FREQUENCY DISCRIMINATION CHARACTERISTICS OF COCHLEAR BASILAR MEMBRANE USING A FLUID/STRUCTURE MODEL

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ABSTRACT

This paper addresses the vibratory mechanics associated with frequency discrimination of basilar membrane within the cochlear of the inner ear. Periodic excitation is provided to the oval window, which results in generation of waves within the fluid-filled cochlear traveling towards the apex. These waves interact with the compliant basilar membrane structure causing its vibratory motion. Solution procedure of the fluid/structure model consists of a two-step process. First, a finite element calculation (ANSYS) solves for the membrane vibration with an initial harmonic pressure distribution. Second, a control volume analysis links the resultant vibratory motion with the fluid pressure acting on the basilar membrane, thus a pressure feedback loop is accomplished. Results show that dominant factors affecting vibratory characteristics of the basilar membrane are its structural geometry and attenuation of pressure wave as it travels away from the oval window. Calculations clearly capture the designed function of the basilar membrane, principally its frequency discrimination behavior.

Keywords : Basilar membrane, Cochlear, Fluid/structure interaction, Vibration mechanics.

1. INTRODUCTION

The cochlear is truly a masterpiece of the Creator; it serves as the vital link between sound and silence. Located in the inner ear about the size of a green pea, the cochlear is the element responsible for converting acoustic wave to neural signal. After being amplified at the outer ear channel (see Fig. 1 from [1]) the incoming acoustic waves excite the tympanic membrane (eardrum). In the middle ear, the three-bone impedance matching ossicular chain structure — malleus, incus and stapes — then transmits the eardrum vibration to the oval window of the curled-up, shell-like cochlear in the inner ear. Once the oval window, located in the cochlear basal turn, is set into motion acoustic wave travels through the fluid-filled canal towards the apex. In the process, the temporally and spatially varying acoustic wave acts as a source of excitation over the compliant basilar membrane structure, which is the dominant structure supporting cells and tissues within the cochlear. In response, the basilar membrane is set into vibratory motion acting as a spatial filter in frequency discrimination. Consequently, local hair cells are excited and cause a chain reaction to send neutral signal to the brain.

Cochlear mechanics began with the ground-breaking work of von Bekesy [2], which earned him a Nobel prize in Medicine. His work can be summarized as follows: stapes periodic motion gives rise to a traveling wave of displacement on the basilar membrane. The characteristic of this wave is very much frequency dependent, i.e., the basilar membrane attains maximum amplitude at a specific frequency dependent location and dies off rapidly thereafter. The low frequency acoustic signal tends to peak near the apex where the high frequency peaks close to the base. This frequency selectivity feature of the basilar membrane is essentially a spatial spectrum analyzer, which converts the amplitude and frequency contents in the acoustic signal to appropriate neural fibers along the length of the basilar membrane. Pickles [3] and Yost [4] both provide an excellent overview of the cochlear.

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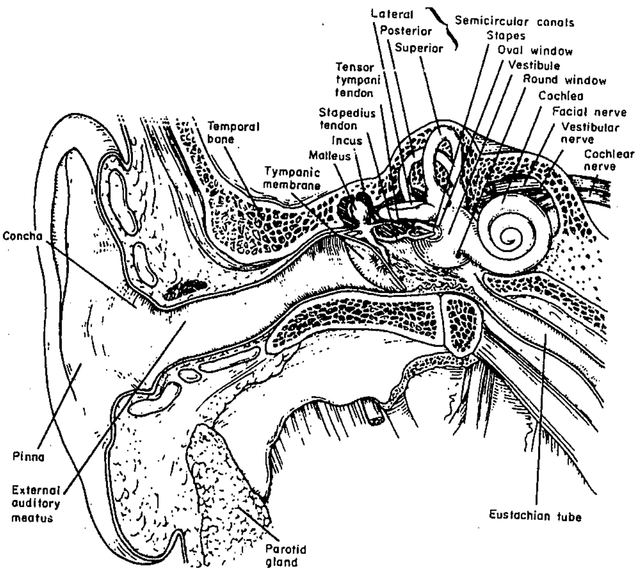

Fig. 1 Sketch of the outer, middle and inner ear [1]. The cochlea, the primary interest in the work, is the coiled structure in the inner ear above the Eustachian tube

The present state-of-the art of the cochlear suggests that it behave in a highly non-linear fashion. This can be readily seen from the human audio range of 20Hz to 20,000Hz in frequency and the approximately 20dB to 100dB in amplitude. The one thousand fold frequency increase and ten thousand fold root-mean-square pressure fluctuation are all, surprisingly, within the manageable range of our ear. The main reason for this lies in the non-linear response of the cochlear to incoming sound. Simply put, the cochlear amplifies low amplitude signal and attenuates high amplitude sound, and, by the way, it is precisely this extremely non-linear feature of the cochlear that modern hearing aid have problem duplicating. The cochlear has been a continual research topic and has been reviewed recently by Dallos [5].

This work aims to study the mechanics of the basilar membrane, particularly its frequency discrimination characteristic, via a fluid/structure coupled model. The fluid model is derived based on first principles within the framework of a control volume analysis. The membrane structural dynamics is studied via the finite element code ANSYS. This two-step procedure is adapted instead of the using ANSYS alone for complete solution stems from the conviction that a problem should be posed as self-evident as possible without comprising on the physics rendered. Details on the coupled model follow.

2. THE FLUID/STRUCTURE MODEL

A fluid/structure feedback model is adapted in this work. The fluid portion uses a control volume analysis to account for both basilar membrane vibration and compressibility effects, therefore, links the acoustic pressure to the vibratory motion of the basilar membrane. The ANSYS code provides the structural dynamics solution for an imposed acoustic traveling wave acting on the membrane. This two-step fluid/structure direct coupled approach proves to be extremely helpful in illuminating the fundamental mechanics of the cochlear basilar membrane.

The cochlear model accounts of anomie fluid filling the scala vestibuli (SV) and the scala media (SM), with the partition — the Reissner's membrane (RM) – assumed not to affect in any the propagation of acoustic wave, see Fig. 2 [6]. This is well justified since the Reissner's membrane is much thinner than the basilar membrane with its associated hair cell structure and confirmed by previous studies [5,7]. Thus traveling acoustic excitation acts directly on the SV side of the basilar membrane and progresses towards the apical end. As is well known, there is no reflection towards the oval window since the round window acts as pressure release for the entire system.

The fluid behavior is modeled by a linearized wave equation in a variable area duct representing the cochlear geometry. Sound propagates into the inner ear serves as the input boundary condition for the cochlear. Two effects contribute to the local pressure variation along the membrane: slightly damped longitudinal traveling waves and local acceleration of the membrane in the transverse direction. The membrane mechanics is modeled by a finite element calculation with the local pressure as input and outputs the membrane vibration. Thus the fluid-membrane coupling provides closure to the problem at hand. The model will be discussed in the following two sections, covering the fluids and the structure features involved.

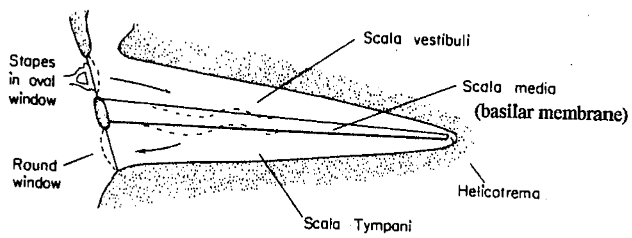

Fig. 2 The cochlea in the uncoiled configuration for modeling purpose [3]. The basilar membrane is located within the scala media structure. The dash lines show displacement initiated by stapes vibration causing motion in the basilar membrane and the round window.

2.1 Fluids Modeling

Fluids filled the cochlear and serves as the primary media of acoustic propagation. In this model, the familiar elongation basilar membrane geometry is used, as shown in Fig. 3. The heart of the model is a control volume analysis of an elementary fluid volume, see Fig. 4, of compressible fluid above the basilar membrane, which allows for the membrane to vibrate. The

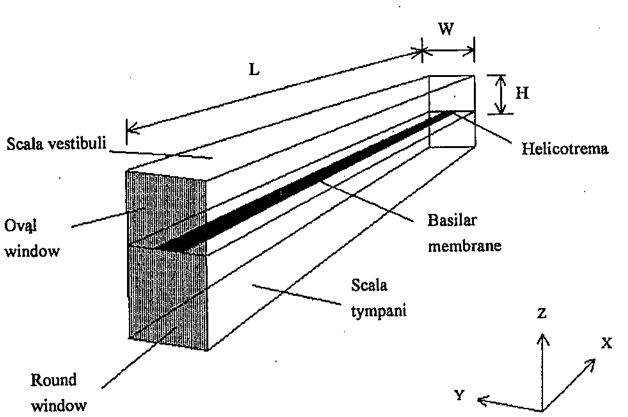

Fig. 3 Model of the cochlea adopted for analysis and computation [6]

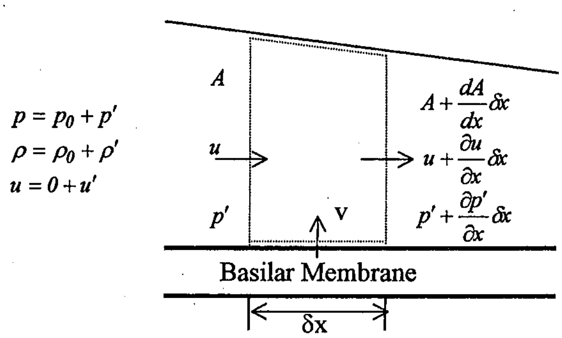

Fig. 4 Sketch of the control volume of fluid (dotted line), located above the basilar membrane, for the fluid analysis. The model accounts for basilar membrane vibration as well as compressibility of the fluid medium

leftward and rightward faces of the control volume represents a differential length along the membrane and of different cross-sectional area due to the nature of the cochlear. The upward face is on the cochlear wall, thus is solid and rigid.

The continuity equation for the control volume is written as

$$0 = \frac{\partial}{\partial t}\int \rho\, dV + \oint \rho(\vec{v}\cdot\vec{n})\, dA \qquad (1)$$

where ρ denotes the fluid density and is considered compressible, dV the elementary control volume of fluid, $\vec{v}$ the velocity at the control surface with outward pointing normal vector $\vec{n}$ and area dA. Applying Eq. (1) for the control volume of Fig. 4 obtains

$$0 = \frac{\partial \rho}{\partial t} A\,\delta x + \rho v b\,\delta x + \left[\frac{d}{dx}(\rho u A)\right]\delta x \qquad (2)$$

where $A(x)$ is the cross-sectional area of the fluid-filled medium, δx the length of the elementary control volume, $v(x)$ the velocity of the elemental basilar membrane (since the control surface is placed on the membrane), $b(x)$ the width of the membrane (in the y-direction), and $u(x)$ the fluid velocity fluctuation. Note that the membrane velocity enters the equation since the control volume is compliant with the lower boundary vibrates with the membrane.

The equation is linearized since physically small, perturbed variables exists along with a large, time-mean value. In other words,

$$\begin{aligned} \rho &= \rho_0 + \rho' \\ p &= p_0 + p' \\ u &= 0 + u' \end{aligned} \qquad (3)$$

After linearizing, Eq. (2) can be written as

$$A\frac{\partial \rho'}{\partial t} = -\rho_0 \frac{\partial}{\partial x}(uA) - \rho_0 b v \qquad (4)$$

To write Eq. (4) in another form, one can equate density perturbation with pressure perturbation via the speed of sound c, namely, $p' = c^2 \rho'$, and obtaining,

$$\frac{A}{c^2}\frac{\partial p'}{\partial t} = -\rho_0 \frac{\partial (uA)}{\partial x} - \rho_0 b v \qquad (5)$$

The conservation of momentum in the direction of the acoustic wave (x) for our control volume takes the form

$$\sum F_x = \frac{\partial}{\partial t}\int \rho u\, dV + \oint \rho u(\vec{v}\cdot\vec{n})\, dA \qquad (6)$$

The sum of forces in the x-direction (left-hand-side of Eq. (6)), is simply differential pressure forces, i.e.,

$$\sum F_x = p' A - \left(p' + \frac{\partial p'}{\partial x}\delta x\right)\cdot\left(A + \frac{dA}{dx}\delta x\right) + \frac{1}{2}\left[p' + \left(p' + \frac{\partial p'}{\partial x}\delta x\right)\right]\cdot\left(\frac{dA}{dx}\right)\delta x \qquad (7)$$

with the last term due to the fact that pressure force also acts on the slanting portion (top surface) of the control surface where an average amount is taken. However, simplifying Eq. (7) along with the linearization approximation, i.e. ignoring quadratic terms, results in only one term,

$$\sum F_x \approx -A\frac{\partial p'}{\partial x}\delta x \qquad (8)$$

The time varying and the flux of momentum term (right-hand-side of Eq. (6)) in full non-linear fashion is

$$\frac{\partial}{\partial t}\int \rho u\, dV + \oint \rho u(\vec{v}\cdot\vec{n})\, dA = \rho\frac{\partial u}{\partial t} A\,\delta x + \rho u v b\,\delta x + \rho\left(u + \frac{\partial u}{\partial x}\delta x\right)^2\left(A + \frac{dA}{dx}\delta x\right) - \rho u^2 A \qquad (9)$$

It is worth noting that the membrane vibration velocity v and the fluid velocity u appear together in the second term. Performing the linearization procedure to Eq. (9), however, results in the only the first term. Finally, the momentum equation, Eq. (6), in linearized form can be written as

$$-A\frac{\partial p'}{\partial x} \approx \rho_0 \frac{\partial (uA)}{\partial t} \quad (10)$$

Combining Eq. (5) with Eq. (10) via the operation $\partial\,[Eq.(5)]/\partial t + \partial\,[Eq.(10)]/\partial x$ thus eliminating the unwanted perturbed flow velocity u results the desired equation governing the perturbed acoustic pressure and membrane vibration velocity ν,

$$\frac{A}{c^2}\frac{\partial^2 p'}{\partial t^2} - \frac{\partial}{\partial x}\left(A\frac{\partial p'}{\partial x}\right) = -\rho_0\, b\frac{\partial v}{\partial t} \quad (11)$$

Physically, this equation states that as the acoustic pressure p' propagates it changes with time and varies along the duct due to its non-uniform cross-sectional area thus the basilar membrane has to respond accordingly.

Noting the physics of Eq. (11) greatly helps to provide insight for its solution [8]. Assume the cross-sectional area takes the form $A(x) = A_0 e^{ax}$, where A_0 is the cross-sectional area of the cochlear channel at the oval window. Substituting into Eq. (11) obtains

$$\frac{A_0\, e^{\alpha x}}{c^2}\frac{\partial^2 p'}{\partial^2 t} - A_0\, e^{\alpha x}\left(\frac{\partial^2 p'}{\partial^2 x} + \alpha\frac{\partial p'}{\partial x}\right) = -\rho_0\, b\frac{\partial v}{\partial t} \quad (12)$$

To solve for the acoustic pressure in harmonic mode with radian frequency ω, one observes that it takes the physical form

$$p'(x,t) = K(x,t) + e^{-\frac{\alpha x}{2}}\,[\mathrm{R}e^{i(\omega t - kx)} + Se^{i(\omega t + kx)}] \quad (13)$$

The first term $K(x,t)$ being the particular solution and the rest as the homogeneous solution. Note that the wave is allowed to decay spatially and propagate in both directions with different amplitude R (right traveling) and S (left traveling). The wave number k takes the particular form

$$k = \sqrt{(\omega/c)^2 - (\alpha/4)^2} \quad (14)$$

Substituting Eqs. (13) and (14) into (12) results

$$\frac{1}{c^2}\frac{\partial^2 K}{\partial t^2} - \frac{\partial^2 K}{\partial x^2} - \alpha\frac{\partial K}{\partial x} = -\frac{\rho_0\, b}{A_0}e^{-\alpha x}\frac{\partial v}{\partial t} \quad (15)$$

Back substitution confirms that the particular solution takes the form

$$K(x,t) = \left(\frac{c^2}{\omega^2}\right)\frac{\rho_0\, b}{A_0}\frac{\partial v}{\partial t}\, e^{-\alpha x}\, e^{i\omega t} \quad (16)$$

It is desired for structural modeling that membrane displacement appears direction in the equation. Let the local membrane displacement be

$$\eta(x,t) = \eta'(x)\, e^{i\omega t} \quad (17)$$

therefore

$$\frac{\partial v(x,t)}{\partial t} = \frac{\partial^2 \eta}{\partial t^2} = -\omega^2\,\eta'\, e^{i\omega t} \quad (18)$$

The particular solution, Eq. (16), then takes the form

$$K(x,t) = -c^2\frac{\rho_0\, b}{A_0}\eta' e^{-\alpha x}\, e^{i\omega t} \quad (19)$$

The perturbed pressure is

$$p'(x,t) = -c^2\frac{\rho_0\, b}{A_0}\eta' e^{-\alpha x}\, e^{i\omega t} + e^{-\frac{\alpha x}{2}}\,[\mathrm{R}e^{i(\omega t - kx)} + Se^{i(\omega t + kx)}] \quad (20)$$

The boundary condition arises physically from vibration of the ossicular bone chain exciting the oval window thus creating a traveling acoustic wave in the anomie fluid. Mathematically, it implies at $x = 0$, $\eta' = 0$ (basilar membrane is held fixed) with pressure perturbation $p'(0, t)$ given. Also, as justified earlier on the basis of no reflected wave, it is reasonable to assume that, for the boundary condition stated, only right-ward traveling wave exists. With this in mind one arrives at the final equation for the fluid model,

$$p'(x,t) = \mathrm{Re}\left\{-c^2\frac{\rho_0\, b}{A_0}\eta'\, e^{-\alpha x}\, e^{i\omega t} + e^{-\frac{\alpha x}{2}}\,[p'(0)\, e^{i(\omega t - kx)}]\right\} \quad (21)$$

where $p'(0)$ is the pressure perturbation amplitude at the oval window and Re denotes the real part.

2.2 Structure Modeling

The desired result of the fluid model, Eq. (21), is a compact means of equating acoustic pressure with basilar membrane displacement, however, with two variables in one equation additional closure is needed. In essence, the finite element (ANSYS) based structure model calculates the membrane displacement with known forcing function, and thus provides the complete solution procedure. The fact that a two-model approach is taken rather than a fluid-structure calculation all within the framework of finite element analysis is stemmed from the conviction that physical phenomena at play would be best illuminated via the two-step process over than of hard-core FEM calculation. And the beauty of the fluid model result, Eq. (21), more than amply justified the approach taken.

The model geometry is shown in Fig. 5, which accounts for the three-dimensional structure of the basilar membrane in its elongated configuration. The structure is of varying cross-sectional area with the thickness decreases and the width increases towards the apex. The dimensions are representative of that of the cochlear of a guinea pig. The system of equation is

$$[M]\,\ddot{\eta} + [C]\,\dot{\eta} + [K]\,\eta = F \quad (22)$$

where $[M]$ is the mass matrix, $[C]$ the damping matrix and $[K]$ the restoring spring force matrix. In actual computation, the damping term is found to be negligible and thus is ignored. As justified post-priori, the physical reasoning seems to suggest that with the

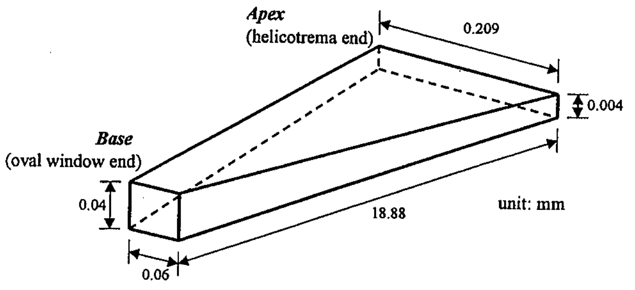

Fig. 5 Detailed geometry of the basilar membrane used in the ANSYS computation

extremely small membrane displacement amplitude, on the order of nano-meter, the damping force due to the embedded fluid and the structure itself is small compared to the other two forces. The Young's modulus of the membrane is taken as 10^9 Pascal, the density 2080kg/m^3, the thickness takes the form of a exponential variation of $H \exp(ax)$, where $H = 0.04$mm and $a = -0.122$/mm.

The boundary conditions are based on the anatomy of the basilar membrane. At the basal portion of the membrane ($x = 0$), it is a fixed end with the fluid-filled portion above being just interior of the oval window, where the excitation is initiated. Between the base and the apex, the two edges of the membrane is fixed to the bony structure of the cochlear. At the apex, it is a free-end, since this is where acoustic pressure enters the scalar tympani.

The forcing acting on the basilar membrane is constant across its width, or the spatial dependence of the acoustic forcing is on x only. This essentially assumes the acoustic propagation travels as plane wave towards the apex, which is amply justified since the cochlear channel dimension is several orders of magnitude smaller than the acoustic wavelength. This excitation propagates from the oval window towards the apex and assumes no reflection, which is justified since the round window essential acts as a pressure release device.

The solution procedure is as follows. A single frequency harmonic excitation enters the cochlear through the oval window with the initial guess being

$$p'(x,t) = \mathrm{Re}\left\{ e^{-\frac{\alpha x}{2}} [p'(0)\, e^{i(\omega t - kx)}] \right\} \qquad (23)$$

which excludes contribution from the membrane vibration as is evident from Eq. (21). The pressure excitation from Eq. (23) is used as input to the structure model using the ANSYS code for computation. The resultant membrane kinematics along its entire length is then substituted into Eq. (21) to update the acoustic pressure excitation and iterated accordingly.

3. RESULTS AND DISCUSSIONS

Principally, the foremost importance in modeling the cochlear is to verify the frequency discrimination characteristics of the basilar membrane. Figure 6 presents the comparison between calculation and data [9] for locations of maximum amplitude along the basilar membrane. The abscissa represents the distance from the stapes foot-plate, or equivalently the oval window, towards the apical direction along the basilar membrane. The ordinate is the excitation frequency driving the stapes. Both data and calculation clearly suggest that the basilar membrane is tuned to high frequency near the base and low frequency towards the apex. A careful observation of this result shows the turning is indeed very sharp — approximately 4kHz decay over 5mm distance — or that the basilar membrane has high spatial resolution. The calculation appears to agree better at high frequencies than at low range.

Another view of frequency discrimination can be depicted from Fig. 7, which shows the response of the 10kHz tuned position to a range of input frequencies. In other words, sound is excited from the stapes foot-plate into the cochlear over the range of frequencies as shown in the abscissa while measuring instrumentation is located at one location, namely the 10kHz tuned position. The ordinate represents the basilar membrane vibration amplitude, Y_{BM}, normalized by the stapes foot-plate amplitude, Y_S, and plotted in logarithmic form. The spread of data, from [10], suggest that this is a very difficult experiment since the nominal basilar membrane vibration amplitude is in the order of nanometer. Computation result from the fluid/structure model is also plotted. The most important aim of this computation is to verify that the calculated basilar membrane amplitude indeed peaks at

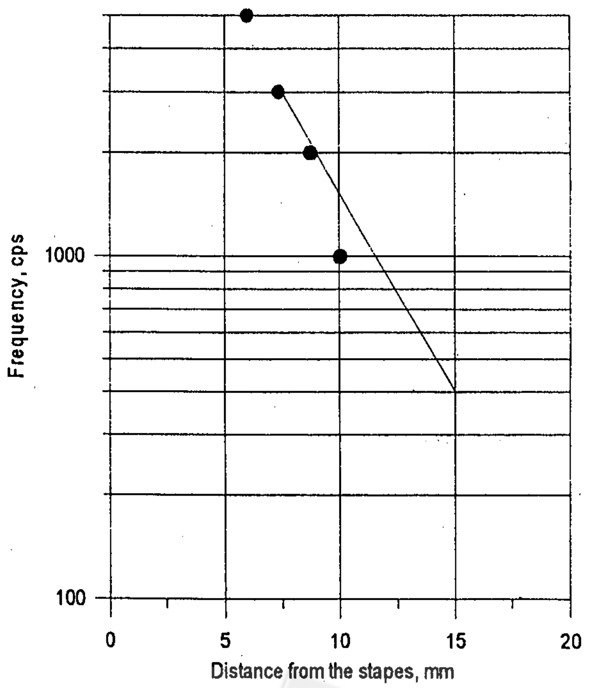

Fig. 6 Comparison of calculated (line) maximum amplitude location along the basilar membrane with data (symbol) [9] at various excitation frequency

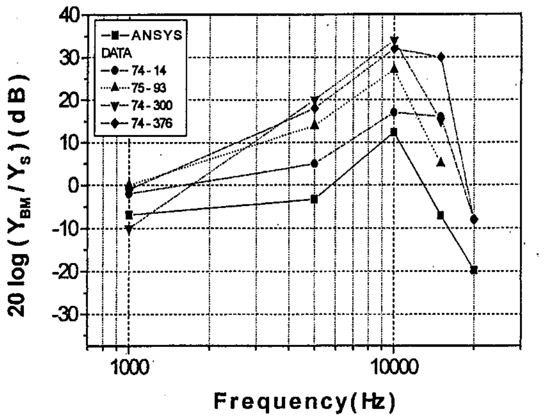

Fig. 7 Comparison of calculated basilar membrane amplitude (normalized by the stapes amplitude) at cochlea first turn location with data [10] for sound input from the outer ear. The transfer function of Fig. 6 is used for this purpose

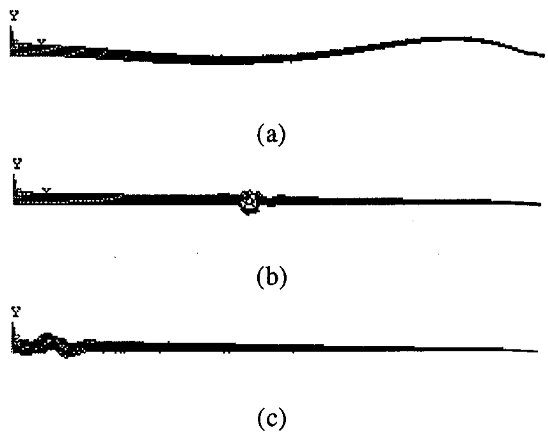

Fig. 8 Basilar membrane mode shape computed at frequency of (a) 213Hz (mode 4), (b) 2,545Hz (mode 18) and (c) 12,039Hz (mode 47). (The basal end is on the left and the apical end is on the right.)

the 10kHz tuned position, which is overwhelmingly shown to be the case. The calculated result reports slightly lowered overall amplitude than the published data, however, this is considered acceptable considering the spread of the data. Another somewhat salient feature but also important characterization of frequency discrimination is the sharp drop-off the membrane vibration beyond the best-tuned location. This feature is generally supported by data (set 74-300 and set 75-93) as well as by overall consensus of the cochlear mechanics community [5], although data set 74-14 and set 74-376 do not wholehearted concur.

A physical feel for the actual basilar membrane amplitude for a range of frequency is always insightful. The ANSYS result shows at the 10kHz best tuned location the response is 12dB, or 4.0 times the stapes amplitude. This is a quadruple amount, 4.0nm, if the stapes amplitude is assumed to be 1nm. However, at 1kHz. the same basilar membrane location reduces to –7dB in normalized amplitude, or 0.45nm, which is about a tenth of the 10kHz value. Incidentally, at the higher frequency of 15kHz the amplitude response is also the same. At much higher audio frequency of 20kHz, the –20dB response is 0.1nm, or about 2.5% of the 10kHz amplitude. All the facts point to one major physic of the basilar membrane — a particular location along its length is designed to vibrate at a specific audio frequency and this location very sharply tuned.

The computed mode shapes for three particular frequencies are presented in Fig. 8. At a very low frequency of 213Hz, Fig. 8a, the basilar membrane is clearly shown to peak close to the apex, in qualitative agreement with the result of Fig. 6. At a higher frequency of 2.5kHz, the response peaks near the middle of the membrane and rapidly dies down in either direction. At 12kHz, the basilar membrane tuning is already very close to the base. These results suggest that the cochlear responds more sharply at the basal end, at higher frequencies, than at the apical end, at lower frequencies.

4. CONCLUDING REMARKS

This work presents results from a fluid/structure coupled model for cochlear basilar membrane vibratory response to sound frequency. A control volume analysis for the fluids contribution is derived linking membrane amplitude with incoming sound pressure. ANSYS solves for the membrane modal vibration for a particular input frequency with a given acoustic pressure imposed on the membrane structure. Results obtained include the following:

(1) The fluid/structure model adequately represents the dominant physics of the basilar membrane, principally its frequency discrimination behavior.
(2) The model shows that dominant factors affecting vibratory characteristics of the basilar membrane are its structural geometry and attenuation of pressure wave as it travels away from the oval window.
(3) Calculated results prove that the basilar membrane behaves like a spatial spectrum analyzer; the basal end responds to high frequency sound while the apical end to low frequency signal.
(4) The basilar membrane poses a non-linear amplifier that amplifies low amplitude sound and attenuates loud ones.
(5) The frequency response is much more sharply tuned at high frequencies than at low as shown by results in Fig. 7.

REFERENCES

1. Kessel, R. G. and Kardon, W. H., *Tissues and Organs: A Text-Atlas of Scanning Electron Microscopy*, Freeman and Co. (1979).
2. von Bèkesy, G., *Experiments in Hearing,*

McGraw-Hill, New York (1960).

3. Pickles, J. O., *An Introduction to the Physiology of Hearing*, London: Academic Press (1982).
4. Yost, W. A., *Fundamentals of Hearing*, San Diego: Academic Press (1994).
5. Dallos, P., Popper, A. N. and Fay, R. R., *The Cochlea*, Chapter 5, New York: Springer (1996).
6. Ryan, A. F. and Dallos, P., "Physiology of the Cochlear," *Hearing Disorders*, Northern, J. L., ed., Little Brown, pp. 253–255 (1984).
7. Neely, S. T. and Kim D. O., "A Model for Active Elements in Cochlear Biomechanics," *J. Acoust. Soc. Am.*, 79, pp. 1472–1480 (1986).
8. Dowling, A. P. and Ffowcs Williams, J. E., *Sound and Sources of Sound*, New York: Halsted Press (1983).
9. De Boer, E., "Auditory Physics: Physical Principles in Hearing Theory," *Physics Reports*, 62(2), pp. 87–174 (1980).
10. Phode, W. S., "Some Observations on Cochlear Mechanics," *J. Acoust. Soc. Am.*, 64, pp. 158–176 (1978).

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