行政院國家科學委員會專題研究計畫　成果報告

以重整群分析耦合場紊流模式(II)

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行政院國家科學委員會專題研究計畫成果報告

以重整群分析與建立耦合磁流場紊流模式

計畫編號：NSC 93-2212-E-002-037

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一、中文摘要

本研究旨在以重整群分析方法研究不可壓縮之耦合磁流場紊流模式，在平均磁導相較於平均流速為可忽略以及 Alfven 效應成立的假設下，透過遞迴重整群程序而建構出的重整群轉換，可以得到一具有換尺不變性的耦合磁流場方程組，此轉換的固定點在數學上可被等價成渦漩黏滯力在 Fourier 空間中的積微分方程組，藉由此積微分方程組的求解，發現流場能量光譜與波數的-5/3 次方呈正比關係。而此關係式可進而推導 Kolmogorov 和 Smagorinsky 係數與流場特徵波數的關係。

關鍵詞：重整群分析，磁流動力紊流、有效渦度黏滯性、磁阻

ABSTRACT

In this study, we continue with a recursive renormalization group (RG) analysis of incompressible turbulence, aiming at investigating various turbulent properties of three-dimensional magneto-hydrodynamics (MHD). In particular, we are able to locate the fixed point (i.e. the invariant effective eddy viscosity) of the RG transformation under the following conditions. (i) The mean magnetic induction is relatively weak compared to the mean flow velocity. (ii) The Alfv´en effect holds, that is, the fluctuating velocity and magnetic induction are nearly parallel and approximately equal in magnitude. It is found under these conditions that re-normalization does not incur an increment of the magnetic resistivity, while the coupling effect tends to reduce the invariant effective eddy viscosity. Both the velocity and magnetic energy spectra are shown to follow the Kolmogorov $k^{-5/3}$ in the inertial subrange; this is consistent with some laboratory measurements and observations in astronomical physics. By assuming further that the velocity and magnetic induction share the same specified form of energy spectrum, we are able to determine the dependence of the (magnetic) Kolmogorov constant CK (CM) and the model constant CS of the Smagorinsky model for large-eddy simulation on some characteristic wavenumbers.

Keywords: Renormalization group analysis, magnetohydrodynamic turbulence, effective eddy viscosity, magnetic resistivity

I. INTRODUCTION

Recently, the authors [1],[2] carried out a recursive renormalization group (RG) analysis of incompressible turbulence for flow turbulence and thermal turbulent transport. In this study, we continue with this previous RG analysis for magneto-hydrodynamic (MHD) turbulence, aiming at investigating various transport properties, in particular, the coupling effects between the flow and magnetic induction fields on the kinetic energy spectrum and the effective eddy viscosity.

The plasma science is widely applied to many areas from laser skill, thin film produce, nuclear rocket, even to astronomical physics (for example, solar wind, solar flares and coronal structures). Like in ordinary Newtonian fluids, MHD turbulence is expected to arise in plasma or magnetized fluids as the Reynolds number is increased beyond some critical value. In spite of the already scarce literature, the interest of MHD turbulence may further be divided into two-dimensional and three-dimensional turbulence. Kim and Yang [3] studied the scaling behavior of the randomly stirred MHD plasma in two dimensions and were able to show existence of the scaling solution at the fixed point of the RG transformation and derive the dependence of the power exponent of the energy spectrum on the driving Gaussian noise. Liang and Diamond [4] also presented their study for two-dimensional MHD turbulence by introducing the velocity stream function and the magnetic flux function in MHD equations. However, the latter authors showed no existence of a fixed point of the RG transformation and especially suggested that the applicability of RG method to turbulent system is intrinsically limited, especially in the case of systems with dual-direction energy transfer.

II. EQUATION OF THE INVARIANT EFFECTIVE EDDY VISCOSITY

The purpose of this section is to look for the invariant effective eddy viscosity by pursuing a differential version of the recursive relationship. Recall that the basic idea underlining the recursive RG analysis is to divide the wavenumber space $(0, k_0)$ to a supergrid region $(0, k_c)$ and a subgrid region (k_c, k_0); the subgrid modes are then removed piece by piece by taking subgrid averaging over a spherical shell (k_{n+1}, k_n). The result will certainly depend on the cutoff ratio $\Lambda = k_{n+1}/k_n$; and thus the invariant (limiting) effective eddy viscosity should be sought by taking the limiting operation $\Lambda \to 1$.

First of all, we rescale the wavenumber by setting $\tilde{k} = k/k_{n+1}$ and have the increment of $\delta\nu$ and $\delta\tau$ in the form:

$$\begin{aligned}\delta\nu_n(k) \;=\; & k_{n+1}^{-8/3}\int_{\tilde{\Omega}_n} d^3\tilde{j}\,\frac{(\tilde{k}^4-2\tilde{k}^3\mu+\tilde{k}\tilde{j}^3\mu)(1-\mu^2)}{2\pi\tilde{j}^2\tilde{k}^2(\tilde{k}^2+\tilde{j}^2-2\tilde{k}\tilde{j}\mu)}\left[\frac{C_K\varepsilon_v^{2/3}\tilde{j}^{-5/3}\phi(\tilde{j}/\tilde{k}_p)}{\hat{\nu}_n(j)\tilde{j}^2+\hat{\nu}_n(|\mathbf{k}-\mathbf{j}|)(\tilde{k}^2+\tilde{j}^2-2\tilde{k}\tilde{j}\mu)}\right.\\ + & \left.\frac{C_M\varepsilon_M^{2/3}\tilde{j}^{-5/3}\varphi(\tilde{j}/\tilde{k}_p)}{\hat{\tau}_n(j)\tilde{j}^2+\hat{\tau}_n(|\mathbf{k}-\mathbf{j}|)(\tilde{k}^2+\tilde{j}^2-2\tilde{k}\tilde{j}\mu)}\right], \end{aligned} \tag{2.1}$$

and

$$\begin{aligned}\delta\tau_n(k) \;=\; & k_{n+1}^{-8/3}\int_{\tilde{\Omega}_n} d^3\tilde{j}\,\frac{(\tilde{k}^4-2\tilde{k}^3\mu+\tilde{k}\tilde{j}^3\mu)(1-\mu^2)}{2\pi\tilde{j}^2\tilde{k}^2(\tilde{k}^2+\tilde{j}^2-2\tilde{k}\tilde{j}\mu)}\left[\frac{C_K\varepsilon_v^{2/3}\tilde{j}^{-5/3}\phi(\tilde{j}/\tilde{k}_p)-C_M\varepsilon_M^{2/3}\tilde{j}^{-5/3}\varphi(\tilde{j}/\tilde{k}_p)}{\hat{\tau}_n(j)\tilde{j}^2+\hat{\nu}_n(|\mathbf{k}-\mathbf{j}|)(\tilde{k}^2+\tilde{j}^2-2\tilde{k}\tilde{j}\mu)}\right.\\ - & \left.\frac{C_K\varepsilon_v^{2/3}\tilde{j}^{-5/3}\phi(\tilde{j}/\tilde{k}_p)-C_M\varepsilon_M^{2/3}\tilde{j}^{-5/3}\varphi(\tilde{j}/\tilde{k}_p)}{\hat{\nu}_n(j)\tilde{j}^2+\hat{\tau}_n(|\mathbf{k}-\mathbf{j}|)(\tilde{k}^2+\tilde{j}^2-2\tilde{k}\tilde{j}\mu)}\right]. \end{aligned} \tag{2.2}$$

According to Eq. (2.1) and Eq. (2.2), we may assume that $\nu_n(k) = k_n^t\tilde{\nu}_n\left(\tilde{k}\right)$, and $\tau_n(k) = k_n^t\tilde{\tau}_n\left(\tilde{k}\right)$ where t is an undetermined parameter. With this scaling law, combining the recursive relationship of viscosity and

Eq. (2.1) gives

$$\begin{cases} k_{n+1}^{t}\tilde{\nu}_{n+1}\left(\tilde{k}\right) = k_n^t \tilde{\nu}_n\left(\tilde{k}\Lambda\right) + k_n^{-8/3-t}\delta\tilde{\nu}_n\left(\tilde{k}\Lambda\right); \\ k_{n+1}^{t}\tilde{\tau}_{n+1}\left(\tilde{k}\right) = k_n^t \tilde{\tau}_n\left(\tilde{k}\Lambda\right) + k_n^{-8/3-t}\delta\tilde{\tau}_n\left(\tilde{k}\Lambda\right). \end{cases}$$

For consistency of the dimension on both sides of Eq. (2.3), we must have $t = -8/3 - t$, and thus $t = -4/3$. It follows by dividing by $k_{n+1}^{-4/3}$ on both sides of Eq. (2.3),

$$\begin{cases} \tilde{\nu}_{n+1}\left(\tilde{k}\right) - \Lambda^{\frac{-4}{3}}\tilde{\nu}_n\left(\tilde{k}\Lambda\right) = \Lambda^{\frac{-4}{3}}\delta\tilde{\nu}_n\left(\tilde{k}\Lambda\right); \\ \tilde{\tau}_{n+1}\left(\tilde{k}\right) - \Lambda^{\frac{-4}{3}}\tilde{\tau}_n\left(\tilde{k}\Lambda\right) = \Lambda^{\frac{-4}{3}}\delta\tilde{\tau}_n\left(\tilde{k}\Lambda\right). \end{cases} \tag{2.3}$$

Now we write $\Lambda = 1 - \lambda$, and let $n \to \infty$, equivalently, we have $\lambda \to 0$, $\tilde{\nu}_n \to \tilde{\nu}$ and $\tilde{\tau}_n \to \tilde{\tau}$. Therefore in the limit of $\lambda \to 0$, we have

$$\begin{cases} \tilde{k}\frac{d\tilde{\nu}(\tilde{k})}{d\tilde{k}} + \frac{4}{3}\tilde{\nu}\left(\tilde{k}\right) = \left[\frac{C_K\varepsilon_v^{2/3}\phi(1/\tilde{k}_p)}{\tilde{\nu}(1)} + \frac{C_M\varepsilon_M^{2/3}\varphi(1/\tilde{k}_p)}{\tilde{\tau}(1)}\right]\frac{\tilde{k}}{4}\left[1 - \left(\frac{\tilde{k}}{2}\right)^2\right]; \\ \tilde{k}\frac{d\tilde{\tau}(\tilde{k})}{d\tilde{k}} + \frac{4}{3}\tilde{\tau}\left(\tilde{k}\right) = 0. \end{cases}$$

Notice that the right hand side of the second equation of (2.4) vanishes, since the two integrands in Eq. (2.2) will cancel out each other exactly in the limit of $n \to \infty$. The two equations in (2.4) can be readily solved to yield

$$\begin{aligned} \nu(k) &= \left\{\nu(k_c)k_c^{\frac{4}{3}} - \frac{135}{364}\left[\frac{C_K\varepsilon_v^{2/3}\phi(k_c/k_p)}{4\nu(k_c)k_c^{4/3}} + \frac{C_M\varepsilon_M^{2/3}\varphi(k_c/k_p)}{4\tau(k_c)k_c^{4/3}}\right]\right\}k^{\frac{-4}{3}} \\ &- \left[\frac{C_K\varepsilon_v^{2/3}\phi(k_c/k_p)}{4\nu(k_c)k_c^{4/3}} + \frac{C_M\varepsilon_M^{2/3}\varphi(k_c/k_p)}{4\tau(k_c)k_c^{4/3}}\right]\left[\frac{3}{52}\left(\frac{k}{k_c}\right)^3 - \frac{3}{7}\left(\frac{k}{k_c}\right)\right]k_c^{\frac{-4}{3}}, \end{aligned} \tag{2.4}$$

and

$$\tau(k) = C_M^{1/2}\varepsilon_M^{1/3}k_c^{4/3}k^{-4/3}\hat{\tau}_n(k_c/k_p), \tag{2.5}$$

where k_c denotes the cutoff wavenumber. It is appropriate to term $\nu(k)$ and $\tau(k)$ the invariant effective eddy viscosity and the invariant effective magnetic resistivity, respectively. It is notable that the RG procedure does not incur an increment of the magnetic resistivity $\tau(k)$, which obeys the second equation of (48) and must scale as in Eq. (2.5) being proportional to $\varepsilon_M^{1/3}k^{-4/3}$. On the other hand, because of the minus sign in front of the terms containing C_M (or ϵ_M) in the expression (49), the effect of the magnetic effect on the effective eddy viscosity is to reduce the latter in magnitude, but not to change its basic behavior.

III. EVALUATION OF THE KOLMOGOROV CONSTANT AND SMAGORINSKY MODEL

The results of Section 5 will be applied here to evaluate the Kolmogorov and Smagorinsky constants. First of all, we set $\nu(k) = C_K^{\frac{1}{2}}\varepsilon_v^{\frac{1}{3}}F(k)$, then (2.4) can be written

$$\begin{aligned} F(k) &= \left\{F(k_c)k_c^{\frac{4}{3}} - \frac{135}{364}\left[\frac{\phi(k_c/k_p)}{4F(k_c)k_c^{4/3}} + \frac{\chi\varphi(k_c/k_p)}{4\hat{\tau}(k_c/k_p)k_c^{4/3}}\right]\right\}k^{\frac{-4}{3}} \\ &- \left[\frac{\phi(k_c/k_p)}{4F(k_c)k_c^{4/3}} + \frac{\chi\varphi(k_c/k_p)}{4\hat{\tau}(k_c/k_p)k_c^{4/3}}\right]\left[\frac{3}{52}\left(\frac{k}{k_c}\right)^3 - \frac{3}{7}\left(\frac{k}{k_c}\right)\right]k_c^{\frac{-4}{3}}, \end{aligned} \tag{3.1}$$

where we used the result of (2.5), and set $\chi = \sqrt{C_M/C_K}\,\sqrt[3]{\varepsilon_M/\varepsilon_v}$. Let us now consider a cutoff k_c and than can yield

$$\begin{aligned}&\int_{k_s}^{k_c} 2k^2\nu(k)E(k)dk\\ =\;& 2C_K^{\frac{3}{2}}\varepsilon_v\int_{k_s}^{k_c} F(k)k^{\frac{1}{3}}\phi(k/k_p)dk\\ =\;& \varepsilon_v,\end{aligned}$$

where k_s denotes the wavenumber of the largest eddy existing in the flow. Canceling out ϵ on both sides, we obtain the Kolmogorov constant C_K in terms of the three characteristic wavenumbers k_c, k_p and k_s:

$$C_K = \left[2\int_{k_s}^{k_c} F(k)k^{\frac{1}{3}}\phi(k/k_p)dk\right]^{\frac{-2}{3}}. \tag{3.2}$$

Similarly, we have

$$C_M = \left[2k_c^{\frac{4}{3}}\hat{\tau}(k_c/k_p)\int_{k_s}^{k_c} k^{-1}\varphi(k/k_p)dk\right]^{\frac{-2}{3}}. \tag{3.3}$$

So far, we have not given a precise form for ϕ and φ. Let us assume further that both the velocity and magnetic induction fields share the same form of the energy spectrum which is a combination form of the scaling laws proposed respectively by Pao [6], and Leslie and Quarini [7], that is,

$$\phi(k/k_p) = A_p\left(\frac{k}{k_p}\right)\exp\left(\frac{-3}{2}C_K^{\frac{-1}{2}}\varepsilon_v^{\frac{-1}{3}}\nu(k)k^{\frac{4}{3}}\right),$$

and

$$\varphi(k/k_p) = A_p\left(\frac{k}{k_p}\right)\exp\left(\frac{-3}{2}C_M^{\frac{-1}{2}}\varepsilon_M^{\frac{-1}{3}}\tau(k)k^{\frac{4}{3}}\right).$$

It follows immediately from (40) that

$$E^v(k) = A_p\left(\frac{k}{k_p}\right)C_K\varepsilon_v^{2/3}k^{-5/3}\exp\left(-\frac{3}{2}C_K^{\frac{-1}{2}}\varepsilon_v^{-1/3}\nu\left(k\right)k^{4/3}\right), \tag{3.4}$$

and

$$E^M(k) = A_p\left(\frac{k}{k_p}\right)C_M\varepsilon_M^{2/3}k^{-5/3}\exp\left(-\frac{3}{2}C_M^{\frac{-1}{2}}\varepsilon_M^{-1/3}\tau\left(k\right)k^{4/3}\right). \tag{3.5}$$

In these formulas, we have the factor

$$A_p\left(x\right) = \frac{x^{s+5/3}}{1+x^{s+5/3}},$$

to take care of energy-containing eddies, where s is a flow parameter. If we consider the leading term of (3.1) and apply (3.4) to (3.2), we may rewrite (3.2) in a more precise form as follows:

$$C_K = \left\{\frac{2\mathcal{M}}{s+5/3}e^{-1.5\mathcal{M}}\log\left[\frac{(k_c/k_p)^{s+5/3}+1}{(k_s/k_p)^{s+5/3}+1}\right]\right\}^{\frac{-2}{3}}, \tag{3.6}$$

where we denote

$$\mathcal{M} \equiv \left\{F(k_c)k_c^{\frac{4}{3}} - \frac{135}{364}\left[\frac{A_p(k/k_p)\exp(-1.5F(k)k^{4/3})}{4F\left(k_c\right)k_c^{4/3}} + \frac{\chi\varphi(k_c/k_p)}{4\hat{\tau}\left(k_c/k_p\right)k_c^{4/3}}\right]\right\}.$$

Following the same calculations as in the above, we may also obtain

$$C_M = \left\{ \frac{2k_c^{\frac{3}{4}}\hat{\tau}\left(\frac{k_c}{k_p}\right)\exp\left[-1.5k_c^{\frac{3}{4}}\hat{\tau}\left(\frac{k_c}{k_p}\right)\right]}{s+5/3} \log\left[\frac{(k_c/k_p)^{s+5/3}+1}{(k_s/k_p)^{s+5/3}+1}\right]\right\}^{\frac{-2}{3}}. \tag{3.7}$$

As a matter of fact, Biskamp [5] indicated that the Kolmogorov constant depends on the precise definition of the averge magnetic induction, and hence on the geometry of the large scale eddies. Here, we have provided two relationships which show how the large-scale eddies can influence not only the Kolmogorov constant C_K but also the magnetic Kolmogorov constant C_M, as shown in (3.6) and (3.7). The large-scale eddies with the wavenumbers k_p and k_s indeed play an important role in deciding both of C_K and C_M. Here we recall that, k_p denotes the $1/($*geometric size*$)$ of the energy-containing eddies and k_s denotes the $1/($*geometric size*$)$ of the largest eddies in fluid.

In order to carry out large eddy simulation (LES) for MHD turbulence, we need to evaluate the Smagorinsky constant for MHD turbulence by using (2.4). First of all, we suppose no matter whenever we perform the RG analysis, the cutoff k_c is always very close to the Kolmogorov scale k_0, that is, we may replace k_c by k_0 in (2.4). In doing so, we evaluate the effective eddy viscosity at $k = k_c$ which is far from k_0. Then (2.4) becomes

$$\begin{aligned}\nu(k_c) &= \left\{\nu_0 k_0^{\frac{4}{3}} - \frac{135}{364}\left[\frac{C_K\varepsilon_v^{2/3}\phi(k_0/k_p)}{4\nu_0 k_0^{4/3}} + \frac{C_M\varepsilon_M^{2/3}\varphi(k_0/k_p)}{4\tau_0 k_0^{4/3}}\right]\right\} k_c^{\frac{-4}{3}} \\ &- \left[\frac{C_K\varepsilon_v^{2/3}\phi(k_0/k_p)}{4\nu_0 k_0^{4/3}} + \frac{C_M\varepsilon_M^{2/3}\varphi(k_0/k_p)}{4\tau_0 k_0^{4/3}}\right]\left[\frac{3}{52}\left(\frac{k_c}{k_0}\right)^3 - \frac{3}{7}\left(\frac{k_c}{k_0}\right)\right] k_0^{\frac{-4}{3}}.\end{aligned} \tag{3.8}$$

Since $k_c/k_0 \ll 1$, we can make the following approximation

$$\begin{aligned}\nu(k_c) &\simeq \left\{\nu_0 k_0^{\frac{4}{3}} - \frac{135}{364}\left[\frac{C_K\varepsilon_v^{2/3}\phi(k_0/k_p)}{4\nu_0 k_0^{4/3}} + \frac{C_M\varepsilon_M^{2/3}\varphi(k_0/k_p)}{4\tau_0 k_0^{4/3}}\right]\right\} k_c^{\frac{-4}{3}} \\ &= C_K^{1/2}\varepsilon_v^{1/3}\left\{\hat{\nu}_0(k_0/k_p) - \frac{135}{364}\left[\frac{\phi(k_0/k_p)}{4\mathcal{H}_1} + \frac{\chi\varphi(k_0/k_p)}{4\mathcal{H}_2}\right]\right\} k_c^{\frac{-4}{3}},\end{aligned} \tag{3.9}$$

where

$$\mathcal{H}_1 = C_K^{-1/2}\varepsilon_v^{-1/3}\nu_0 k_0^{4/3} \quad \text{and} \quad \mathcal{H}_2 = C_M^{-1/2}\varepsilon_M^{-1/3}\tau_0 k_0^{4/3}.$$

Next, we express ε_v in the resolvable velocity,

$$\varepsilon_v = \frac{\nu(k_c)}{2}\left(\frac{\partial \mathbf{u}_i^<}{\partial \mathbf{x}_j} + \frac{\partial \mathbf{u}_j^<}{\partial \mathbf{x}_i}\right)^2.$$

Substituting it in Eq. (3.9) yields

$$\nu(k_c) = \left\{\hat{\nu}_0(k_0/k_p) - \frac{135}{364}\left[\frac{\phi(k_0/k_p)}{4\mathcal{H}_1} + \frac{\chi\varphi(k_0/k_p)}{4\mathcal{H}_2}\right]\right\} C_K^{\frac{1}{2}}\left[\frac{\nu(k_c)}{2}\left(\frac{\partial \mathbf{u}_i^<}{\partial \mathbf{x}_j} + \frac{\partial \mathbf{u}_j^<}{\partial \mathbf{x}_i}\right)^2\right]^{\frac{1}{3}} k_c^{\frac{-4}{3}}.$$

Solving the above algebraic equation for $\nu(k_c)$ and replacing k_c by $2\pi/\Delta$ where Δ denotes the cutoff size, we obtain

$$\begin{aligned}\nu(k_c) &= \frac{1}{4\sqrt{2}\pi^2}\left(\left\{\hat{\nu}_0(k_0/k_p) - \frac{135}{364}\left[\frac{\phi(k_0/k_p)}{4\mathcal{H}_1} + \frac{\chi\varphi(k_0/k_p)}{4\mathcal{H}_2}\right]\right\} C_K^{\frac{1}{2}}\right)^{\frac{3}{2}} \Delta^2\left|\frac{\partial \mathbf{u}_i^<}{\partial \mathbf{x}_j} + \frac{\partial \mathbf{u}_j^<}{\partial \mathbf{x}_i}\right| \\ &\equiv C_S\Delta^2\left|\frac{\partial \mathbf{u}_i^<}{\partial \mathbf{x}_j} + \frac{\partial \mathbf{u}_j^<}{\partial \mathbf{x}_i}\right|,\end{aligned}$$

where

$$C_S = \frac{1}{4\sqrt{2}\pi^2}\left\{\hat{\nu}_0(k_0/k_p) - \frac{135}{364}\left[\frac{\phi(k_0/k_p)}{4\mathcal{H}_1} + \frac{\chi\varphi(k_0/k_p)}{4\mathcal{H}_2}\right]\right\}^{\frac{3}{2}} C_K^{\frac{3}{4}}. \quad (3.10)$$

This is the Smagorinsky constant, where we have left two undetermined parameters $\mathcal{H}_1$ and $\mathcal{H}_2$, which require two additional conditions to be fully determined.

IV. CONCLUDING REMARKS

In this study, we have extended our previous RG analysis of incompressible flow turbulence to incompressible MHD turbulence.

The Elsässer variables are introduced to write the MHD equations for the velocity and magnetic induction fields in a symmetric form. RG analysis is then performed in the wavenumber domain. To proceed further with the RG transformation, we have to the following two assumptions. (i) The mean magnetic induction is relatively weak compared to the mean flow velocity. (ii) The Alfvén effect holds, that is, the fluctuating velocity and magnetic induction are nearly parallel and approximately equal in magnitude. Under these conditions, renormalization does not incur an increment of the magnetic resistivity τ, while the coupling effect tends to reduce the invariant effective eddy viscosity $\nu(k)$. Both the velocity and magnetic energy spectra are shown to follow the Kolmogorov $k^{-5/3}$ in the inertial subrange; this is consistent with some available laboratory measurements and observations in astronomical physics. Furthermore, by assuming that the velocity and magnetic induction fields share the same combined form of the energy spectra proposed respectively by Pao, and Leslie and Quarini, we are able to determine the dependence of the Kolmogorov constant C_K and the magnetic Kolmogorov constant C_M on the characteristic wavenumbers k_c, k_p and k_s. The results are applied to obtain the dependence of the Smagorinsky constant C_S for large-eddy simulation, which however contains two undetermined constants to be resolved.

In spite of the present success, it must be stressed upon that the imposed conditions (i) and (ii) imply a negligible effect of the subgrid cross helicity between the velocity and magnetic fields. There are cases where the effect is important and which may lead to quite different energy spectrum. In an early study, Kraichnan [9] derived a $k^{-3/2}$ energy spectrum of the inertial subrange when the magnetic energy in the sub-inertial wavenumbers exceeds the total energy in the inertial subrange. Pouquet et al. [10] had an intensive study on strong MHD helical turbulence and the nonlinear dynamo effect. Recently, Nakayama [11],[12] obtained also the $k^{-3/2}$ energy spectrum in the inertial subrange by constructing a spectral theory of strong shear Alfvén turbulence anisotropized by the presence of a uniform mean magnetic field. Of particular interest, we refer to Yoshizawa et al. [13] for reviewing the importance of the cross-helicity effect, and more generally for an extensive review of turbulence theories and modeling of fluids and plasmas.

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