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Accuracy Examination of the Multiple Interacting Continua Approximation

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ABSTRACT

This study examines the accuracy of the multiple interacting continua (MINC) model. A regularly shaped rock block with uniform initial conditions and a step change in boundary conditions on the surface is evaluated. The analysis results show that pressures (or temperatures) predicted by MINC approximation may deviate from exact solutions by as much as 10 – 15% at certain points within the blocks. However, when fluid (or heat) flow rates are integrated over the entire block surface, the MINC approximation and exact solution agree well, deviating by less than 1% at all points. This finding indicates that MINC approximation can accurately represent the transient inter-porosity flow in fractured porous media when the matrix blocks are indeed continuously subjected to nearly uniform boundary conditions.

Key Words: MINC, fractured porous media, approximation

I. Introduction

Modeling of transport processes in fractured porous media is mathematically very difficult because of the involvement of complicated geometries and transport processes. Pruess and Narasimhan (1982) developed an extension of the double-porosity method (Barenblatt *et al*., 1960; Warren and Root, 1963; Liu and Huang, 1994), referred to as the "multiple interacting continua" (MINC) method, which can handle the transient inter-porosity flow of fluid and heat in fractured porous media. MINC approximation assumes that, due to high permeability and low storativity of the fractures, any thermodynamic changes in a fractured porous medium will propagate rapidly in the fracture network while migrating only slowly into the low-permeable rock matrix blocks. Therefore, the thermodynamic changes in the rock matrix blocks depend primarily on the distance to the nearest fracture. In light of this concept and neglecting of gravity effects, fluid and heat flows in rock matrix blocks may be treated by means of a one-dimensional approximation. This concept is applicable to regular and irregular matrix blocks (Pruess and Karasaki, 1983; Pruess, 1983).

In numerical simulations, the MINC method partitions rock matrix blocks into sets of nested volume elements (Figs. 1 and 2). Thus, the interactions between the fractures and rock matrix can be described by one-dimensional mass and energy conservation equations. When applicable, this approximation can save substantial amounts of computer time and storage space in comparison with the more detailed discretization in conventional finite difference approaches. However, the accuracy of the MINC method employed needs to be tested and justified (Pruess, 1983). Generally, the mass and heat flows are not perpendicular to the fracture surfaces, especially near fracture intersections (corners); thus, these flows cannot strictly be considered in only one dimension. Therefore, two idealized geometrical configurations with simple boundary conditions, for which both exact solutions and solutions based on MINC approximation are available in analytical and semi-analytical forms, are considered herein to evaluate this "corner" effect.

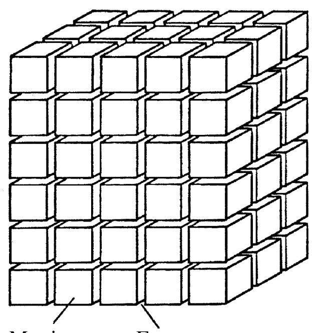

Fig. 1. Idealized model of a fractured porous medium.

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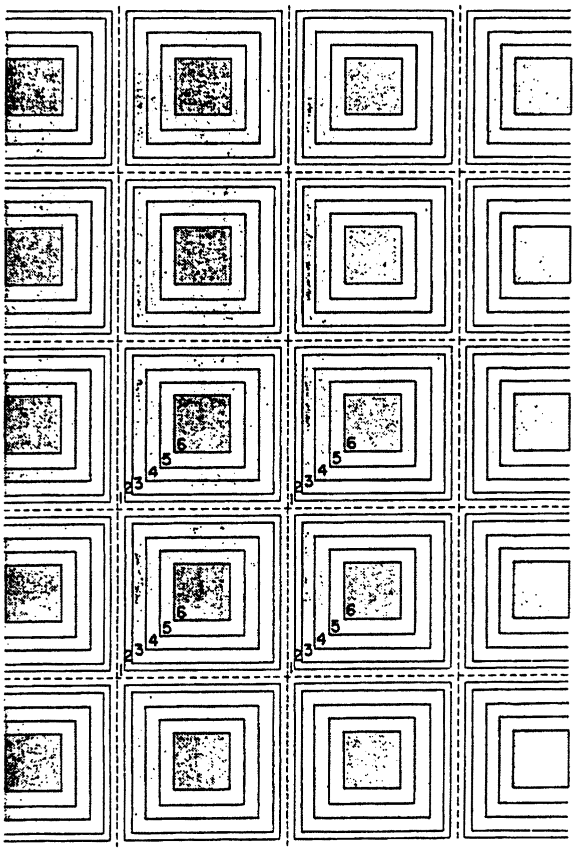

Fig. 2. Computational mesh for modeling transport processes in fractured porous media when MINC approximation is employed (after Pruess, 1983).

II. Fluid Flow in a Porous Cube (Case 1)

The test considered in Case 1 is an isothermal and slightly compressible fluid flow in a porous cube (or, equivalently, heat conduction in an impermeable cube). A constant pressure, P_b, is maintained on the cube surfaces with an initial pressure of zero being assumed everywhere. With MINC approximation, the fluid flow in a cube can be approximated by a one-dimensional model, as shown in Fig. 3. The basic model represents one-sixth of a cube, with the surface area used for flow decreasing from D^2 (D is the length of the cube) at the edges of the cube to zero at the center. Thus, the total mass flow on the cube surfaces will be six multiplied by the mass given in the one-dimensional model. This one-dimensional approximation leads to a differential equation, which is identical in form to the heat conduction equation for a system with spherical geometry. Carslaw and Jaeger (1959) gave the dimensionless pressure and flow rate for this problem using one-dimensional approximation:

$$P_D = \frac{P}{P_b} = 1 - \frac{D}{\pi z}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\frac{2n\pi z}{D}\exp\{-\frac{4n^2\pi^2 kt}{\phi\mu c D^2}\}\,, \quad (1)$$

$$q_D = \frac{q\mu}{kP_b} = 24\sum_{n=1}^{\infty}\exp\{-\frac{n^2\pi^2 kt}{\phi\mu c(D/2)^2}\}\,. \quad (2)$$

The same problem can be solved in three dimensions. Carslaw and Jaeger (1959) provided the dimensionless transient pressure and flow rate:

$$P_D = \frac{P}{P_b} = 1 - \frac{64}{\pi^3}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^{l+m+n-3}}{(2l-1)(2m-1)(2n-1)}$$
$$\times\cos\frac{(2l-1)\pi x}{D}\cos\frac{(2l-1)\pi y}{D}\cos\frac{(2l-1)\pi z}{D}$$
$$\times\exp\{-\frac{k\pi^2 t}{\phi\mu c D^2}[(2l-1)^2+(2m-1)^2+(2n-1)^2]\} \quad (3)$$

and

$$q_D = \frac{q\mu}{kP_f D} = \frac{24\times 64}{\pi^4}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{(2l-1)^2(2m-1)^2}$$
$$\times\exp\{-\frac{\pi^2}{4}[(2l-1)^2+(2m-1)^2+(2n-1)^2]$$
$$\times\frac{kt}{\phi c\mu(D/2)^2}\}\,. \quad (4)$$

In the above equations, the x-, y-, and z-coordinates are measured from the center of the cube and parallel to the corresponding edges. Figure 4 presents the dimensionless pressures at a distance of $z = 0.3D$ for MINC approximation and the exact solution plotted versus dimensionless time. The pressures obtained using MINC approximation are clearly somewhat higher than the exact pressures at the center of the plane of $z = 0.3D$ (i.e., for $x' = x/D = 0$, $y' = y/D = 0$) while the pressures obtained using MINC approximation in the

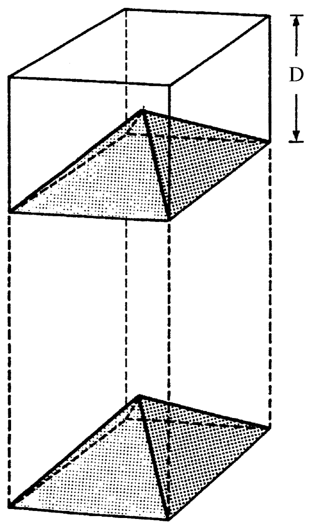

Fig. 3. Basic model for a porous cube using one-dimensional approximation.

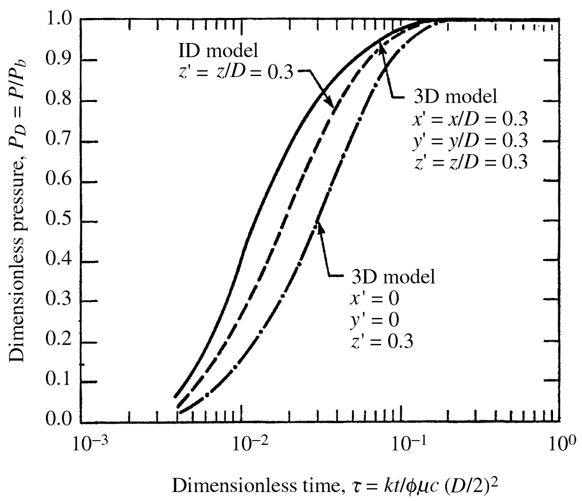

Fig. 4. Pressure distributions in a porous cube calculated using one-dimensional approximation and the exact solution (labeled as the 3D model).

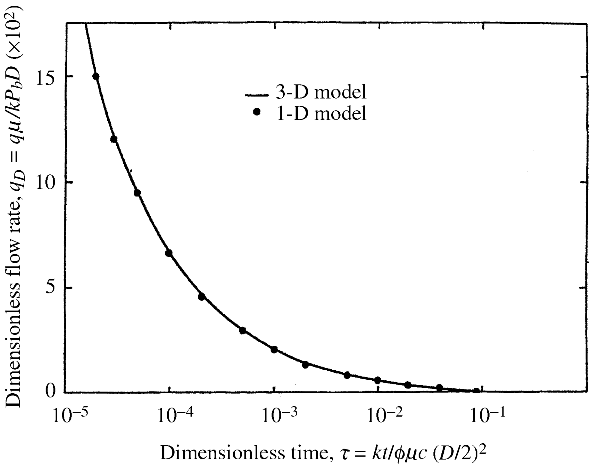

Fig. 5. Comparison of the flow rate from the cube using one-dimensional solution and the exact solution (labeled as the 3D model).

corner of that plane ($x' = y' = 0.3$) are somewhat lower than the exact pressures. The discrepancies are comparatively small (approximately 10 – 15%). Meanwhile, as shown in Fig. 5, good correlation exists between the MINC approximation results and the exact solution for the total (areally integrated) flow rate on the cube surface, which is more important than the distribution of the pressure field.

III. Fluid Flow in a Rectangular Porous Slab (Case 2)

To further test MINC approximation, the fluid flow rate can be compared in two-dimensional rectangular matrix blocks with the lengths A and B for different aspect ratios $\beta = A/B$. The same initial and boundary conditions are applied in Case 2 as in Case 1. Meanwhile, the basic model (Fig. 6) of a rectangle can be solved using MINC approximation. The governing equation describing the mass conservation in the domain of the basic model can be expressed as

$$qS - \{qS + \frac{\partial}{\partial z}(qS)dz\} = \frac{\partial(Sdz\phi\rho)}{\partial t}, \tag{5}$$

where q, ϕ, ρ, and t represent the mass flow rate, porosity, fluid density, and time, respectively. Meanwhile, S denotes the cross-sectional surface area in the z direction and can be expressed as

$$S = 4z + A - B. \tag{6}$$

By substituting Eq. (6) and Darcy's law ($q = -\frac{\rho k}{\mu}\nabla P$) into Eq. (5), the governing equation describing slightly compressible fluid flow in the domain of the basic model can be expressed as

$$\frac{\partial^2 P}{\partial z^2} + \frac{1}{z + 1/4(A-B)}\frac{\partial P}{\partial z} = \frac{\phi\mu c}{k}\frac{\partial P}{\partial t}, \tag{7}$$

where c is the fluid compressibility and k is the intrinsic permeability of the medium. The initial and boundary conditions are

$$P(z, 0) = 0, \tag{8}$$

$$P(\frac{B}{2}, t) = P_b, \tag{9}$$

$$\left.\frac{\partial P}{\partial z}\right|_{z=0} = 0. \tag{10}$$

In terms of dimensionless parameters, the governing equation and the initial and boundary conditions can be rewritten as

$$\frac{\partial^2 P_D}{\partial \eta^2} + \frac{1}{\eta}\frac{\partial P_D}{\partial \eta} = \frac{\partial P_D}{\partial t_D}, \tag{11}$$

$$P_D = 0, \tag{12}$$

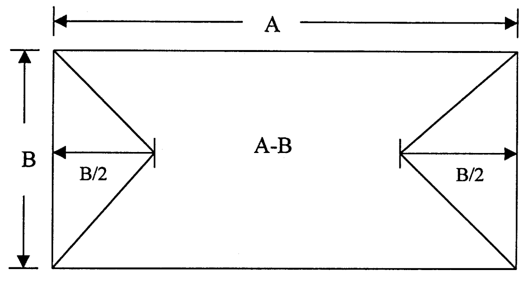

Fig. 6. Basic model for a rectangular porous slab using one-dimensional approximation.

$$P_D(\eta = \frac{A+B}{4B}, t_D) = 1\,, \quad \textbf{(13)}$$

$$\left.\frac{\partial P_D}{\partial \eta}\right|_{\eta = \frac{A-B}{4B}} = 0\,, \quad \textbf{(14)}$$

where

$$P_D = \frac{P}{P_b}\,, \quad \textbf{(15)}$$

$$\eta = \frac{z + 1/4(A-B)}{B}\,, \quad \textbf{(16)}$$

$$t_D = \frac{kt}{\phi\mu c B^2}\,. \quad \textbf{(17)}$$

In the Laplace domain, the solution of Eq. (7) subject to the specified initial and boundary conditions is

$$\overline{P_D} = \frac{1}{p}\frac{K_1(\sqrt{p}\frac{A-B}{4B})I_0(\sqrt{p}\,\eta) + K_0(\sqrt{p}\,\eta)I_1(\sqrt{p}\frac{A-B}{4B})}{K_1(\sqrt{p}\frac{A-B}{4B})I_0(\sqrt{p}\frac{A+B}{4B}) + K_0(\sqrt{p}\frac{A+B}{4B})I_1(\sqrt{p}\frac{A-B}{4B})}\,, \quad \textbf{(18)}$$

where p signifies the Laplace parameter while I_0, K_0, I_1 and K_1 are the modified Bessel function of the first kind and second kind of order 0 and 1, respectively. The dimensionless mass flow rate at the surface of the rectangle can be obtained from Eq. (18) by evaluating the pressure gradient on the surface and can be expressed as

$$\overline{q_D} = 2(\frac{A}{B}+1)\frac{1}{\sqrt{p}}\frac{K_1(\sqrt{p}\frac{A-B}{4B})I_1(\sqrt{p}\frac{A+B}{4B}) - K_1(\sqrt{p}\frac{A+B}{4B})I_1(\sqrt{p}\frac{A-B}{4B})}{K_1(\sqrt{p}\frac{A-B}{4B})I_0(\sqrt{p}\frac{A+B}{4B}) + K_0(\sqrt{p}\frac{A+B}{4B})I_1(\sqrt{p}\frac{A-B}{4B})}\,, \quad \textbf{(19)}$$

where $\overline{q_D}$ is equal to $q\mu/kP_b$. In this study, the real solution for the dimensionless flow rate is obtained by means of numerical inversion of Eq. (19) (Stehfest, 1970). The exact solution for this two-dimensional problem was provided by Carslaw and Jaeger (1959):

$$P_D = 1 - \frac{16}{\pi^2}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\frac{(-1)^{l+m-2}}{(2l-1)(2m-1)}\cos\frac{(2l-1)\pi x}{A} \times \cos\frac{(2m-1)\pi y}{B}\exp\{-\frac{l\pi^2 t}{\phi\mu c}[\frac{(2l-1)^2}{A^2}+\frac{(2m-1)^2}{B^2}]\} \quad \textbf{(20)}$$

and

$$q_D = \frac{64}{\pi^2}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\{\frac{A}{B}\frac{1}{(2l-1)^2}+\frac{B}{A}\frac{1}{(2m-1)^2}\} \times \exp\{-\frac{k\pi^2 t}{\phi\mu c}[\frac{(2l-1)^2}{A^2}+\frac{(2m-1)^2}{B^2}]\}\,. \quad \textbf{(21)}$$

Figure 7 indicates that the dimensionless mass flow rates across the surface of the rectangle obtained using MINC approximation for different aspect ratios (β) agree well with the exact solutions. The difference between the MINC approximation result and the exact solution decreases as the aspect ratio increases because the "corner" effect diminishes in MINC approximation.

IV. Conclusions

The "multiple interacting continua" method is based on the assumption that changes in the thermodynamic conditions of rock matrix blocks are primarily determined by the distance from the nearest fracture. The accuracy of this assumption has been evaluated herein for regularly shaped (cubic and rectangular) rock blocks with uniform initial conditions, which are afterward subjected to a stepwise change in the boundary conditions on the surface. The analysis results show that pressures (or temperatures) predicted via MINC approximation may deviate from exact solutions by as much as 10 – 15% at certain points within the blocks. However, when fluid

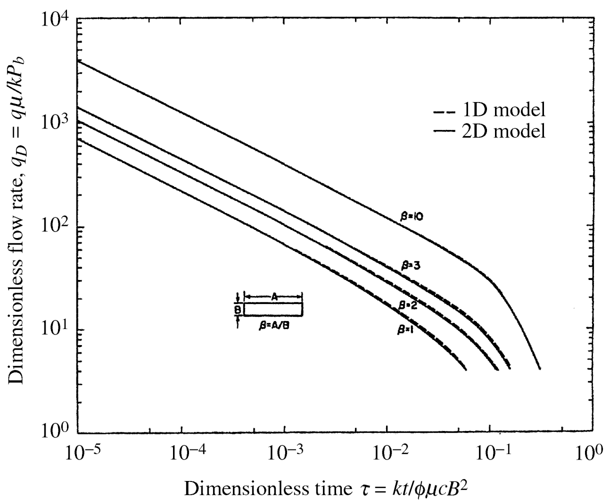

Fig. 7. Comparison of the flow rate from a rectangular porous slab between the one-dimensional solution and the exact solution (labeled as the 2D model).

(or heat) flow rates are integrated over the entire block surface, the results obtained using MINC-approximation agree well with the exact solutions, differing by less than 1%. This finding indicates that MINC approximation can accurately represent the transient inter-porosity flow in fractured porous media when the matrix blocks are indeed continuously subjected to nearly uniform boundary conditions.

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多層交互介質模式正確性之探討

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摘　要

本文探討多層交互介質模式之正確性，選擇一形狀規則之岩體配合均勻之起始條件及驟變之邊界條件作爲評估對象，研究結果顯示多層交互介質模式預測岩體內部壓力（或溫度）之變化與解析解之誤差在10 – 15%，但整塊岩體表面之水流（或熱流）通量在1%以下之誤差，此一結果進一步闡明出多層交互介質模式能準確預估水流（或熱流）在破碎孔隙介質中岩基與裂隙間之暫態變化，並且岩基表面長期處於近似均勻之邊界情況。