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以重整群分析紊流模式(I)

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一、中文摘要

本研究旨在以重整群分析方法研究不可壓縮之紊流場，在大尺度渦漩與小尺度渦漩為統計之獨立的假設下，我們可以建立一個遞迴重整群的程序，而建構出重整群轉換，在此轉換下得到一具有換尺不變性的 Navior-Stokes 方程，此轉換的固定點在數學上可被等價成渦漩黏滯力在 Fourier 空間中的積微分方程式，藉由此積微分方程式的求解，發現渦漩黏滯力光譜與流場波數的-4/3 次方呈正比的關係，以及流場能量光譜與波數的-5/3 次方呈正比關係，此外，此解亦可進而推導出 Smagorinsky 模型，並且精確指出 Smagorinsky 常數與隔點大小以及流場特徵波數的關係。

關鍵詞：紊流、重整群、渦漩黏滯力、Kolmogorov 常數、流場能量光譜、大尺度渦漩模擬、Smagorinsky 模型。

Abstract

The study starts with a brief review on recent development of renormalization group analysis for incompressible turbulence. It is found fruitful to take the simple hypothesis that large-scale eddies are statistically independent of those of smaller scales. A recursive renormalization procedure is then proposed for turbulence governed by the Navier-Stokes equation in an exact manner that a nonlinear triple term appearing in early treatment can be dispensed with in the present formulation. By employing the combined form of the scaling laws proposed respectively by Pao and Leslie \& Quarini for the energy spectrum, the relevant exponents for the spectrum are completely determined. Furthermore, the limiting operation of renormalization group analysis yields an inhomogeneous ordinary differential equation for the invariant effective eddy viscosity. The closed-form solution of the equation facilitates derivation of the Smagorinsky model for large-eddy simulation of turbulent flow, which reveals the explicit dependence of the model constant on the cutoff size and other characteristic wavenumbers.

Keywords: turbulence, renormalization group, effective eddy viscosity, Kolmogorov constant, energy spectrum, large-eddy simulation, Smagorinsky model.

1. Introduction

Renormalization group (RG) analysis of turbulence has aroused a considerable interest in fluid society since Wilson (1974) and his colleagues developed the method and won a wonderful success in the study of critical phenomena. But as Frisch said in his notable book (1995), Twenty years later turbulence remains unsolved. However, RG methods stand a good chance of playing a role in the solution of problem of turbulence.

Nelkin could possibly be among the earliest pioneers who studied the renormalization group theory of turbulence. But there are now basically two different RG approaches to fluid turbulence: one originated by Forster, Nelson and Stephen (FNS) , in which, the critical development has been done by Yakhot and Orszag ; the other based on the work of Rose , Rose Sulem ,known as recursive RG, which was recently studied extensively by Zhou, Vahala Hossain . FNS introduced the Navier-Stokes equation with a forcing term, which is regarded as the driving mechanism of turbulence. The eps-RG approach, in particular the work of Yakhot Orszag (YO) has received critical reviews by several authors in that their analysis lacks rigor in dealing with the usage of the parameter eps consistently, and in that YO's theory, if valid, can only hold true in the small wavenumber limit. On the other hand, the recursive RG approach dispenses with the driving force, and one may work on a finite range of wavenumbers by carrying out the renormalization procedure in a recursive manner. Although the recursive RG approach sounds a more reasonable one, there is no warrant that the existing procedure for carrying out renormalization is in its most adequate form. Like YO's theory being critically reviewed subsequently, the detailed procedure of the recursive RG approach is certainly worthy of further examination. For example, that existing recursive RG analysis shows appearance of a triple nonlinear term in the renormalized Navier-Stokes equation is an unnecessary complexity and may even cause inconsistency in the development of the whole theory. Moreover, Zhou, Vahala Hossain reported a cusp behavior of the effective eddy viscosity in the large wavenumber limit, which in turn depends on the refinement ratio in the renormalization procedure, while most of other studies did not show a similar behavior. Resolution of the latter point naturally requires limiting operation of the full recursive renormalization to locate the invariant effective eddy viscosity.

2. RG Procedure

In the present study, the flow turbulence considered is isotropic, stationary and homogeneous, and is assumed to be governed by the incompressible Navier-Stokes equation. The correlation of velocity is invariant under spatial translation:

$$\mathbf{u}_\alpha(\mathrm{k},\mathbf{t}) = \begin{cases} \mathbf{u}_\alpha^<(\mathrm{k},\mathbf{t}) \quad , & |\mathrm{k}| < \mathbf{k_1} \quad (\text{sup}\mathbf{ergrid}) \\ \mathbf{u}_\alpha^>(\mathrm{k},\mathbf{t}) \quad , & \mathbf{k_1} < |\mathrm{k}| < \mathbf{k_0} \quad (\mathbf{subgrid}) \end{cases}$$

For supergrid $|\mathbf{k}| < \mathbf{k_1}$ Navier-Stokes equation becomes**:**

$$\mathbf{u}_\alpha^<(\mathbf{k},\omega) = \left(\mathbf{i}\omega + \nu_\mathbf{0}\mathbf{k}^\mathbf{2}\right)^{-\mathbf{1}}\mathbf{M}_{\alpha\beta\gamma}(\mathbf{k})\int \Big[\mathbf{u}_\beta^<(\mathbf{j},\omega)\mathbf{u}_\gamma^<(\mathbf{k}-\mathbf{j},\omega) + 2\mathbf{u}_\beta^<(\mathbf{j},\omega)\mathbf{u}_\gamma^>(\mathbf{k}-\mathbf{j},\omega) + \mathbf{u}_\beta^>(\mathbf{j},\omega)\mathbf{u}_\gamma^>(\mathbf{k}-\mathbf{j},\omega)\Big]\mathbf{d^3j}$$

For subgrid $\mathbf{k_1} < |\mathbf{j}| < \mathbf{k_0}$: assume Markovian approximation.

$$\mathbf{u}_\beta^>(\mathbf{j},\omega) = (\nu_\mathbf{0}\mathbf{j}^\mathbf{2})^{-\mathbf{1}}\mathbf{M}_{\beta\beta'\gamma'}(\mathbf{j})\int \Big[\mathbf{u}_{\beta'}^<(\mathbf{j}',\omega)\mathbf{u}_{\gamma'}^<(\mathbf{j}-\mathbf{j}',\omega) + 2\mathbf{u}_{\beta'}^<(\mathbf{j}',\omega)\mathbf{u}_{\gamma'}^>(\mathbf{j}-\mathbf{j}',\omega) + \mathbf{u}_{\beta'}^>(\mathbf{j}',\omega)\mathbf{u}_{\gamma'}^>(\mathbf{j}-\mathbf{j}',\omega\Big]\mathbf{d^3j'}$$

Take the ensemble averge over the subgrid modes to obtain,

$$\mathbf{u}_\alpha^<(\mathbf{k},\omega) = \left(\mathbf{i}\omega + \nu_0\mathbf{k}^2\right)^{-1}\mathbf{M}_{\alpha\beta\gamma}(\mathbf{k})\int\Big[\mathbf{u}_\beta^<(\mathbf{j},\omega)\mathbf{u}_\gamma^<(\mathbf{k}-\mathbf{j},\omega) + \left\langle\mathbf{u}_\beta^>(\mathbf{j},\omega)\mathbf{u}_\gamma^>(\mathbf{k}-\mathbf{j},\omega)\right\rangle\Big]\mathbf{d}^3\mathbf{j}$$

Multiply both sides of $\mathbf{u}_\beta^>(\mathbf{j},\omega)$ by $\mathbf{u}_\gamma^>(\mathbf{k}-\mathbf{j},\omega)$, and take average,

$$\left(\nu_\mathbf{0}|\mathbf{j}|^\mathbf{2}\right)\left\langle\mathbf{u}_\beta^>(\mathbf{j},\omega)\mathbf{u}_\gamma^>(\mathbf{k}-\mathbf{j},\omega)\right\rangle = \mathbf{2M}_{\beta\beta'\gamma'}(\mathbf{j})\int \left\langle\mathbf{u}_\gamma^>(\mathbf{k}-\mathbf{j},\omega)\mathbf{u}_{\gamma'}^>(\mathbf{j}-\mathbf{j}',\omega)\right\rangle\mathbf{u}_{\beta'}^<(\mathbf{j}',\omega)\mathbf{d^3j'}$$

we make use of $\mathbf{u}_\beta^>(\mathbf{j},\omega)$ by renaming β by γ and $\mathbf{j}$ by $\mathbf{k}-\mathbf{j}$, followed by multiplying on both sides $\mathbf{u}_\beta^>(\mathbf{j},\omega)$,

$$\left(\nu_\mathbf{0}|\mathbf{k}-\mathbf{j}|^\mathbf{2}\right)\left\langle\mathbf{u}_\beta^>(\mathbf{j},\omega)\mathbf{u}_\gamma^>(\mathbf{k}-\mathbf{j},\omega)\right\rangle = \mathbf{2M}_{\gamma\beta'\gamma'}(\mathbf{k}-\mathbf{j})\int \left\langle\mathbf{u}_\gamma^>(\mathbf{k}-\mathbf{j}-\mathbf{p},\omega)\mathbf{u}_\beta^>(\mathbf{j},\omega)\right\rangle\mathbf{u}_{\beta'}^<(\mathbf{p},\omega)\mathbf{d^3p}$$

Adding those two equations together,

$$\left(\nu_\mathbf{0}|\mathbf{j}|^\mathbf{2} + \nu_\mathbf{0}|\mathbf{k}-\mathbf{j}|^\mathbf{2}\right)\left\langle\mathbf{u}_\beta^>(\mathbf{j},\omega)\mathbf{u}_\gamma^>(\mathbf{k}-\mathbf{j},\omega)\right\rangle$$
$$= \mathbf{4M}_{\gamma\beta'\gamma'}(\mathbf{k}-\mathbf{j})\int \left\langle\mathbf{u}_\gamma^>(\mathbf{k}-\mathbf{j}-\mathbf{p},\omega)\mathbf{u}_\beta^>(\mathbf{j},\omega)\right\rangle\mathbf{u}_{\beta'}^<(\mathbf{p},\omega)\mathbf{d^3p}$$
$$= \mathbf{4M}_{\gamma\beta'\gamma'}(\mathbf{k}-\mathbf{j})\mathbf{D}_{\gamma'\beta}(\mathbf{j})\mathbf{D}_{\alpha\beta'}(\mathbf{k})\mathbf{Q}(\mathbf{j})\mathbf{u}_\alpha^<(\mathbf{k},\omega)$$

Substitute in subgrid mode,

$$\left(\mathbf{i}\omega + \nu_\mathbf{1}(\mathbf{k})\mathbf{k}^\mathbf{2}\right)\mathbf{u}_\alpha^<(\mathbf{k},\omega) = \mathbf{M}_{\alpha\beta\gamma}(\mathbf{k})\int \mathbf{u}_\beta(\mathbf{j},\omega)\mathbf{u}_\gamma(\mathbf{k}-\mathbf{j},\omega)\,\mathbf{d^3j}$$

where $\nu_\mathbf{1} = \nu_\mathbf{0} + \delta\nu_\mathbf{0}(\mathbf{k})$

The increment of the effective eddy viscosity is given in the integral form:

$$\delta\nu_\mathbf{0}(\mathbf{k}) = \frac{\mathbf{1}}{2\pi}\int_{\Omega_0}\frac{\mathbf{E_0}(\mathbf{j})\mathbf{L}(\mathbf{k},\mathbf{k}-\mathbf{j})}{(\nu_\mathbf{0}|\mathbf{j}|^\mathbf{2} + \nu_\mathbf{0}|\mathbf{k}-\mathbf{j}|^\mathbf{2})\mathbf{j}^\mathbf{2}\mathbf{k}^\mathbf{2}}\mathbf{d^3j}\,,$$

where $\Omega_\mathbf{0} = \left\{\,\mathbf{j} \mid \mathbf{0} < |\mathbf{k}| < \mathrm{k}_1\,,\ \mathrm{k}_1 < |\mathbf{j}|\,,\ |\mathbf{k}-\mathbf{j}| < \mathrm{k}_0\right\}$ .To obtain the general relationship for the effective eddy viscosity between two successive renormalization steps, exactly the same procedure is applicable to the next shell (k_2,k_1), the third shell and so on. After removing the n-th shell, we have the relationship

$$\nu_\mathbf{n+1} = \nu_\mathbf{n} + \delta\nu_\mathbf{n}(\mathbf{k})$$

where

$$\delta\nu_\mathbf{n}(\mathbf{k}) = \frac{\mathbf{1}}{2\pi}\int_{\Omega_\mathbf{n}}\frac{\mathbf{E_n}(\mathbf{j})\mathbf{L}(\mathbf{k},\mathbf{k}-\mathbf{j})}{(\nu_\mathbf{n}(\mathbf{j})|\mathbf{j}|^\mathbf{2} + \nu_\mathbf{n}(\mathbf{k}-\mathbf{j})|\mathbf{k}-\mathbf{j}|^\mathbf{2})\mathbf{j}^\mathbf{2}\mathbf{k}^\mathbf{2}}\mathbf{d^3j}$$

where $\Omega_\mathbf{n} = \left\{\,\mathbf{j} \mid \mathbf{0} < |\mathbf{k}| < \mathrm{k}_{n+1}\,,\ \mathrm{k}_{n+1} < |\mathbf{j}|\,,\ |\mathbf{k}-\mathbf{j}| < \mathrm{k}_n\right\}$

The increment of eddy viscosity at lower avenumbers are relatively insignificant in magnitude, compared to that at higher wavenumbers which exhibits a rapid increase near the cutoff wavenumber. This behavior is in good consistency with that obtained by Kraichnan based on his testing field model (TFM). In fact, what Kraichnan obtained could be appropriately interpreted as the result of one-step renormalization.Assume Pao's energy model (1965)

$$\mathbf{E_0}(\mathbf{j}) = \mathbf{A_s}\left(\frac{\mathrm{j}}{\mathrm{k_p}}\right)\mathbf{C_K}\varepsilon^\mathbf{a}\mathbf{j}^\mathbf{y}\exp\left(-\frac{\mathbf{3}}{\mathbf{2}}\mathbf{C}_\mathbf{K}^{-\frac{\mathbf{1}}{\mathbf{2}}}\nu_\mathbf{0}\varepsilon^\mathbf{b}\mathbf{j}^\mathbf{z}\right)$$

where $\mathbf{A_s}(\mathbf{x}) = \dfrac{\mathbf{x}^{\frac{\mathbf{17}}{\mathbf{3}}}}{\mathbf{1}+\mathbf{x}^{\frac{\mathbf{17}}{\mathbf{3}}}}$, and $\mathrm{k_p}$ denotes the wavenumber corresponding to the peak of energy-containing eddies. Substitution in RG-transformation gives and the dissipation relation

$\varepsilon = \int_0^{k_{n+1}} 2\nu_n(k) k^2 E_n(k) dk$, then obtain the renormalized energy spectrum

$$E_n(j) = A_s C_K \varepsilon^{\frac{2}{3}} j^{\frac{-5}{3}} \exp\left(-\frac{3}{2} C_K^{-\frac{1}{2}} \varepsilon^{\frac{-1}{3}} \nu_n(j) j^{\frac{4}{3}}\right)$$

Rescale the wavenumber by setting $\tilde{k} = \frac{k}{k_n}$

Take the limit of $n \to \infty \quad \Rightarrow \quad \xi \to 0$ and $\tilde{\nu}_n \to \tilde{\nu}$, $\tilde{\Omega}_n \to \tilde{\Omega}$

$$\left[\tilde{k}\frac{d\tilde{\nu}(\tilde{k})}{d\tilde{k}} + \frac{4}{3}\tilde{\nu}(\tilde{k})\right]\xi = \frac{(1-\xi)^{\frac{-4}{3}}}{2\pi} C_K \varepsilon^{\frac{2}{3}}$$

$$\int_{\tilde{\Omega}} \frac{A_s \tilde{j}^{\frac{-5}{3}} \exp\left(-\frac{3}{2} C_K^{-\frac{1}{2}} \tilde{\nu} \varepsilon^{\frac{-1}{3}} \tilde{j}^{\frac{4}{3}}\right)}{\left(\tilde{\nu}(\tilde{j})\left|\tilde{j}\right|^2 + \tilde{\nu}\ (\tilde{k}-\tilde{j})(\tilde{k}^2 + \tilde{j}^2 - 2\tilde{k}\tilde{j}\mu)\right)\tilde{j}^2\tilde{k}^2} \times \frac{(\tilde{k}^4 - 2\tilde{k}^3\tilde{j}\mu + \tilde{k}\tilde{j}^3\mu)(1-\mu^2)}{\tilde{k}^2 + \tilde{j}^2 - 2\tilde{k}\tilde{j}\mu} d^3\tilde{j}$$

where $\tilde{\Omega}$ is the intersection part of two unit spheres, and have the limiting result

$$\Xi(\tilde{\Omega}_n) \to \Xi(\tilde{\Omega}) = 2\pi\sqrt{1-(\frac{\tilde{k}}{2})^2}\,\frac{\tilde{k}}{\sqrt{1-(\frac{\tilde{k}}{2})^2}}\xi = 2\pi\tilde{k}\xi$$

Set $B(\tilde{k}) = C_K^{\frac{-1}{2}} \varepsilon^{\frac{-1}{3}} \tilde{\nu}(\tilde{k})$, then

$$\tilde{k}\frac{dB}{d\tilde{k}} + \frac{4}{3}B = A_s\left(\frac{1}{\tilde{k}_p}\right)\frac{\exp(-1.5B(1))}{B(1)}\frac{\tilde{k}}{4}\left[1-\left(\frac{\tilde{k}}{2}\right)^2\right]$$

Solution: (Fixed Point of RG-transformation)

$$B(\tilde{k}) = \left(B(1) - \frac{135}{364}\sigma\right)\tilde{k}^{\frac{-4}{3}} - \sigma\left(\frac{3}{52}\tilde{k}^3 - \frac{3}{7}\tilde{k}\right)$$

where, the constant $\sigma = A_s\left(\frac{1}{\tilde{k}_p}\right)\frac{\exp(-1.5B(1))}{4B(1)}$

Substituting $F(k) = \nu(k) C_K^{\frac{-1}{2}} \varepsilon^{\frac{-1}{3}}$ in the dissipation equation gives

$$\int_{k_s}^{k_c} 2\nu(k)E(k)dk = 2C_K^{\frac{3}{2}}\varepsilon\int_{k_s}^{k_c} F(k)A_s\left(\frac{k_c}{k_p}\right)k^{\frac{1}{3}}\exp\left(-1.5F(k)k^{\frac{4}{3}}\right)dk = \varepsilon$$

where, k_s means the wavenumber of the largest eddy existing in the fluid.Therefore,

$$C_K = \left\{2\int_{k_s}^{k_c} F(k)A_s\left(\frac{k_c}{k_p}\right)k^{\frac{1}{3}}\exp\left(-1.5F(k)k^{\frac{4}{3}}\right)dk\right\}^{\frac{-2}{3}}$$

Apply $\nu(k)$ formula for $k_c = k_0$, and evaluate $\nu(k)$ at $k = k_c$ then

$$\nu(k_c) \cong C_K^{\frac{1}{2}}\varepsilon^{\frac{1}{3}}\left[F(k_0)k_0^{\frac{4}{3}} - \frac{135}{364}\frac{\exp\left(-1.5F(k_0)k_0^{\frac{4}{3}}\right)}{4F(k_0)k_0^{\frac{4}{3}}}\right]k_c^{\frac{-4}{3}}$$

On the other hand $\nu(k_c) \equiv C_K^{\frac{1}{2}}\varepsilon^{\frac{1}{3}}F(k_c)$,thus,

$$F(k_c)k_c^{\frac{4}{3}} \cong F(k_0)k_0^{\frac{4}{3}} - \frac{135}{364}\frac{\exp\left(-1.5F(k_0)k_0^{\frac{4}{3}}\right)}{4F(k_0)k_0^{\frac{4}{3}}}$$

Recall $B(\tilde{k}) = C_K^{\frac{-1}{2}}\varepsilon^{\frac{-1}{3}}\nu(k)k^{\frac{4}{3}}$

$$\Rightarrow \quad \nu_0 = \nu(k_0) = B(1)C_K^{\frac{1}{2}}\varepsilon^{\frac{1}{3}}k_0^{\frac{-4}{3}} = C_K^{\frac{1}{2}}\varepsilon^{\frac{1}{3}}F(k_0)$$

$\therefore \quad F(k_0)k_0^{\frac{-4}{3}} = B(1)$,From Kraichnan (1976),

$\frac{C_K D''}{\left(B(1)C_K^{\frac{1}{2}}\right)^2} = 1$ and $D'' = 0.44$

We obtain $B(1) = \sqrt{0.44} = 0.6633$

Thus, $F(k_c)k_c^{\frac{4}{3}} \cong B(1) - \frac{135}{364}\frac{e^{-1.5B(1)}}{4B(1)} = 0.6116$

Using this value for Kolmogorov constant, we finally construct a relation between Kolmogorov constant and the physical parameter of fluid k_s and mesh size Δ . Substituting C_K in $\nu(k_c)$, we may express ε in the resolvable velocity $\varepsilon = \frac{\nu(k_c)}{2}\left(\frac{\partial u_i^<}{\partial x_j} + \frac{\partial u_j^<}{\partial x_i}\right)^2$ which yields

$$\nu(k_c) = \frac{1}{4\sqrt{2}\pi^2}\left\{C_K^{\frac{1}{2}}\left[0.6116 - \frac{135}{364}A_s\frac{\exp(-1.5\times 0.6116)}{4\times 0.6116}\right]\right\}^{\frac{3}{2}} \times\Delta^2\left|\frac{\partial u_i^<}{\partial x_j} + \frac{\partial u_j^<}{\partial x_i}\right| \equiv C_s\Delta^2\left|\frac{\partial u_i^<}{\partial x_j} + \frac{\partial u_j^<}{\partial x_i}\right|$$

which is exactly the Smagorinsky model, and the

Smagorinsky constant is given by

$$C_s = \frac{1}{4\sqrt{2}\pi^2}\left\{C_K^{\frac{1}{2}}\left[0.6116 - \frac{135}{364}A_s\frac{\exp(-1.5\times 0.6116)}{4\times 0.6116}\right]\right\}^{\frac{3}{2}}$$
$$\cong 0.0097C_K^{\frac{3}{4}}$$

3. Concluding remarks

In the present study, we have developed a consistent approach to recursive RG analysis of incompressible turbulence. Salient features of flow properties are obtained in this approach, and can be compared/contrasted to early theoretical and experimentally measured results. The renormalized Navier-Stokes equation bears a close analogy with the original one by merely introducing one additional quadratic term which contributes to the increment of the effective eddy viscosity. The rapid increase of the increment of the effective eddy viscosity near the cutoff wavenumber is in good consistency with Kraichnan's result based on his testing field model (TFM). Furthermore, we determine all the exponents for the energy spectrum, which is supposed to be a combination of the scaling laws proposed respectively by Pao and by Leslie Quarini. In particular,

the k^{-5/3} power dependence of the energy spectrum is found to be dominating in the inertial subrange..The limiting operation of recursive renormalization yields an inhomogeneous ordinary differential equation for the invariantb effective eddy viscosity. The equation is much simple, compared to the series form that appeared in Rose and Zhou, Vahala Hossain. Simplicity of the equation enables us to derive a closed-form solution, which shows k^{-4/3}power dependence of the invariant effective eddy viscosity at the small wavenumber limit; this is consistent with the result of Yahkot Orszag's theory. The invariant effective eddy viscosity shows also a mild cusp behavior near the cutoff wavenumber. The closed-form solution of the invariant effective eddy viscosity further facilitates derivation of the Smagorinsky model for large-eddy simulation of turbulence. In particular, we are able to show that the

Smagorinsky constant is proportional to the power 3/4 of the Kolmogorov constant, which in turn depends on the cutoff size, the wavenumber of the largest eddy existing in the flow and the wavenumber at the peak of the energy spectrum. The Kolmogorov constant is illustrated to vary between 1.35 and 2.06; this is in close agreement with the generally accepted range of experimental values: 1.2--2.2 McComb.The derived Smagorinsky model along with other ideas will be implemented to compute several complicated turbulent flows; the results will be reported elsewhere.

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