{18}$$

By including u^* in the system (10) and (11), we obtain

$$y^{(n)} = x^{(n)} = f(\mathbf{x}) + g(\mathbf{x})[u - u^* + u^*]$$

= $f(\mathbf{x}) + g(\mathbf{x}) \Big[u - u^* + g^{-1}(\mathbf{x})(-f(\mathbf{x}) + y_d^{(n)} - \mathbf{c}^{\mathsf{T}} \tilde{\mathbf{y}}) \Big]$
= $y_d^{(n)} - \mathbf{c}^{\mathsf{T}} \tilde{\mathbf{y}} + g(\mathbf{x})[u - u^*]$ (19)

which implies that

$$\tilde{\tilde{y}} = F\tilde{y} + g[u - u^*]$$
⁽²⁰⁾

where

$$\boldsymbol{F} = \begin{bmatrix} \boldsymbol{0}_{(n-1)\times 1} & \boldsymbol{I}_{n-1} \\ \cdots \cdots \cdots \\ -\boldsymbol{c}^{\mathrm{T}} \end{bmatrix}, \quad \boldsymbol{g} = \begin{bmatrix} \boldsymbol{0}_{(n-1)\times 1} \\ \cdots \cdots \\ \boldsymbol{g}(\boldsymbol{x}) \end{bmatrix}$$
(21)

Replacing u^* in (20) by (17), the error equations become

$$\tilde{\mathbf{y}} = \mathbf{F}\tilde{\mathbf{y}} + \mathbf{g}[u - u_{\text{FCMAC}}^* - \varepsilon]$$
(22)

As $\Delta(\lambda)$ is Hurwitz, so then is the $n \times n$ matrix F. Hence for any $n \times n$ symmetric positive definite matrix Q, there exists an $n \times n$ symmetric positive definite matrix P, so that the following Lyapunov matrix equation holds

$$\boldsymbol{P}\boldsymbol{F} + \boldsymbol{F}^{\mathrm{T}}\boldsymbol{P} = -\boldsymbol{Q} \tag{23}$$

The solution **P** of (23) can then be used to construct a Lyapunov function $V \equiv (1/2)\tilde{y}^{T} P \tilde{y}$, with its rate being computed as

$$\dot{V} = \frac{1}{2}\tilde{\mathbf{y}}^{\mathrm{T}}(\mathbf{P}\mathbf{F} + \mathbf{F}^{\mathrm{T}}\mathbf{P})\tilde{\mathbf{y}} + \tilde{\mathbf{y}}^{\mathrm{T}}\mathbf{P}\mathbf{g}(u - u_{\mathrm{FCMAC}}^{*} - \varepsilon)$$
$$= -\frac{1}{2}\tilde{\mathbf{y}}^{\mathrm{T}}\mathbf{Q}\tilde{\mathbf{y}} + e(u - u_{\mathrm{FCMAC}}^{*} - \varepsilon)$$
(24)

where $e = \tilde{y}^{T} P g$. Now if w^{*} and *D* are available, we may use the control

$$u = u_{\rm FCMAC}^* + u_r \tag{25}$$

to steer the system, where u_r is designed as

$$u_r = -D \cdot \operatorname{sgn}(e) \tag{26}$$

with $sgn(\cdot)$ being the signum function [1, 2]

$$\operatorname{sgn}(e) \equiv \begin{cases} 1, & e > 0 \\ 0, & e = 0 \\ -1, & e < 0 \end{cases}$$
(27)

It can be then shown that the tracking error (13) will converge exponentially to zero. In this design, the switching mechanism of the compensation u_r is developed to accommodate the approximation error using FCMAC. Nevertheless, the optimal weight and the bound are mostly unknown in applications. When f is uncertain, the optimal weight vector w^* is conventionally trained by some off-line learning process, which may take a long period of time. Alternatively, the adaptive laws described subsequently can be used to estimate w^* and D online.

3.2 AFCMAC with switching compensation

As stated earlier, the unknown optimal weights raise a serious problem in the implementation of the FCMACbased scheme. To solve this problem, the adaptive laws to estimate the optimal weights and the bound are incorporated to yield the AFCMAC. We shall use the following laws to find the estimates \hat{w} and \hat{D}

$$\dot{\hat{w}} = -\gamma_1 e A^{\mathrm{T}} h \tag{28}$$

$$\hat{D} = \gamma_2 |e|, \quad \hat{D}(0) > 0$$
 (29)

where the constants γ_1 and γ_2 are the design parameters. As g(x) is known, the variable *e* is available and hence the previous laws are well-defined. With these estimates, the AFCMAC control scheme is designed as

$$u = u_{\rm AFCMAC} + u_{ar} \tag{30}$$

where

$$u_{\rm AFCMAC} = \boldsymbol{h}^{\rm T} \boldsymbol{A} \hat{\boldsymbol{w}} \tag{31}$$

$$u_{ar} = -\hat{D} \cdot \operatorname{sgn}(e) \tag{32}$$

The 'Thm. 1' part of Fig. 3 exhibits the nonlinear system subject to the AFCMAC control scheme (28)-(32), whose performance is summarised in the following theorem.

Theorem 1: If the design parameters γ_1 and γ_2 are both positive, the application of the AFCMAC control scheme (28)–(32) to the nonlinear system (10) and (11) yields a closed-loop system in which

1. all signals are bounded;

2. the tracking error \tilde{y} converges asymptotically to zero (i.e. $\tilde{y}(t) \rightarrow 0$ as $t \rightarrow \infty$).



Fig. 3 Architecture of the AFCMAC

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Proof: Consider the Lyapunov function candidate [1]

$$V \equiv \frac{1}{2}\tilde{\boldsymbol{y}}^{\mathrm{T}}\boldsymbol{P}\tilde{\boldsymbol{y}} + \frac{1}{2\gamma_{1}}\tilde{\boldsymbol{w}}^{\mathrm{T}}\tilde{\boldsymbol{w}} + \frac{1}{2\gamma_{2}}\tilde{D}^{2}$$
(33)

where $\tilde{w} \equiv \hat{w} - w^*$ and $\tilde{D} \equiv \hat{D} - D$. The time derivative of V along the trajectory of the closed-loop system derived from (22) is found as

$$\dot{V} = \frac{1}{2} \tilde{\mathbf{y}}^{\mathrm{T}} (\mathbf{P} \mathbf{F} + \mathbf{F}^{\mathrm{T}} \mathbf{P}) \tilde{\mathbf{y}} + e(u - u_{\mathrm{FCMAC}}^{*} - \varepsilon) + \frac{1}{\gamma_{1}} \dot{\tilde{\mathbf{w}}}^{\mathrm{T}} \tilde{\mathbf{w}} + \frac{1}{\gamma_{2}} \tilde{D} \dot{\tilde{D}} = -\frac{1}{2} \tilde{\mathbf{y}}^{\mathrm{T}} \mathbf{Q} \tilde{\mathbf{y}} + e(u_{\mathrm{AFCMAC}} - u_{\mathrm{FCMAC}}^{*} + u_{ar} - \varepsilon) + \frac{1}{\gamma_{1}} \dot{\tilde{\mathbf{w}}}^{\mathrm{T}} \tilde{\mathbf{w}} + \frac{1}{\gamma_{2}} \tilde{D} \dot{\tilde{D}}$$
(34)

where (23) and the control law (30) have been applied. Next, from (28) and (29), and from (31) and (32), it follows that

$$\dot{V} = -\frac{1}{2}\tilde{\mathbf{y}}^{\mathrm{T}}\mathbf{Q}\tilde{\mathbf{y}} + e\mathbf{h}^{\mathrm{T}}A\tilde{\mathbf{w}} - \hat{D}|e| - \varepsilon e - e\mathbf{h}^{\mathrm{T}}A\tilde{\mathbf{w}} + (\hat{D} - D)|e|$$
$$= -\frac{1}{2}\tilde{\mathbf{y}}^{\mathrm{T}}\mathbf{Q}\tilde{\mathbf{y}} - \varepsilon e - D|e|$$
(35)

By invoking the inequality

$$\pm \varepsilon e \le |\varepsilon e| \le D|e| \tag{36}$$

we then obtain

$$\dot{V} \le -\frac{1}{2}\tilde{\boldsymbol{y}}^{\mathrm{T}}\boldsymbol{Q}\tilde{\boldsymbol{y}}$$
(37)

which shows that V is non-increasing. Therefore V is bounded, that is, $V \in \mathcal{L}_{\infty}$, which implies that all signals in the closed-loop system are bounded (\tilde{y} , \tilde{w} , $\tilde{D} \in \mathcal{L}_{\infty}$). Next, as the right-hand side of (22) is bounded, we have $\tilde{y} \in \mathcal{L}_{\infty}$. Integrating and rearranging (37) yields

$$\lim_{t\to\infty}\int_0^t \tilde{\mathbf{y}}^{\mathrm{T}}(\tau)\tilde{\mathbf{y}}(\tau)\,\mathrm{d}\tau \le \frac{2}{\lambda_{\min}(\mathbf{Q})}[V(0) - \lim_{t\to\infty}V(t)] < \infty \quad (38)$$

where $\lambda_{\min}(Q)$ (>0) denotes the minimum eigenvalue of the matrix Q. Therefore $\tilde{y} \in \mathcal{L}_2$. Now the Barbălat lemma [1, 2, 20] can be invoked to conclude that the tracking error \tilde{y} converges to zero asymptotically.

Although the AFCMAC control scheme described earlier can be used to track the reference trajectory, the switching compensation (32) may cause chattering in the control input, which, in turn, may lead to undesirable effects [1, 2]. To deal with this problem, a modified AFCMAC scheme is proposed, in which the switching compensation is replaced by a smooth compensation, so that chattering can be eliminated, while keeping the performance satisfactory.

3.3 AFCMAC with smooth compensation (modified AFCMAC)

Basically, the chattering of control input comes from the discontinuity of the switching function. To prevent chattering, one may replace the switching by other continuous

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maps. Owing to its similarity to the switching function such that the performance can be preserved, the saturation function would be a good choice. As a result, the compensation u_{ar} in (32) of the previous AFCMAC control scheme is modified to

$$u_{ar} = -\hat{D} \cdot \operatorname{sat}_b(e) \tag{39}$$

where $sat_b(\cdot)$ denotes the saturation function [1, 2]

$$\operatorname{sat}_{b}(e) \equiv \begin{cases} \operatorname{sgn}(e) & \text{if } |e| > b \\ e/b & \text{otherwise} \end{cases}$$
(40)

with the constant b > 0 specifying the boundary layer. It is seen that as $b \rightarrow 0$, the saturation function approaches the switching function. Moreover, to assure the convergence rates of the estimation, the adaptive laws (28) and (29) are replaced by

$$\hat{\boldsymbol{w}} = -\sigma_1 \hat{\boldsymbol{w}} - \gamma_1 \boldsymbol{e} \boldsymbol{A}^{\mathrm{T}} \boldsymbol{h}$$

$$\tag{41}$$

$$\hat{D} = -\sigma_2 \hat{D} + \gamma_2 |e|, \quad \hat{D}(0) > 0$$
 (42)

where the constants γ_1 , γ_2 , σ_1 and σ_2 are the design parameters. The first terms on the right-hand side of (41) and (42) represent the σ -modification, increasing the robustness of the adaptive law (28) and (29) [20, 21]. The closed-loop system of the nonlinear system subject to this modified AFCMAC control scheme (30) and (31) and (39)–(42) is depicted by the 'Thm. 2' part in Fig. 3. The following theorem summarises the main result of this study.

Theorem 2: Consider the nonlinear system (10) and (11) controlled by the modified AFCMAC scheme (30) and (31) and (39)–(42). If the design parameters (γ_1 , γ_2 , σ_1 , σ_2 , b) are positive, and the parameters (σ_1 , σ_2) are chosen such that max(σ_1 , (3/4) σ_2) $< \lambda_{\min}(\mathbf{Q})/\lambda_{\max}(\mathbf{P})$, where $\lambda_{\max}(\mathbf{P})$ (>0) denotes the maximal eigenvalue of the matrix \mathbf{P} , then

1. all signals in the closed-loop system are bounded; 2. the tracking error \tilde{y} converges exponentially to a residual set that can be made small by adjusting the parameters σ_1 , σ_2 and b.

Proof: The same Lyapunov function candidate (33) as in Theorem 1 shall be used to perform the analysis. Referring to (34), the time derivative of V along the trajectory of the closed-loop system by using the modified laws (39)–(42) becomes

$$\dot{V} = -\frac{1}{2}\tilde{\mathbf{y}}^{\mathrm{T}}\mathbf{Q}\tilde{\mathbf{y}} - \hat{D}e \cdot \mathrm{sat}_{b}(e) - \varepsilon e + (\hat{D} - D)|e| - \frac{\sigma_{1}}{\gamma_{1}}\tilde{\mathbf{w}}^{\mathrm{T}}\hat{\mathbf{w}} - \frac{\sigma_{2}}{\gamma_{2}}\tilde{D}\hat{D}$$
(43)

Depending on the values of |e|, the following two cases are considered separately

Case 1: |e| > b. As sat_b(e) = sgn(e), (43) yields

$$\dot{V} = -\frac{1}{2}\tilde{\mathbf{y}}^{\mathrm{T}}\mathbf{Q}\tilde{\mathbf{y}} - \varepsilon e - D|e| - \frac{\sigma_{1}}{\gamma_{1}}\tilde{\mathbf{w}}^{\mathrm{T}}\hat{\mathbf{w}} - \frac{\sigma_{2}}{\gamma_{2}}\tilde{D}\hat{D} \qquad (44)$$

The application of the inequality (36) leads to

$$\dot{V} \leq -\frac{1}{2}\tilde{\boldsymbol{y}}^{\mathrm{T}}\boldsymbol{Q}\tilde{\boldsymbol{y}} - \frac{\sigma_{1}}{\gamma_{1}}\tilde{\boldsymbol{w}}^{\mathrm{T}}(\tilde{\boldsymbol{w}} + \boldsymbol{w}^{*}) - \frac{\sigma_{2}}{\gamma_{2}}\tilde{D}(\tilde{D} + D) \qquad (45)$$

As for $\boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{R}^n$, we have $\pm \boldsymbol{a} \cdot \boldsymbol{b} \le (1/2) \|\boldsymbol{a}\|^2 + (1/2) \|\boldsymbol{b}\|^2$, it follows that

$$\dot{V} \leq -\frac{1}{2} \tilde{\boldsymbol{y}}^{\mathsf{T}} \boldsymbol{Q} \tilde{\boldsymbol{y}} - \frac{\sigma_1}{2\gamma_1} \tilde{\boldsymbol{w}}^{\mathsf{T}} \tilde{\boldsymbol{w}} - \frac{\sigma_2}{2\gamma_2} \tilde{\boldsymbol{D}}^2 + \frac{\sigma_1}{2\gamma_1} \|\boldsymbol{w}^*\|^2 + \frac{\sigma_2}{2\gamma_2} D^2$$
(46)

Now for any $\beta_1 > 0$, the previous inequality may be rewritten as

$$\dot{V} \leq -\beta_{1}V + \beta_{1} \left(\frac{1}{2} \tilde{\boldsymbol{y}}^{\mathrm{T}} \boldsymbol{P} \tilde{\boldsymbol{y}} + \frac{\tilde{\boldsymbol{w}}^{\mathrm{T}} \tilde{\boldsymbol{w}}}{2\gamma_{1}} + \frac{\tilde{D}^{2}}{2\gamma_{2}} \right) - \frac{1}{2} \tilde{\boldsymbol{y}}^{\mathrm{T}} \boldsymbol{Q} \tilde{\boldsymbol{y}}$$

$$- \frac{\sigma_{1}}{2\gamma_{1}} \tilde{\boldsymbol{w}}^{\mathrm{T}} \tilde{\boldsymbol{w}} - \frac{\sigma_{2}}{2\gamma_{2}} \tilde{D}^{2} + \frac{\sigma_{1}}{2\gamma_{1}} \|\boldsymbol{w}^{*}\|^{2} + \frac{\sigma_{2}}{2\gamma_{2}} D^{2}$$

$$\leq -\beta_{1}V + (\beta_{1}\lambda_{\max}(\boldsymbol{P}) - \lambda_{\min}(\boldsymbol{Q})) \frac{\|\tilde{\boldsymbol{y}}\|^{2}}{2}$$

$$+ (\beta_{1} - \sigma_{1}) \frac{\tilde{\boldsymbol{w}}^{\mathrm{T}} \tilde{\boldsymbol{w}}}{2\gamma_{1}} + (\beta_{1} - \sigma_{2}) \frac{\tilde{D}^{2}}{2\gamma_{2}} + \alpha_{1} \qquad (47)$$

in which the inequalities $\tilde{y}^T P \tilde{y} \leq \lambda_{\max}(P) \|\tilde{y}\|^2$ and $-\tilde{y}^T Q \tilde{y} \leq -\lambda_{\min}(Q) \|\tilde{y}\|^2$ have been used, and the constant α_1 is defined by

$$\alpha_1 \equiv \frac{\sigma_1}{2\gamma_1} \| \mathbf{w}^* \|^2 + \frac{\sigma_2}{2\gamma_2} D^2 > 0$$
 (48)

Now, if

$$\beta_1 = \min\left\{\frac{\lambda_{\min}(\boldsymbol{Q})}{\lambda_{\max}(\boldsymbol{P})}, \, \sigma_1, \, \sigma_2\right\}$$
(49)

is selected, we have

$$\dot{V} \le -\beta_1 V + \alpha_1 \tag{50}$$

which implies that all the signals are bounded according to Ioannou and Kokotovic [21]. Moreover, if the parameters σ_1 and σ_2 are chosen such that

$$\max(\sigma_1, \sigma_2) < \frac{\lambda_{\min}(\boldsymbol{Q})}{\lambda_{\max}(\boldsymbol{P})}$$
(51)

and β_1 is selected as

$$\beta_1 = \min(\sigma_1, \sigma_2) \tag{52}$$

the inequailty (47) implies that

$$\dot{V} \le -\beta_1 V + (\beta_1 \lambda_{\max}(\boldsymbol{P}) - \lambda_{\min}(\boldsymbol{Q})) \frac{\|\tilde{\boldsymbol{y}}\|^2}{2} + \alpha_1 \qquad (53)$$

Now, define the residual set as

$$\Gamma_{1} \equiv \left\{ \tilde{\boldsymbol{y}} : \| \tilde{\boldsymbol{y}} \|^{2} < \frac{2\alpha_{1}}{\lambda_{\min}(\boldsymbol{Q}) - \beta_{1}\lambda_{\max}(\boldsymbol{P})} \right\}$$
(54)

It is seen that outside the residual set, we have

$$\dot{V} \le -\beta_1 V \tag{55}$$

such that the tracking error \tilde{y} converges exponentially [21].

Case 2: $|e| \le b$. For this case, sat_b(e) = e/b, and (43) yields

$$\dot{V} = -\frac{1}{2}\tilde{\mathbf{y}}^{\mathrm{T}}\mathbf{Q}\tilde{\mathbf{y}} - \frac{\hat{D}}{b}|e|^{2} + \hat{D}|e| - \varepsilon e - D|e|$$

$$-\frac{\sigma_{1}}{\gamma_{1}}\tilde{\mathbf{w}}^{\mathrm{T}}\hat{\mathbf{w}} - \frac{\sigma_{2}}{\gamma_{2}}\tilde{D}\hat{D}$$

$$\leq -\frac{1}{2}\tilde{\mathbf{y}}^{\mathrm{T}}\mathbf{Q}\tilde{\mathbf{y}} - \frac{\hat{D}}{b}\left[\left(|e| - \frac{b}{2}\right)^{2} - \frac{b^{2}}{4}\right] - \frac{\sigma_{1}}{\gamma_{1}}\tilde{\mathbf{w}}^{\mathrm{T}}\hat{\mathbf{w}}$$

$$-\frac{\sigma_{2}}{\gamma_{2}}\tilde{D}\hat{D}$$
(56)

by applying (36). As $(|e| - b/2)^2 \ge 0$, using similar techniques as in Case 1, we find

$$\dot{V} \leq -\frac{1}{2}\tilde{\boldsymbol{y}}^{\mathrm{T}}\boldsymbol{Q}\tilde{\boldsymbol{y}} + \frac{b}{4}(\tilde{D}+D) - \frac{\sigma_{1}}{2\gamma_{1}}\tilde{\boldsymbol{w}}^{\mathrm{T}}\tilde{\boldsymbol{w}} - \frac{\sigma_{2}}{2\gamma_{2}}\tilde{D}^{2} + \frac{\sigma_{1}}{2\gamma_{1}}\|\boldsymbol{w}^{*}\|^{2} + \frac{\sigma_{2}}{2\gamma_{2}}D^{2}$$

$$(57)$$

Owing to the following inequality

$$\begin{split} \frac{b}{4}(\tilde{D}+D) &= \frac{1}{4} \left[\sqrt{\frac{\gamma_2}{\sigma_2}} \ b \cdot \sqrt{\frac{\sigma_2}{\gamma_2}} \ \tilde{D} + \sqrt{\frac{\gamma_2}{\sigma_2}} \ b \cdot \sqrt{\frac{\sigma_2}{\gamma_2}} \ D \right] \\ &\leq \frac{\sigma_2}{8\gamma_2} \tilde{D}^2 + \frac{\sigma_2}{8\gamma_2} D^2 + \frac{\gamma_2}{4\sigma_2} b^2 \end{split}$$
(58)

Equation (57) can be further expressed as

$$\dot{V} \leq -\frac{1}{2}\tilde{\boldsymbol{y}}^{\mathrm{T}}\boldsymbol{Q}\tilde{\boldsymbol{y}} - \sigma_{1}\frac{\tilde{\boldsymbol{w}}^{\mathrm{T}}\tilde{\boldsymbol{w}}}{2\gamma_{1}} - \left(\frac{3}{4}\sigma_{2}\right)\frac{\tilde{D}^{2}}{2\gamma_{2}} + \frac{\sigma_{1}}{2\gamma_{1}}\|\boldsymbol{w}^{*}\|^{2} + \frac{5\sigma_{2}}{8\gamma_{2}}D^{2} + \frac{\gamma_{2}}{4\sigma_{2}}b^{2}$$

$$(59)$$

It is observed that the inequality (59) is similar to (46), and we may use an analogous method to perform the analysis. In particular, by defining

$$\alpha_2 \equiv \frac{\sigma_1}{2\gamma_1} \| \boldsymbol{w}^* \|^2 + \frac{5\sigma_2}{8\gamma_2} D^2 + \frac{\gamma_2}{4\sigma_2} b^2 > 0 \qquad (60)$$

choosing σ_1 and σ_2 such that

$$\max(\sigma_1, \frac{3}{4}\sigma_2) < \frac{\lambda_{\min}(\boldsymbol{Q})}{\lambda_{\max}(\boldsymbol{P})}$$
(61)

and selecting β_2 to be

$$\beta_2 = \min\left(\sigma_1, \frac{3}{4}\sigma_2\right) \tag{62}$$

we can prove that all the signals are bounded, and \tilde{y} converges exponentially to the residual set given by

$$\Gamma_2 \equiv \left\{ \tilde{\boldsymbol{y}} : \| \tilde{\boldsymbol{y}} \|^2 < \frac{2\alpha_2}{\lambda_{\min}(\boldsymbol{Q}) - \beta_2 \lambda_{\max}(\boldsymbol{P})} \right\}$$
(63)

In order to have the modified AFCMAC perform satisfactory, it is further required that for both cases, the residual sets can be made arbitrarily small. This can be achieved by adjusting the parameters $(\gamma_1, \gamma_2, \sigma_1, \sigma_2, b)$ such that (α_1, α_2) are small enough, as $||w^*||$ and *D* are fixed. In particular, the residual set Γ_1 can be made small and contained in the region $\{\tilde{y}: ||\tilde{y}^T Pg|| \leq b\}$, which corresponds to the region of Case 2. For that setting, the exponential convergence is assured for Case 1. When the errors enter the regime of Case 2, they are driven exponentially to the small Γ_2 , which guarantees the performance of the proposed modified AFCMAC scheme.

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Fig. 4 Diagram of the inverted pendulum

Applications of the modified AFCMAC 4

To illustrate the performance of the proposed modified AFCMAC, three applications shall be discussed. The first one is on the tracking of an inverted pendulum with friction. The second is on the one-link robotic manipulator [5, 18]. For the third example, the tracking problem for a thirdorder highly nonlinear system is attacked.

4.1 Example 1: inverted pendulum with friction

The inverted pendulum consists of a thin homogeneous rod of mass m and length l, with a load of point mass $m_{\rm L}$ attached to the end, as depicted in Fig. 4. Assume that friction torque exists in the joint that can be modelled [22] as

$$b_{\rm v}(\dot{q}) = \begin{cases} \tau_{\rm c} \, {\rm sgn}(\dot{q}) + c_{\rm v} \dot{q}, & \dot{q} \neq 0\\ \tau_{\rm e}, & \dot{q} = 0, & |\tau_{\rm e}| < \tau_{\rm s}\\ \tau_{\rm s} \, {\rm sgn}(\tau_{\rm e}), & \dot{q} = 0, & |\tau_{\rm e}| \ge \tau_{\rm s} \end{cases}$$
(64)

where $\tau_{\rm e}$ denotes the external torque, $\tau_{\rm c}$ is the Coulomb friction, $\tau_{\rm s}$ is the breakaway torque and $c_{\rm v}$ is the coefficient of viscous friction. Let q denote the joint angle. Euler's law can be applied to find the equations of



Fig. 5 Numerical results of Example 1

e Control input a Tracking response

- b Tracking error
- Updated CMAC weights f Updated approximation error bound c Control input g
- d Tracking error h Tracking error

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motion as

$$\ddot{q} = -\frac{3}{(m+3m_{\rm L})l^2} b_{\rm v}(\dot{q}) + \frac{3(m+2m_{\rm L})g_0}{2(m+3m_{\rm L})l} \sin(q) + \frac{3}{(m+3m_{\rm L})l^2} \tau$$
(65)

$$y = q \tag{66}$$

where τ is the torque applied to the joint and $g_0 = 9.8026$ is the gravitational constant. Here the external torque in the friction model (64) is given by $\tau_{\rm e} = \tau + (m/2 + m_{\rm L}) lg_0 \sin(q)$. As a specific example, it is assumed that m = l = 1, $m_{\rm L} = 2/3$ and $\tau_{\rm c} = 0.023$, $v_{\rm c} = 0.001$, $\tau_{\rm s} = 0.035$.

The objective is to drive the inverted pendulum and follow the desired trajectory y_d , defined by the reference model

$$\ddot{q}_d = -5\dot{q}_d - 5q_d + 5r \tag{67}$$

$$y_d = q_d \tag{68}$$

with $q_d(0) = 0$, $\dot{q}_d(0) = 0$, where the reference input *r* is a unit periodic rectangular signal with period T = 32 s. Now suppose that in designing the controller, the friction model is unknown, and thus the proposed AFCMAC control schemes are applicable. First, the AFCMAC control scheme discussed in Section 3.2 is applied. By choosing the following design parameters

$$\hat{D}(0) = 10, \quad \gamma_1 = 30, \quad \gamma_2 = 0.05, \quad \boldsymbol{d} = \begin{bmatrix} 1 & 0.001 \end{bmatrix}^{\mathrm{T}},$$
 $\boldsymbol{c} = \begin{bmatrix} 1 & 2 \end{bmatrix}^{\mathrm{T}} \text{ and } \boldsymbol{Q} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ (69)

the corresponding *F*, *P* and $\lambda_{\min}(Q)/\lambda_{\max}(P)$ can be found to be

$$\boldsymbol{F} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad \boldsymbol{P} = \begin{bmatrix} 15 & 5 \\ 5 & 5 \end{bmatrix} \text{ and}$$
$$\frac{\lambda_{\min}(\boldsymbol{Q})}{\lambda_{\max}(\boldsymbol{P})} = 0.5858 \tag{70}$$

The simulation was then performed with the results shown in Figs. 5a and b for the tracking response and the tracking error, respectively. It is seen that the tracking error indeed converges asymptotically to zero. However, as shown in Fig. 5c, the control input is bounded, but suffers from severe chattering.

To resolve the chattering problem, the modified AFCMAC control scheme is then applied. In addition to selecting the same design parameters as before, we choose

$$\sigma_1 = 0.5, \quad \sigma_2 = 0.01 \quad \text{and} \quad b = 1$$
 (71)

that meet with the requirement given in Theorem 2. The simulation results show that the response remains rapid and the tracking error (Fig. 5d) converges exponentially to a residual set whose size can be roughly estimated to be ± 0.0096 rad. It is further seen that the control input (Fig. 5e) is now bounded without chattering because of a smooth compensation. In Figs. 5f and g, the updated weights and the upper bound of approximation error are given, respectively, which indicates that all of the updated control gains are bounded.

To further appreciate the effects of the parameters on the performance, a different setting of parameters is chosen in the modified AFCMAC as

 $\sigma_1 = 0.05, \quad \sigma_2 = 0.001 \quad \text{and} \quad b = 0.1$ (72)

From the simulation results, it is seen that the control performance is the same as that obtained previously, except that the residual set of the tracking error is now about ± 0.0037 rad (Fig. 5*h*), which is smaller than that in the previous case. This comparison justifies that the residual set can be made smaller by using smaller σ_1 , σ_2 and *b*.

Note that a large $\hat{D}(0)$ is chosen here, as the bound on the approximation error is not clear. Through the σ -modification in Theorem 2, the actual bound can be attained presumably, so that the chattering can be attenuated. One may argue that with small $\hat{D}(0)$, the chattering phenomenon may not appear by using the adaptive law with Theorem 1. However, as shown in Figs. 6a-d, the chattering for $\hat{D}(0) = 0.01$, $\gamma_1 = 300$, $\gamma_2 = 2$, still exists. On the contrary, if the modified AFCMAC is used (Theorem 2), the chattering can be suppressed by choosing $\hat{D}(0) = 10$, $\sigma_1 = 0.5$, $\sigma_2 = 0.001$, b = 1, as shown in Figs. 6e, f.

4.2 Example 2: one-link rigid robotic manipulator

To compare the proposed modified AFCMAC control scheme with the methods used by Zhihong *et al.* [5] and Lin and Peng [18], the same one-link rigid robotic manipulator is chosen as

$$ml^2\ddot{q} + b\dot{q} + mlg_v\cos(q) = u \tag{73}$$

where *l* is the link length, *m* is the mass and *q* is the angular position with initial conditions q(0) = -0.1 and $\dot{q}(0) = 0$. Let the state variables be $x_1 = q$ and $x_2 = \dot{q}$, so that the model (73) can be expressed as

$$\begin{bmatrix} \dot{x}_1\\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1\\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1\\ x_2 \end{bmatrix} + \begin{bmatrix} 0\\ 1 \end{bmatrix} (f + gu + d)$$
(74)

where $f = (-b/ml^2)x_2 - (g_v/l) \cos(x_1)$ and $g = (1/ml^2)$. The parameters are given by $m = l = b = g_v = 1$, and it is assumed that d is the external square wave disturbance with magnitude 0.1 and period 2π , which is the same as in Zhihong [5] and Lin and Peng [18]. The reference signal x_d is generated by the following model

$$\begin{bmatrix} \dot{x}_{d1} \\ \dot{x}_{d2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -16 & -8 \end{bmatrix} \begin{bmatrix} x_{d1} \\ x_{d2} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t)$$
(75)

with initial condition $[x_{d1}, x_{d2}]^{T} = [0, 0]^{T}$, where r(t) is a periodic rectangular signal with a period of 6 s.

The proposed modified AFCMAC control scheme is now used to perform the tracking. The design parameters are selected as

$$\hat{D}(0) = 30, \quad \gamma_1 = 30, \quad \gamma_2 = 0.01, \quad \sigma_1 = 0.001,$$

 $\sigma_2 = 0.0001, \quad b = 0.1, \quad \boldsymbol{d} = \begin{bmatrix} 1 & 0.001 \end{bmatrix}^T,$
 $\boldsymbol{c} = \begin{bmatrix} 9 & 6 \end{bmatrix}^T \text{ and } \quad \boldsymbol{Q} = \begin{bmatrix} 90 & 9 \\ 9 & 2 \end{bmatrix}$
(76)

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Fig. 6 Numerical results of Example 1

a Tracking error

b Control input $(u = u_{\text{FCMAC}} + u_r)$ c Control input (u_{FCMAC}) d Control input (u_r) e Tracking error

f Control input $(u = u_{\text{FCMAC}} + u_r)$

such that

$$F = \begin{bmatrix} 0 & 1 \\ -9 & -6 \end{bmatrix}, \quad P = \begin{bmatrix} 30 & 5 \\ 5 & 1 \end{bmatrix} \text{ and}$$
$$\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} = 0.0353 \tag{77}$$

The simulation was then performed with results shown in Figs. 7a-c. Although the system performances (Figs. 7a-c) are similar to that given by Zhihong [5] and Lin and Peng [18], the tracking error (Fig. 7b) is smaller than that by Zhihong [5], and the choice of the initial bound of approximation error $\hat{D}(0) = 30$ is more reasonable than that of $\hat{\delta}(0) = 0.01$ in Lin and Peng [18].

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4.3 Example 3: third-order nonlinear system

To demonstrate that the proposed control schemes can handle a higher order nonlinear system, a third-order one is constructed as

$$\ddot{x} = -4\ddot{x}^2 - 3e^{-|x|} - 2\sin(x) + u \tag{78}$$

$$y = x \tag{79}$$

where y is the system output, u is the control input. Now, consider the following reference model

$$\ddot{x_d} = -6\ddot{x_d} - 11\dot{x_d} - 6x_d + r \tag{80}$$

$$y_d = x_d \tag{81}$$

where *r* is a piecewise constant signal. Suppose that $f = -4\ddot{x}^2 - 3e^{-|\ddot{x}|} - 2\sin(x)$ is uncertain. We now apply the proposed control scheme.



Fig. 7 Numerical results of Example 2

- a Tracking response
- b Tracking error
- c Control input

The modified AFCMAC control scheme is first used with the following parameters

$$\hat{D}(0) = 10, \quad \gamma_1 = 1.5, \quad \gamma_2 = 0.5, \quad \sigma_1 = 0.1,$$

$$\sigma_2 = 0.01, \quad b = 1, \quad \boldsymbol{d} = \begin{bmatrix} 3 \ 2 \ 1 \end{bmatrix}^{\mathrm{T}}, \quad \boldsymbol{c} = \begin{bmatrix} 1 \ 2 \ 3 \end{bmatrix}^{\mathrm{T}}$$

$$\boldsymbol{Q} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}, \quad \boldsymbol{F} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix},$$

$$\boldsymbol{P} = \begin{bmatrix} 23 & 21 & 5 \\ 21 & 46 & 13 \\ 5 & 13 & 6 \end{bmatrix} \quad \text{and} \quad \frac{\lambda_{\min}(\boldsymbol{Q})}{\lambda_{\max}(\boldsymbol{P})} = 0.1617 \quad (82)$$

Figs. 8a and b show the tracking response and tracking error, respectively. Obviously, the tracking error converges exponentially to a residual set whose size is about 0.0036. The control input is shown in Fig. 8c, which is seen to be bounded without chattering. Simulation was conducted



Fig. 8 Numerical results of Example 3

a Tracking response

b Tracking error

c Control input

d Tracking error

with some adjusting of the parameters as

$$\sigma_1 = 0.01, \quad \sigma_2 = 0.001 \quad \text{and} \quad b = 0.5$$
 (83)

which also satisfies the requirement in (62). The performances are similar, but the residual set of the tracking error

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becomes about 0.00036 (Fig. 8*d*) that is smaller than that in the previous case. This example shows that the proposed control scheme is applicable to deal with a class of complicated, higher-order nonlinear systems.

5 Conclusions

In this study, a modified AFCMAC scheme was developed to solve the tracking problem for a class of nonlinear systems. The proposed method was based on the CMAC technique that was integrated into the THEN part of a fuzzy reasoning mechanism. The resulting architecture, the FCMAC, was simpler than that of the basic CMAC. The FCMAC method was then combined with the adaptive law, so that the entire controller gains or weights could be adjusted online without preliminary off-line learning. To accommodate the approximation error of the control from the ideal control input, two different compensations were considered. Although the AFCMAC control scheme using the switching compensation drives the tracking error to converge asymptotically to zero, severe chattering resulted that could cause undesirable effects. Alternatively, the modified AFCMAC scheme with the saturation compensation prevented the chattering and steered the tracking error to converge exponentially to a residual set whose size could be adjusted. The application of the method to three examples demonstrated the effectiveness of the proposed modified AFCMAC scheme, as shown by the corresponding simulation results.

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