Forest Sci., Vol. 32, No. 4, 1986, pp. 845–854

Market Value of Timber When Some Offerings Are Not Sold: Implications for Appraisal and Demand Analysis

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Abstract. A model of market value was developed for tracts of timber sold at auction for a minimum appraised value, when a substantial number of the tracts offered do not receive a bid. In this model, market value is a function of the characteristics of the timber and of market conditions, but market value is observed and equal to the high bid only if it is greater than the appraised value. This tobit model leads to the probability of selling a tract of given characteristics, conditional on a certain appraised value. Symmetrically, the model yields the appraised value that ensures a certain probability of selling. The expected high bid, conditional or unconditional on the occurrence of a sale, can also be computed. When this is done for all tracts offered and for various probability targets, a schedule of expected volume sold against expected price is obtained that corresponds to the aggregate demand for timber from the forest of interest. The model was estimated by maximum likelihood with data from the Chequamegon National Forest (WI) from 1976 to 1980. Variables influencing market value were: volume and quality of timber, hauling distance, hardwood lumber price, and whether the sale was a salvage operation. The corresponding parameters were used to predict the effect of setting appraised values that would have sold 95, 75, or 65% of the tracts offered for sale on the Chequamegon in 1981 and 1982. The results led to an estimate of the price elasticity of the demand for timber from that forest of −1.56. Forest Sci. 32:845–854.

Additional key words. Transaction evidence, stumpage markets, national forests, bidding, demand, elasticities, prices, tobit, econometrics.

The National Forest Management Act of 1976 states that "the Secretary of Agriculture, under such rules and regulations as he may prescribe, may sell, *at not less than appraised value,* trees, portions of trees, or forest products located on National Forest system lands" (USDA Forest Service 1978, emphasis added).

In implementing this mandate, the Forest Service has defined the appraised value as "the value based on the forestry industry firm of average efficiency, which will interest sufficient purchasers to harvest the allowable cut under multiple-use and sustained yield principles" (USDA Forest Service 1977). This definition implies that the basis for appraising timber is the competitive market, but also that the appraised value may be adjusted to reflect the various goals of the agency, including equity, multiple-use, and sustained yield.

The practical application of this policy has used either the residual-value method or the transaction-evidence method of appraisal (Wiener 1981). The former con-

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sists in estimating the value of final products likely to result from a particular tract of timber, subtracting all processing costs and a reasonable profit to arrive at the value of stumpage (USDA Forest Service 1982). The latter relies instead almost entirely on the price paid in the past for sales similar to that being offered (USDA Forest Service 1981).

Recent applications of the transaction-evidence approach have used multiple regression techniques to compute the expected high bid on a tract of timber in function of different variables that describe the tract and market conditions. The appraised value is then set equal to the predicted high bid, minus some fraction of the standard error of estimates. The database to compute the regression equations consists of past sales, usually on the same forest (USDI Bureau of Land Management 1981, USDA Forest Service 1984, Buongiorno and Young 1984, Merzenich 1985).

A problem arises in estimating these regressions when a substantial fraction of the tracts of timber offered for sale in the past have not been sold. In that case, the unsold offerings cannot just be removed from the database. This would lead to a biased prediction of the market value of a tract yet to be sold. Clearly, the fact that nobody bought some timber is important market information. Conversely, one cannot assign a value of zero to the offerings not sold and use the data in the regression. This also would lead to predictions of market value that are biased, in addition to residuals that are heteroscedastic.

The main purpose of this paper was to develop a rigorous transaction-evidence appraisal model that used all available information. When an offering is not sold we do have information on market value, but it is limited. All we know is that the true market value lies between zero and the appraised value.

In the next section, a theoretical model consistent with this information is presented. The model yields successively the probability that an offering will be sold conditional on a certain appraised value, the expected high bid if the offering is sold, and the unconditional market value. We will show how the model can be used to estimate the demand schedule for timber from a forest. The paper then reports the results of applying the model to the Chequamegon National Forest, in northern Wisconsin.

THE MODEL

The basic hypothesis of the model is that the market value of a tract of timber, exclusive of land, is a function of the characteristics of the tract, such as timber volume, species, size, location of sale, and market conditions. For simplicity, in the remainder of the paper we refer to all these variables as "timber sale (or tract) characteristics." If the market value is at least equal to the appraised value, then the market value is measured by the highest bid. If the tract is not sold then the market value is not observed; it lies somewhere below the appraised value. In that sense, the market value is a latent variable. Formally, the model can be expressed in the following system of equations:

$$MV_i^* = B'X_i + e_i \tag{1}$$
$$THB_i = MV_i^* \quad \text{if} \quad MV_i^* \geq AV_i \tag{2}$$
$$THB_i = 0 \quad \text{if} \quad MV_i^* < AV_i \tag{3}$$

where

MV_i^* is the market value of tract i, the star showing that this latent variable is observed only in circumstances defined by equation (2).

X_i is a vector of variables defining the characteristics of tract i, or suitable transformations of these variables.

B is a vector of coefficients corresponding to the variables X_i.

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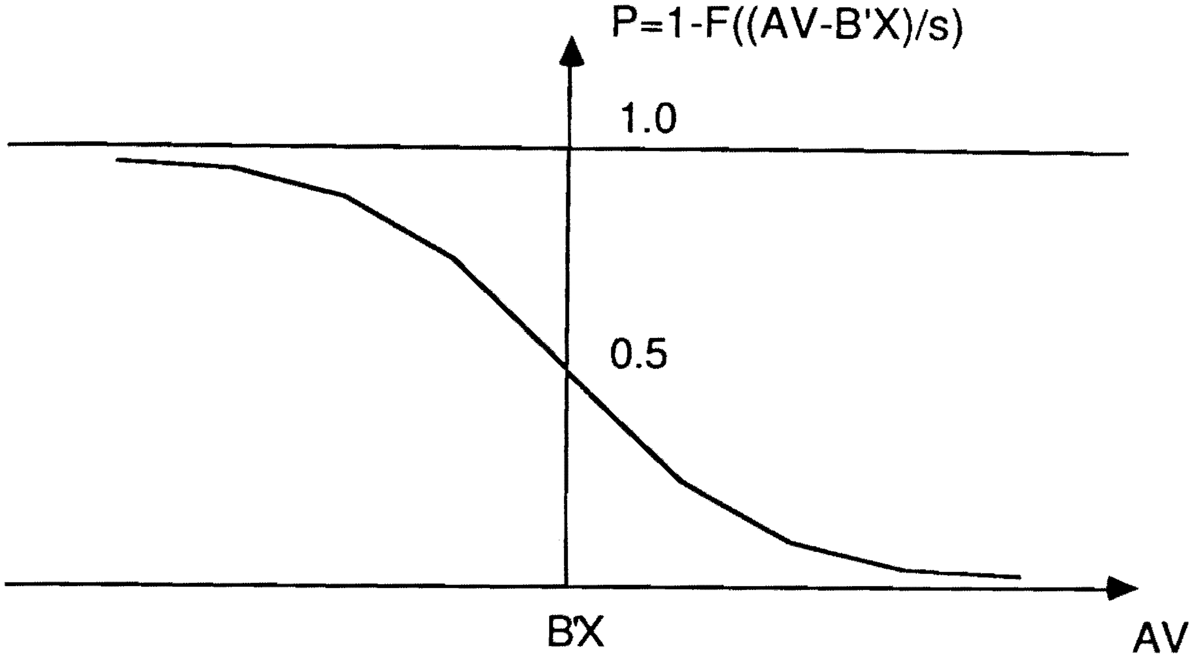

FIGURE 1. The probability (P) of selling a tract of timber conditional on timber characteristics (X) decreases monotonically as the appraised value (AV) increases.

THB_i is the highest bid for tract i.
AV_i is the appraised value for tract i.
e_i is a residual distributed normally and independently across offerings, conditional on X_i, with mean zero and variance σ^2.

In equation (3) the high bid is arbitrarily assigned a value of zero when the market value is less than the appraised value; any dummy value to record the fact that the tract is not sold would be suitable.

The set of equations (1), (2), (3), forms a tobit model, named after the seminal work of James Tobin on household demand equations (Tobin 1958). Based on this model, we can derive the probability of selling a tract, conditional on a certain appraised value and specific sale characteristics, namely (Amemiya 1984):

$$\begin{aligned} P(MV_i^* \geq AV_i|X_i) &= 1 - P(MV_i^* < AV_i|X_i) \\ &= 1 - F[(AV_i - B'X_i)/\sigma] \\ &= F[(-AV_i + B'X_i)/\sigma] \end{aligned} \quad (4)$$

where $F(\cdot)$ is the cumulative distribution function for a standardized normal variable. The variation of the probability of a sale in function of the appraised value, AV_i, for a given set of sale characteristics, X_i, is shown in Figure 1. The probability decreases in monotonous fashion as the appraised value increases. It is equal to 0.50 when the appraised value is just equal to $B'X_i$, the expected market value.

This same equation (4) can be used in a symmetric manner to determine what appraised value will ensure a certain probability of sale. Let us assume that the seller decides that only a certain fraction, u, of the offerings can remain unsold. Then, from equation (4) we must have:

$$1 - u = F[(-AV_i + B'X_i)/\sigma],$$

or

$$AV_i = -z(1 - u)\,\sigma + B'X_i \quad (5)$$

where $z(1 - u)$ is the standard normal deviate such that the probability of it being exceeded is $1 - u$. For example, if $u = 0.10$ then $z(1 - u) = 1.282$.

What the value of u should be is a policy decision. However, the consequences of different values of u on expected receipts can be determined readily as follows. As shown in Appendix A, the expected high bid on a tract of given characteristics is, if it is sold:

$$E(THB_i \mid X_i;\ MV_i^* \geq AV_i) = B'X_i + \sigma h[(AV_i - B'X_i)/\sigma] \quad (6)$$

where the function h is defined as $h(\cdot) = f(\cdot)/[1 - F(\cdot)]$, where f is the probability density function of a standardized normal variable. The function h is usually referred to as the hazard rate, and its reciprocal as the Mill's ratio (Amemiya 1984). Since h is a monotonically increasing function within the range of interest in this paper, the conditional high bid rises as the appraised value, AV_i, increases.

Then, the unconditional expected high bid for a tract of given characteristics is the product of the probability of selling and of the expected high bid, given a sale. That is, using equations (4) and (6):

$$\begin{aligned} E(THB_i \mid X_i) &= E(THB_i \mid X_i;\ MV_i^* \geq AV_i)P(MV_i^* \geq AV_i \mid X_i) \\ &= B'X_i F[(-AV_i + B'X_i)/\sigma] + \sigma f[(-AV_i + B'X_i)/\sigma] \end{aligned} \quad (7)$$

Here, increases of appraised value, AV_i, act in opposite directions. They increase the conditional high bid, but decrease the probability of selling.

Summing $E(THB_i \mid X_i)$ over all offerings will then give the expected receipts corresponding to a specific value of u: $R(u) = \Sigma_i E(THB_i \mid X_i)$. The corresponding expected volume sold is: $Q(u) = (1 - u)\ \Sigma_i Q_i$, where Q_i is the volume of timber on the ith tract. Therefore, the expected average price received is:

$$P(u) = R(u)/Q(u) \quad (8)$$

A sequence of prices $P(u)$ and corresponding quantities $Q(u)$ for different values of u describes the demand schedule for timber on the forest of interest.

ESTIMATION METHOD

The selected sample regression function (6) depends on the sale characteristics X_i and on h_i. Omitting the second term as a regressor in the subsample regression will result in biased and inconsistent estimates for B. This is exactly what happens when high bid is regressed on sale characteristics for the offerings that were sold only. The bias is seen to arise from the ordinary problem of omitted variables.

Clearly, ordinary least squares is not appropriate to estimate B and σ over the subsample. Several methods have been proposed to do the estimation, including Heckman's two-step estimator (Heckman 1976) and nonlinear least squares and weighted least square procedures (Amemiya 1984). However, in the case of model (1), (2), (3), a fully specified likelihood function based on the conditional distribution of THB_i given X_i and AV_i can be derived. As shown in Appendix B, the logarithmic likelihood function is:

$$L = \sum_{i \in s_1} [-\log \sigma - (1/2\sigma^2)(THB_i - B'X_i)^2] + \sum_{i \in s_2} \{\log F[(AV_i - B'X_i)/\sigma]\} \quad (9)$$

where

$s_1 = \{i\text{: } MV_i^* \geq AV_i\}$ is the set of offerings that were sold.
$s_2 = \{i\text{: } MV_i^* < AV_i\}$ is the set of offerings that were not sold.
$\in$ is the inclusion operator.

Maximizing L with respect to the parameters B and σ yields consistent, asymptotically unbiased and efficient estimates that are asymptotically normally distributed (Amemiya 1984). Since the log-likelihood function is nonlinear, an iterative procedure such as Newton-Raphson must be used.

TABLE 1. Definition of variables used in market value model.

Variable	Definition
THB	Total highest bid ($).
AV	Total appraised value ($).
SPV	Volume of softwood pulpwood (CCF).
SSV	Volume of softwood sawtimber (CCF).
HPV	Volume of hardwood pulpwood (CCF).
HSV	Volume of hardwood sawtimber (CCF).
SLP	Price index of softwood lumber in the quarter before the sale.
HLP	Price index of hardwood lumber in the quarter before the sale.
HAUVOL	Product of estimated hauling distance to market and of volume sold (miles × CCF).
SRC	Specified length of permanent road construction (miles).
ACRES	Area of the tract of timber sold (acres).
SALV	Dummy variable = 1 for salvage sales, 0 otherwise.
METH	Dummy variable = 1 for sealed bids, 0 otherwise.

Note: CCF = hundred cubic feet.

APPLICATION

The principles outlined above were applied to the Chequamegon National Forest in northern Wisconsin. This forest comprises some 840,000 acres of aspen-birch and maple-beech-birch forest. The administrators of the forest offered 234 tracts of timber for sale from 1976 to 1980. Of these, 191 were sold and 43 were not. The data on high bid (THB_i), appraised value (AV_i) and sale characteristics (X_i) came mostly from Forest Service Appraisal summary sheets (R9-2400-17) and timber sale reports (FSM-2490); all variables used are listed in Table 1.

In order to keep the number of variables in X_i reasonably small, while including all timber species on the forest, individual species volumes were aggregated into the following groups: softwood pulpwood (*SPV*), softwood sawtimber (*SSV*), hardwood pulpwood (*HPV*), and hardwood sawtimber (*HSV*). The coefficients of these variables in equation (1) can be interpreted as the unit price of each group. We expected this unit price to increase with the volume offered. Specifically, the unit price of a species group was assumed to vary linearly with the volume offered. This led to the following expression for the returns originating from a particular species group, R_s:

$$R_s = (a_s + b_sQ_s)Q_s = a_sQ_s + b_sQ_s^2$$

where Q_s is the volume of timber of species/quality *s*, and a_s and b_s are constants. Therefore, equation (1) was quadratic with respect to *SPV*, *SSV*, and *HSV*.

Other timber sale characteristics that we thought might influence market value included the cost of road building, the cost of hauling, and the density of timber. Cost of road construction was approximated by the length of road construction specified by the Forest Service, *SRC*. This variable was expected to have a negative coefficient measuring a cost per mile. The cost of hauling was reflected by the product of the estimated length of haul multiplied by the total volume offered, *HAUVOL*. This variable was expected to have a negative coefficient, reflecting a cost per mile, per unit of volume. The effect of timber density was represented by the area of the sale, *ACRES*. One would expect unit costs to be lower for dense than sparse stands. Consequently, other things being equal, larger areas should correspond to lower values.

Two variables were used to reflect market conditions, the price of softwood

TABLE 2. Tobit maximum-likelihood estimates of market value equation coefficients for tracts of timber offered on the Chequamegon National Forest from 1976 to 1980.

Variable	ML estimate		Standard error	t ratio
SPV	7.0	***	2.1	3.34
SPV^2	0.0034	***	0.0006	5.78
HPV	−5.0	**	2.5	−2.18
HPV^2	0.0035	***	0.0009	3.86
SSV^2	0.066	***	0.007	9.67
HSV^2	0.024	***	0.003	7.89
SRC	−26.0		31.0	−0.83
HAUVOL	−0.09	***	0.03	−2.89
SLP	−40.0		42.0	−0.95
HLP	129.0	**	67.0	1.93
SALV	4,237.0	***	1,433.0	2.96
METH	2,995.0		2,512.0	1.19
ACRES	0.8		1.8	0.42
Constant	−16,931.0	***	5,318.0	−3.18
σ	7,787.0	***	429.0	18.16

Notes: *** and ** indicate coefficients that were significantly different from zero at the 0.99 and 0.95 confidence level, respectively.

lumber in the quarter before the timber was offered, *SLP*, and that of hardwood lumber, *HLP*. Other things being equal we expected the market value of tracts to increase as these two variables increased. A dummy variable was used for the method of auction, *METH* (sealed or oral bid). Haynes (1980) found that sealed bidding led to higher bid prices in areas of low competition among buyers. Another dummy variable, *SALV*, was used to distinguish salvage sales, which were expected to have less value.

The results of estimation of equation (1) are presented in Table 2. The estimation was done by maximizing the log likelihood function (9), over the parameter vectors B and σ, using the data set and variables just described. The variables that had a significant influence on the market value of a tract, at the 95% confidence level, were those measuring the volume of timber of different species and size: *SPV*, *HPV*, *SSV*, *HSV*, or the square thereof. All squared terms had a positive parameter, supporting the hypothesis that the unit value of timber increased as the volume increased. Also significant were the variables measuring the distance times the volume hauled (*HAUVOL*), the hardwood lumber price (*HLP*), and the salvage dummy (*SALV*). All three variables had the expected signs.

On the other hand, the area of the sale (*ACRES*), and the method of sale (*METH*) did not seem to influence market value significantly; nor did the price of softwood lumber (*SLP*) and the specified length of road construction (*SRC*). These negative results are plausible since: (1) most of the timber sold on the Chequamegon consists of hardwoods; (2) after 1976 few sales were made at oral auctions; and (3) the Forest Service refunds the buyer for a large part of road building expenditures.

The estimates of σ and B in Table 2 can be inserted in equation (4) to predict the probability that an offering of specific characteristics would be sold, conditional on a preset appraised value. The effect of increases in the variables X_i on the probability of selling is in the same direction as the effect on market value. For example, an increase in the volume of hardwood sawlog volume, *HSV*, leads to an increase in the probability of selling, other things being equal. So does an increase in the price of hardwood lumber. On the other hand, the probability of

TABLE 3. Effects of different methods of appraising timber on timber sales, receipts, and prices on the Chequamegon National Forest in 1981 and 1982.

Appraisal method	Sales		Volume sold		Sale receipts ($)	
	No.	%	CCF	%	Total	CCF
Current	55	85	99,159	80	1,080,141	10.89
Alternative I	61	95	115,089	95	762,870	6.63
Alternative II	48	75	90,859	75	713,778	7.86
Alternative III	42	65	78,745	65	664,539	8.44

Notes: Effects of the current method are those observed in 1981 and 1982. Effects of alternative methods are those predicted, had they been applied in 1981 and 1982. Alternatives I, II, and III set appraised value so that the probability of selling is 95, 75, and 65%, respectively. CCF = hundred cubic feet.

selling declines as the distance-volume hauled, *HAUVOL*, increases. It also declines with increases of the appraised value.

Alternatively, the parameters in Table 2 can be used in equation (5) to set the appraised value, given a desired probability of selling. Here, the decision-maker faces a dilemma. Setting the probability of sale too high would result in appraised values that are much below market value. This would defeat the main purpose of appraising timber, which is to ensure that no timber is bought at token prices even when competition is weak. On the other hand, a low probability of selling would lead to high appraised values and, therefore, costly sales when they occur [equation (6)] and high timber prices. Besides, a traditional goal of the Forest Service, which is to dispose of the allowable cut, would not be reached.

Fortunately, the equations developed above can be used to predict some of the consequences of the probability of selling at any level. As an example, we have used sales data on the Chequamegon National Forest in 1981 and 1982. For each sale we computed the appraised value that would have ensured a probability of selling of 95, 75, or 65%. This was done using equation (5) and the parameters in Table 2. We then determined the expected high bid, if the offering was sold, with equation (6), and the unconditional high bid with equation (7). Adding this over all tracts gave the total expected income for these two years. This, divided by the expected volume sold, gave the expected average price [equation (8)].

The results are shown in Table 3. Had appraised values been set so that the probability of selling any tract was 95%, 61 sales would have been made, instead of the 55 that actually occurred. But total predicted receipts would have been less than those observed, and so would have been the average price paid per unit (CCF). The predicted unit price rose as the target probability, and the corresponding volume offered declined. Nevertheless, the predicted price was always smaller than the one observed. It seems that demand was unusually high in 1981 and 1982, compared to the years 1976 to 1980 that were used to calibrate the model, and that the variables used could not capture this rise in demand.

Keeping in mind these limitations of the model, the data in Table 3 give some indication of the shape of the demand for timber faced by the Chequamegon National Forest. Plotting the volume of timber sold against the average price received, for each one of the three alternatives in Table 3, revealed a nearly linear demand curve. The arc elasticity is:

$$E = [(Q_1 - Q_3)/(Q1 + Q3)/(P1 - P3)/(P1 + P3)] = -1.56$$

where *Wi* and *Pi* are the volume and price for alternative *i* in Table 3. $E = -1.56$ indicates an elastic demand curve such that a decline in the volume sold from the National Forest of 10% leads to an increase in price of some 6.4%.

SUMMARY AND CONCLUSIONS

The purpose of this paper was to develop a model of the market value of stumpage that would be applicable when a significant number of the tracts of timber offered in the past were not sold. The tobit model used was such that (1) sale value depended on sale characteristics and (2) sale value was observed only if it was higher than the appraised value. The model led to equations that predicted the probability of selling, given the characteristics of the tract offered and the appraised value. The expected high bid, conditional on the occurrence of the sale, and the unconditional high bid were also derived.

The most direct application of these equations is in calculating appraised values that are consistent with the multiple goals of the selling agency. For example, given the desire to sell a particular tract of timber, reflected by a certain target probability of selling, it is a simple matter to determine the appraised value consistent with that target, given the sale characteristics and the history of past sales on that forest.

We have also shown how, based on individual sale data, one could develop a schedule relating total volume sold on the forest to the price per unit of timber. This way of estimating the demand for timber is worthy of further study, as an alternative to the standard approaches that rely on aggregate data from a forest (Adams 1983, Connaughton and Haynes 1983). It should be kept in mind, however, that this alternative is possible only if a substantial fraction of the volume of timber that is offered on the forest of interest is not sold.

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APPENDIX A

Derivation of equation (6):

$$E(THB_i | X_i, MV_i^* \geq AV_i) = E(B'X_i + e_i | X_i, MV_i^* \geq AV_i)$$
$$= B'X_i + \sigma E(e_i/\sigma | X_i, MV_i^*/\sigma \geq AV_i/\sigma),$$

due to the hortogonality of e_i and X_i.

Then, using (1) yields:

$$E(THB_i | X_i, MV_i^* \geq AV_i) = B'X_i + \sigma E(e_i/\sigma | e_i/\sigma \geq [(AV_i - B'X_i)/\sigma]. \quad \text{(A1)}$$

Now, let

$$e_i/\sigma = V \text{ and } (AV_i - B'X_i)/\sigma = a,$$

then

$$E(V | V \geq a) = \int_{-\infty}^{\infty} tf(t | V \geq a)\, dt$$

where

$$f(t | V \geq a) = \partial P(V \leq t | V \geq a)/\partial t$$

and

$$P(V \leq t | V \geq a) = P(a \leq V \leq t)/P(V \geq a)$$
$$= [F(t) - F(a)]/[1 - F(a)] \quad \text{if} \quad t > a, \text{ 0 otherwise,}$$

so that

$$f(t | V \geq a) = \{1/[1 - F(a)]\}\partial[F(t) - F(a)]/\partial t$$
$$f(t | V \geq a) = f(t)/[1 - F(a)] \quad \text{if} \quad t > a,$$

where

$$f(t) = (2\pi)^{-1}\exp(-½t^2).$$

Therefore

$$E(V | V \geq a) = \int_a^{\infty} tf(t)/[1 - F(a)]\, dt$$

and since

$$(2\pi)^{-½} \int_a^{\infty} t \exp(-½t^2)\, dt = -(2\pi)^{½}\exp(-½t^2)\Big|_a^{\infty} = f(a)$$

we get

$$E(V | V \geq a) = f(a)/[1 - F(a)],$$

which, substituted in equation (A1), yields the desired equation (6).

APPENDIX B

Derivation of the equation (9):

If $THB_i = 0$, the contribution to the likelihood function is:

$$P(THB_i = 0 | X_i) = P(B'X_i + e_i < AV_i | X_i)$$
$$= P[e_i/\sigma < (AV_i - B'X_i)/\sigma | X_i]$$
$$= F[(AV_i - B'X_i)/\sigma] \text{ due to the normality of } e_i.$$

If $THB_i > 0$, the contribution to the likelihood function is

$$(2\pi)^{-1/2}\sigma^{-1}\exp\{-1/2[(THB_i - B'X_i)/\sigma]^2\}.$$

Thus, letting s_1 and s_2 be the set of observations for which $THB_i > 0$ and $THB_i = 0$, respectively, the total likelihood function is:

$$1 = \prod_{i \in s_1} (2\pi)^{-1/2}\sigma^{-1}\exp\{-1/2[(THB_i - B'X_i)/\sigma]^2\} \prod_{i \in s_2} F[(AV_i - B'X_i)/\sigma]$$

which, after taking the logarithms, leads to the log-likelihood function (9).