# The effect of sequential information releases on trading volume and price behaviour

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Abstract – This paper examines a two-period setting in which each trader receives a private signal, possibly different, in each period before he trades. The principal objectives are threefold. First, we describe the risky asset demands and price reactions in a noisy rational expectations equilibrium where the time 1 average private signal is not revealed by the price sequence but the time 2 average private signal is. Secondly, we analyse how informed trading volume is affected by the revealed information and supply shocks when pure noise trading volume is uncorrelated with observable market variables. Our result indicates that no trade occurs for informed traders when net supply remains fixed across rounds of trade. And, when supply shocks are random, trading volume is induced by the informed and the noise traders, but noise trading is not predictable. Finally, we investigate these properties in the case when pure noise trading volume is correlated with observable market variables. It is shown that no informed trading takes place when there is no supply shock. However, when net supply contains random shocks, trading volume consists of noise and informed trading, both of which can be estimated.

### 1. Introduction

In the framework of rational expectations models, most papers focus more on price change. But price change, as Beaver (1968) points out, reflects the average change in traders' beliefs attributable to the public disclosure, whereas trading volume reflects traders' idiosyncratic reactions. We examine how the revelation of information influences the price change as well as the trading behaviour of market participants in a multiperiod setting. According to the literature on noisy rational expectations, an exogenous supply shock prevents price from fully revealing the average private signal in a one-period model. Grossman and Stiglitz (1980), and Verrecchia (1982) provide a one-trading-date model with private information acquisition prior to trade. Information acquisition in our paper is similar to these articles, but we discuss two rounds of trade, and a costly signal is acquired by some investors privately prior to trade in both rounds.

Some recent papers, Demski and Feltham (1994), McNichols and Trueman (1994), Kim and Verrecchia (1991b), and Grundy and McNichols (1989), are based on a two-period (three-date) noisy rational expectations model with endoge-nous private information acquisition and trading at

time 1, a public information release and trading at time 2, and consumption at time 3. Our paper examines a different setting adapted from Brown and Jennings (1989). Brown and Jennings assume that traders' times 1 and 2 signals are private and uncorrelated. They show that technical analysis has value in a two-period model in which traders have rational expectations about the relation between signals and prices. However, they do not consider how information revealed by the price sequence affects trading volume.

In our model, we assume that just prior to the opening of the market at times 1 and 2, each trader receives a private signal regarding the accounting earnings of the time  $3^{1}$ . This implies that traders begin with homogeneous beliefs about the forthcoming accounting earnings and then receive private signals that cause their beliefs to diverge. The two private signals, at two different dates, have uncorrelated (or correlated) errors. In the real world we can observe many other examples of private signal acquisition. One example is related to dealers' trading. The security analysts hired by dealers forecast accounting earnings for dealers' trade. These forecast data are private signals because they are only provided to dealers, and not disclosed to outside investors. Consider another example. Suppose firm A has a business relationship with firm B and through business activity firm A has private information about firm B's prospect.

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<sup>&</sup>lt;sup>1</sup> We assume that the accounting earnings is equal to the cash flow. This is because in our model earnings will not be measured until time 3, the end of the time line, and hence the earnings based on accrual accounting is the same as the cash flow at time 3. Note, when the cash flow is disclosed, the time 3 payoff of the risky asset is common knowledge.

Therefore, firm A can earn profit by buying or selling firm B's shares.

Furthermore, we suppose that per capita supply of the risky asset is uncertain at times 1 and 2. Each trader cannot fully infer the time 1 average private signal from the price sequence because all variables are unobservable. Lin, Wang and Tsai (1995) show that there exists a noisy rational expectations equilibrium price of and demand for the risky asset in which the average private signal at dates 1 and 2 are not revealed by the price sequence. In that case, all historical prices are used in decision making, and trading volume is related to the differences between individual trader's beliefs and to the average belief among all traders. On the contrary, in our model of no supply shock, we analyse the equilibrium price of and demand for the risky asset when the sequence of prices does not reveal the time 1 average private signal but the time 2 equilibrium price reveals the time 2 average private signal. This setting implies the information content of a forthcoming accounting earnings contained in the time 1's private signal has only been partially impounded in both times' prices, but the information contained in the time 2's private signal to be released in the forthcoming earnings announcement has been fully impounded in current price. Grundy and McNichols (1989) assume that traders receive private and public signals at times 1 and 2, respectively. They show that no trade takes place in the second round when the time 1 average private signal is not revealed by the price sequence. In the case of supply without random shock, we show that the informed traders achieve a Pareto-optimal allocation at time 1 and they do not trade at time 2. That is, in this equilibrium, the time 2 price contains the information of the time 2 average private signal, and we can think of it as a public signal. Hence, the time 2 average private signal is redundant. The model of Grundy and McNichols is therefore a special case of ours.

Grundy and McNichols also demonstrate the existence of and examine the characteristics of linear and rational price functions when there is no time 2 variation in supply, with discussion about the effects of the addition of a second supply variation. Similarly, we first examine the properties of price functions without the time 2 supply shocks, and then relax this assumption. As developed by Lang, Litzenberger and Madrigal (1992), we have two groups of traders participating in the market. The traders of one group, referred to as the informed traders, privately receive imperfect information about the firm's end-of-period liquidating value prior to trade. The noise traders are the second group of traders, and their trades are exogenously random variables with distributions that are independent of their information and the risky asset's price. That is, their demand for the risky asset is unrelated to the information content of the public or private signals, such as the information content of an earnings announcement. This noise trading is a convenient device to ensure price is not fully informative. We also investigate the patterns of trading volume in the setting where pure noise trading volume is correlated or uncorrelated with observable market variables, such as prices and public releases.

Our results indicate that when pure noise trading volume is uncorrelated with observable market variables, no informed trading occurs with constant net supply. And when net supply is with random shock, the time 2 holding units of the risky asset by the informed and the noise traders are different from the time 1 holding units, but only the informed trading volume is predictable. In the case of pure noise trading volume being correlated with observable market variables, informed traders also do not trade when there is no supply shock. On the other hand, when net supply contains random shocks, volume consists of noise trading and informed trading. They both constitute the predictable components of trading volume. However, when supply contains random shock, the time 2 average private information is only partially revealed irrespective of pure noise trading being uncorrelated or correlated with market variables. Moreover, to reveal the empirical implications of a theoretical model of trading volume, we also derive a model to describe what factors would influence the total trading volume. With supply shocks, no matter whether pure noise trading is correlated or uncorrelated with observable variables, our model shows that total trading volume is related to the market price of the risky asset and informed traders' individual expectations as well as the average expectation of the liquidating value of the risky asset in both periods.

In the following sections, a two-period model of asset prices and demands is introduced. Section 2 describes the equilibrium price in which the time 1 average private signal is not fully revealed by the sequence of prices, but the time 2 average private signal is. Section 3 shows and compares the patterns of trading volume and their empirical implications in the settings of pure noise trading being correlated and uncorrelated with observable market variables. Conclusions and suggestions for future research are contained in Section 4.

### 2. The basic model

The basic framework for our analysis follows Grundy and McNichols (1989), and Brown and Jennings (1989). There are two time periods (three dates). A riskless asset and one risky asset are exchanged in markets opening at times t = 1 and t = 2. Consumption occurs only at t = 3 when each share of the riskless asset pays 1 unit and the risky

asset provides a random payoff  $\tilde{F}$ <sup>2</sup>. The risk-free rate is assumed to be 0. A set of N informed traders indexed by i = 1, 2, ..., N own random endowments  $\tilde{\xi}_i$  of the risky asset. The endowments are independently and identically normally distributed with mean  $\mu_r$  and variance  $N\sigma_r^2$ . Because total supply of the risky asset from informed traders is equal to total endowment of the risky asset from informed traders, total supply of the risky asset is calculated by  $\Sigma_i \tilde{\xi}_i$ . Hence, the per capita supply of the risky asset from informed traders,  $\tilde{X}$ , is  $(\Sigma_i \tilde{\xi}_i)/N^3$  We consider the limiting results when traders are countably infinite in number, and per capita supply of informed traders,  $\tilde{X}$ , is then normally distributed with mean  $\mu_x$  and variance  $\sigma_x^{2,4}$  The correlation between  $\tilde{\xi}_i$ , and  $\tilde{X}$  converges to zero as N goes to in-finity. Thus, trader *i*'s observation of  $\tilde{\xi}_i$  cannot provide any information about the realisation of  $\tilde{X}$ under the limiting economy.

At the first trading date, informed trader *i* receives a private signal

$$\tilde{Y}_i = \tilde{F} + \tilde{w} + \tilde{e}_i \tag{1}$$

where  $\tilde{F}$  is the per-unit payoff from the risky asset that has a normal distribution with mean zero and variance  $\sigma_F^2$ ,  $\tilde{w}$  is a common noise term that has a normal distribution with mean zero and variance  $\sigma_w^2$ , and  $\tilde{e}_i$  is an idiosyncratic noise term that has a normal distribution with mean zero and variance  $\sigma_e^2$ . The random variables  $\tilde{F}$ ,  $\tilde{w}$  and  $\tilde{e}_i$  are independent of one another. Furthermore, the idiosyncratic noise terms  $\tilde{e}_i$  are uncorrelated, and hence independent across traders, that is  $E[\tilde{e}_i \cdot \tilde{e}_i] = 0$  for all  $i \neq j$ .

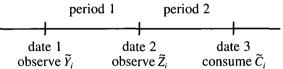
At the second trading date, informed trader i observes a private signal

$$\tilde{Z}_i = \tilde{F} + \tilde{\nu} + \tilde{\eta}_i \tag{2}$$

where  $\tilde{v}$  is a common noise term that has a normal

<sup>3</sup> We denote  $\tilde{X}_t$  as the random per capita supply of the risky asset from informed traders at time t, t=1,2. If  $\tilde{X}_1=\tilde{X}_2$ , we denote both periods' per capita supply of informed traders by  $\tilde{X}$ . <sup>4</sup> Grossman (1976, 1978) argues that the equilibrium price

<sup>4</sup> Grossman (1976, 1978) argues that the equilibrium price aggregates the available information perfectly. Hellwig (1980) indicates that Grossman's agents are slightly schizophrenic. The covariance between 'noise' in individual information and 'noise' in the price is non-zero because the number of agents is finite, and each agent exerts a non-negligible influence on the price. Therefore, although the equilibrium prices of the limit economy are not generally correct for economies with finitely many traders, it can somewhat solve part of the Grossman's problem. Following Lang, Litzenberger, and Madrigal (1992), and Hellwig (1980), our paper considers thelimit economy in a large market. This analysis will lead to an alternate view of the aggregation of information through the price. distribution with mean zero and variance  $\sigma_{\nu}^2$ , and  $\tilde{\eta}_i$ is an idiosyncratic noise term that has a normal distribution with mean zero and variance  $\sigma_{\eta}^2$ . The random variables  $\tilde{F}$ ,  $\tilde{\nu}$  and  $\tilde{\eta}_i$  are independent of one another. The signals' idiosyncratic errors are independent across traders, which means  $E[\tilde{\eta}_i, \tilde{\eta}_j] = 0$  for all  $i \neq j$ . Moreover, the common errors  $\tilde{w}$  and  $\tilde{\nu}$  are uncorrelated. Similarly,  $\tilde{\eta}_i$  is independent of  $\tilde{e}_i$ . We refer to this case as intertemporal independence, a situation in which, say, two accounting earnings forecasts, at two different dates, have uncorrelated errors. The random variables are assumed to be independent and multivariate normal. In sum, the flow of information in the market is as follows.



We denote by  $I_{it}$  the time *t* information set on which informed trader *i* conditions his investment decisions, and  $\tilde{P}_i$ , the time *t* market-clearing price of the risky asset. The trader *i*'s initial wealth in the first round is  $\tilde{W}_{il}$ , where  $\tilde{W}_{il} = \tilde{\xi}_i \tilde{P}_1 \cdot \tilde{W}_{i2}$  is trader *i*'s initial wealth in the second round. Also, the cost of information acquisition is  $K_i$  (*t* = 1,2).<sup>5</sup> Traders are assumed to have negative exponential utility of final consumption  $\tilde{C}_i$ , and informed trader (or speculative trader) *i* chooses a feasible trading strategy to maximise the expected utility of consumption at time 3:

$$E\left[U\left(\tilde{C}_{i}\right)\right] = E\left[-\exp\left(-r_{i}\tilde{C}_{i}\right)\right]$$

where  $r_i$  is trader *i*'s coefficient of absolute risk aversion. For all traders,  $r_i \in [r_L, r_U]$ , with  $r_L > 0$ and  $r_U < \infty$ . Following Lang, Litzenberger, and Madrigal (1992), the speculative trading volume at time *t* is defined as<sup>6</sup>

$$\tilde{V}_i = \frac{1}{2} \sum_i \left| \Delta \tilde{x}_{ii} \right|$$

where  $\tilde{x}_{it}$  is trader *i*'s holding units of the risky asset at time *t* and  $\Delta \tilde{x}_{it}$  is the change in trader *i*'s holdings at time *t*.<sup>7</sup> The unmodeled noise traders now enter to inject randomness in the supply of the

<sup>&</sup>lt;sup>2</sup> At time 3, consumption occurs, as does an accounting earnings announcement. Because our model assumes that the cash flow is equal to the accounting earnings at time 3, the time 3 payoff (liquidating value) of the risky asset,  $\tilde{F}$ , is also the accounting earnings.

<sup>&</sup>lt;sup>5</sup> We assume that there is no connection between the precision of private signals and information acquisition costs.

<sup>&</sup>lt;sup>6</sup> Leone, Nelson and Nottingham (1961) indicate that measurements are frequently recorded without their algebraic sign. The effect of dropping the sign is to add the otherwise negative values to the positive values. Geometrically, this amounts to folding the negative side of the distribution onto the positive side. When the underlying distribution (of the algebraic values) is normal, the distribution of the absolute measurement is here described as the folded normal distribution. Therefore, in our model,  $|\Delta \tilde{x}_{it}|$  is folded-normally distributed.

risky asset. That is, in addition to the informed traders, we assume that there exists a population of noise traders in the market whose aggregate supply of the risky asset in period *t* is given by the random variable  $\tilde{\Theta}_t$ . The trade of (unmodeled) noise traders is an exogenously specified random variable with a distribution that is independent of the risky asset's price and traders' information. Therefore, the units of the risky asset of informed trader *i*,  $\tilde{x}_{it}$ , is not held by the noise traders at time *t*. If *S* is the total market supply of the risky asset, the net supply of the informed traders is then given in period *t* by  $\sum_i \tilde{x}_{it} = S - \tilde{\Theta}_t$ , t = 1,2. We assume for the time being that total market supply remains constant in both periods.

Our model assumes that each informed trader receives a private signal containing a common error and an idiosyncratic error, possibly different, in each period before he trades. We describe a noisy rational expectations equilibrium where the supply of one trader is unobservable by the others and a second trading date is allowed. Informed trader *i* bases his investment decisions on both private information and market price. This implies that the information sets of trader *i* for the first and second trading dates are  $I_{i1} = \{\tilde{P}_1, \tilde{Y}_i\}$  and  $I_{i2} = \{\tilde{P}_1, \tilde{P}_2, \tilde{Y}_i, \tilde{Z}_i\},\$ respectively. Because the realisation of per capita supply of informed traders at time t,  $\tilde{X}_t$  (t = 1,2), is unobservable, the equilibrium price  $\tilde{P}_1$  is a linear function of the per capita supply  $\tilde{X}$  and the average private signal  $\overline{Y}$ .<sup>8</sup> Moreover, the time 2 price  $\widetilde{P}_2$  is a linear function of the supply  $\tilde{X}$ , the average signal  $\overline{Y}$ , the price history  $\widetilde{P}_1$ , and the time 2 average private signal  $\overline{Z}$ . That is, a linear conjecture of  $\widetilde{P}_1$  and  $\tilde{P}_2$  can be written as:

$$\tilde{P}_1 = \alpha_0 + \alpha_1 \overline{Y} + \alpha_2 \tilde{X}$$
(3)

$$\tilde{P}_2 = \beta_0 + \beta_1 \overline{Y} + \beta_2 \tilde{X} + \beta_3 \overline{Z} + \beta_4 \tilde{P}_1$$
(4)

where  $\overline{Y} \equiv \lim_{N \to \infty} (\Sigma \widetilde{Y}_i / N) = \widetilde{F} + \widetilde{w}, \overline{Z} \equiv \lim_{N \to \infty} (\Sigma \widetilde{Z}_i / N)$ =  $\widetilde{F} + \widetilde{v}$ . The limiting results of  $\widetilde{Y}_i$  and  $\widetilde{Z}_i$  imply that

<sup>10</sup> Because equation (4) can be rewritten as

$$\tilde{P}_2 = \beta_0 + \frac{\beta_2}{\alpha_2} (\tilde{P}_1 - \alpha_0) + \beta_3 \overline{Z} ,$$

the time 2 information set  $\{\tilde{P}_1, \tilde{P}_2, \tilde{Y}_i, \overline{Z}\}$  is equivalent to  $\{\tilde{P}_2, \tilde{Y}_i, \overline{Z}\}$ . Informed traders can infer  $\tilde{P}_1$  by observing  $\tilde{P}_2$  and  $\overline{Z}$ , that is, traders can ignore  $\tilde{P}_1$  in the second period.

the time t price  $\tilde{P}_t$  is not influenced by trader i's idiosyncratic error as  $N \rightarrow \infty$ . Since in the second period  $\tilde{P}_1$  and  $\tilde{P}_2$  are observable, the condition ensuring that the average private signal  $\overline{Y}$  is not revealed (is revealed) by the sequence of prices is  $\alpha_1/\alpha_2 = \beta_1/\beta_2$  ( $\alpha_1/\alpha_2 \neq \beta_1/\beta_2$ ). Moreover, the price  $\tilde{P}_2$  can reveal the average signal  $\bar{Z}$  when the two equations are linearly dependent, that is, when  $\alpha_1/\alpha_2 = \beta_1/\beta_2$ .<sup>9</sup> In this case, the information set of informed trader *i* in period 2 can be rewritten as  $I_{i2} = \{\tilde{P}_1, \tilde{P}_2, \tilde{Y}_i, \bar{Z}\}$ . Because the price  $\tilde{P}_1$  is perfectly correlated with the linear combination  $\beta_1 \overline{Y} + \beta_2 \tilde{X}$ , we can set  $\beta_4 = 0$  without loss of generality.<sup>10</sup> As a result, the period 2 information set of trader *i* can be further simplified as  $I_{i2} = \{\tilde{P}_2, \tilde{Y}_i, \bar{Z}\}$ . In sum,  $\bar{Y}$  is not revealed and  $\overline{Z}$  is revealed by the price sequence  $\{\tilde{P}_1, \tilde{P}_2\}$  if  $\alpha_1/\alpha_2 = \beta_1/\beta_2$  (see Appendix 1 for details). We examine some properties of this particular linear rational expectations equilibrium as follows.

# 2.1. The equilibrium price and demand functions at time 2

Using the backward solution technique of dynamic programming, we first solve the equilibrium price of and demand (holding units) for the risky asset in the second period. Secondly, we solve the time 1 equilibrium price and demand. The expected payoff (accounting earnings) is  $E[\tilde{P}_2|\tilde{P}_1,\tilde{Y}_i]$  and  $E[\tilde{F}|\tilde{P}_2,\tilde{Y}_i,\bar{Z}]$  in the first and second periods, respectively. Informed trader *i* chooses  $\tilde{x}_{it}$  to maximise the expected utility of his final consumption. Then the optimal trading strategy in the second period is

$$M_{\tilde{x}_{i_2}}^{AX} E\left[-e^{-r_i\tilde{C}_i}|I_{i_2}\right]$$
  
s.t.  $\tilde{C}_i = \tilde{W}_{i_2} - \tilde{x}_{i_2}(\tilde{P}_2 - \tilde{F}) - K_2$  (5)

The optimal  $\tilde{x}_{i2}$  and per capita demand of informed traders,  $\tilde{X}$ , can be expressed as (see Appendix 2 for details)<sup>11</sup>

$$\tilde{x}_{i2}^{*} = \frac{E[\tilde{F}|I_{i2}] - \tilde{P}_{2}}{r_{i}\sigma^{2}(\tilde{F}|I_{i2})} = \frac{b_{0} + b_{1}\tilde{P}_{2} + b_{2}\tilde{Y}_{i} + b_{3}\overline{Z} - \tilde{P}_{2}}{r_{i}\sigma^{2}(\tilde{F}|I_{i2})}$$
(6)

and

$$\tilde{X} = \frac{b_0 + b_1 \tilde{P}_2 + b_2 \overline{Y} + b_3 \overline{Z} - \tilde{P}_2}{\bar{r} \sigma^2 (\tilde{F} | I_{i2})}$$
(7)

<sup>&</sup>lt;sup>7</sup> Kim and Verrecchia (1991) indicate the definition of period 2 trading volume is  $(1/2) \int |\tilde{D}_{2i} - \tilde{D}_{1i}| d_i$ , where  $\tilde{D}_{ii}$  is trader *i*'s desired holding (gross demand) of the risky asset in period *t*, *t*=1,2.

<sup>&</sup>lt;sup>8</sup> As  $\tilde{X}_1 = \tilde{X}_2$ , we denote both periods' per capita supply of informed traders by  $\tilde{X}$ ; see footnote 3.

<sup>&</sup>lt;sup>9</sup> Lin, Wang and Tsai (1995) show that if the respective average private signals  $\overline{Y}$  and  $\overline{Z}$  at times 1 and 2 are not revealed by the price sequence, that is,  $\alpha_1/\alpha_2 \neq \beta_1/\beta_2$ , the market will have trading volume in both periods. Intuitively, if equilibrium price cannot fully reveal the private signal, informed traders must retrade based on the private signal.

<sup>&</sup>lt;sup>11</sup> In equilibrium, per capita supply is equal to per capita demand. Hence, we also denote by  $\tilde{X}_i$  per capital demand of the risky asset from informed traders at time t, t=1,2. We consider the limiting results when traders are countably infinite in number, and thus  $\tilde{X}_i = \lim_{N \to \infty} ((\Sigma_i \tilde{x}_{it})/N)$ , where  $\tilde{x}_{it}$  is trader *i*'s holding units of the risky asset at time *t*. If  $\tilde{X}_1 = \tilde{X}_2$ , we denote both periods' per capita demand of informed traders by  $\tilde{X}$ .

where  $\bar{r} = \frac{1}{\Sigma(N/r_i)}$ . We rewrite (6) as

$$\tilde{x}_{i2}^{*} = \frac{\bar{r}}{r_i} \tilde{X} + \frac{b_2}{r_i \sigma^2 \left(\tilde{F} \middle| \tilde{P}_2, \tilde{Y}_i, \overline{Z} \right)} \tilde{e}_i$$
(8)

Rewriting (7) makes clear the relation between the  $\beta$  in (4) and b in (7). We can then express the time 2 equilibrium price of the risky asset as

$$\tilde{P}_{2} = \frac{b_{0}}{1-b_{1}} + \frac{b_{2}}{1-b_{1}}\overline{Y} + \frac{b_{3}}{1-b_{1}}\overline{Z} - \frac{\overline{r}\sigma^{2}(\overline{F}|I_{i2})}{1-b_{1}}\widetilde{X}$$
(9)

# 2.2. The equilibrium price and demand functions at time 1

Now consider the determination of  $\tilde{x}_{i1}$ . In the first period, informed trader *i* seeks to allocate his initial wealth between the riskless and the risky assets in order to maximise the expected utility of final consumption.

$$\begin{aligned} & \underset{\tilde{x}_{i2}}{MAX} \quad E\left[-e^{-r_{i}\tilde{C}_{i}} | I_{i1}\right] \\ & s.t. \quad \tilde{C}_{i} = \tilde{W}_{i1} - \tilde{x}_{i1} \left(\tilde{P}_{1} - \tilde{P}_{2}\right) - K_{1} - x_{i2}^{*} \left(\tilde{P}_{2} - \tilde{F}\right) - K_{2} \end{aligned}$$

Since  $x_{i2}^*$  is not a random variable in period 1,<sup>12</sup> the solution of the first period maximisation problem is (see Appendix 3 for details)

$$\tilde{x}_{i1}^{*} = \frac{E[\tilde{P}_{2}|I_{i1}] - \tilde{P}_{1}}{r_{i}\sigma^{2}(\tilde{P}_{2}|I_{i1})} = \frac{a_{0} + a_{1}\tilde{P}_{1} + a_{2}\tilde{Y}_{i} - \tilde{P}_{1}}{r_{i}\sigma^{2}(\tilde{P}_{2}|I_{i1})}$$
(10)

and averaging over all *i* gives

$$\tilde{X} = \frac{a_0 + a_1 \tilde{P}_1 + a_2 \overline{Y} - \tilde{P}_1}{\bar{r} \sigma^2 (\tilde{P}_2 | I_{i_1})}$$
(11)

We can then express  $\tilde{x}_{i1}^*$  as

$$\tilde{x}_{i1}^* = \frac{\bar{r}}{r_i} \tilde{X} + \frac{a_2}{r_i \sigma^2 \left(\tilde{P}_2 | I_{i1}\right)} \tilde{e}_i$$
(12)

Rewriting (11) makes clear the relation between the  $\alpha$  in (3) and *a* in (11). Therefore, the equilibri-

<sup>12</sup> Because  $\alpha_1/\alpha_2 = \beta_1/\beta_2$  and  $\widetilde{X} = (\widetilde{P}_1 - \alpha_0 - \alpha_1 \overline{Y})/\alpha_2$ , we can then obtain

$$x_{i2}^{*} = \frac{\overline{r}}{r_{i}} \widetilde{X} - \frac{\beta_{1}}{\beta_{2}} \cdot \frac{\overline{r}}{r_{i}} \widetilde{e}_{i} = \frac{\overline{r}}{r_{i}} \left[ \left( \frac{\widetilde{P}_{1} - \alpha_{0}}{\alpha_{2}} \right) - \frac{\alpha_{1}}{\alpha_{2}} \widetilde{Y}_{i} \right]$$

Hence,  $x_{i2}^*$  is not a random variable in period 1.

<sup>13</sup> The amount of volume due to noise trading can emerge from a variety of sources. For our purposes, it is sufficient to think of idiosyncratic liquidity demands as the cause. Also, all non-speculative trades belong to noise trades. um price at time 1,  $\tilde{P}_1$ , can be written as follows.

$$\tilde{P}_{1} = \frac{a_{0}}{1-a_{1}} + \frac{a_{2}}{1-a_{1}}\overline{Y} - \frac{\bar{r}\sigma^{2}(\tilde{P}_{2}|I_{i1})}{1-a_{1}}\tilde{X}$$
(13)

Price changes around the time of earnings announcements are well documented. In our model, price changes occur prior to the release of an anticipated earnings announcement. The price changes prior to the earnings announcement are the result of endogenous acquisition of information about the forthcoming earnings announcement. The magnitude of the price changes depends on both the extent to which prior information acquisition has resulted in the forthcoming earnings announcement being impounded in  $\tilde{P}_1$  and  $\tilde{P}_2$ , and the extent to which the amount of variation in prices is attributable to 'noise'.

#### 3. Trading volume analysis

Under noisy rational expectations, the noise in prices is induced by uncertain supply, and thus traders' posterior expectations still remain divergent even after they observe prices. Trading volume arises from the heterogeneity of traders' assessments of the risky asset's future value as well as from the impact of net supply change. This implies that supply shocks and price changes have the effect of moving traders along their demand curve for the risky asset. Our paper assumes that there are two groups of traders in this economy, each of whom may trade assets at the beginning of periods 1 and 2. One group of traders, referred to as the informed traders (or speculative traders), are induced to trade from an analysis of optimal trade strategies, given their information and preferences. Noise traders are the other group. Their demands for the risky asset at the beginning of each period are unrelated to any information in the market. The supply shocks of the risky asset are assumed due to the nonspeculative trade,<sup>13</sup> that is, this component is not induced by information. Hence, if supply contains random shocks, the total trading volume may result from both the informed and the noise traders.

Section 2 shows how the prices would change when the time 2 average private signal is fully revealed but the time 1 average private signal is not. This section examines the patterns of trading volume. We first investigate the setting where pure noise trading volume is uncorrelated with the net trade between the noise and the informed traders. In the second setting, this assumption is relaxed.

# 3.1. Pure noise trading volume uncorrelated with observable variables

Denote by  $\tilde{V}_{nt}$  (t=1,2) the noise trading volume

at time t. From its definition, informed trading volume  $\tilde{V}_i$  contains the trade among informed traders and half of the trade between the informed and the noise traders; another half is contained in noise trading volume  $\tilde{V}_{nt}$ . We assume  $\tilde{V}_{nt} = \tilde{G}_t + \tilde{H}_t$ , where  $\tilde{G}_t$  is the pure noise trading and  $\tilde{H}_t$  is half of the trade between the informed and the noise traders. The two components of noise trading,  $\tilde{G}_t$  and  $\tilde{H}_t$ , have their respective empirical intuitions. The empirical intuition of the component  $\tilde{H}_t$  is that informed traders trade with noise traders based on superior information about the firm's true cash flows, and noise traders trade with informed traders due to liquidity reasons.  $\tilde{G}_t$  reveals the fact that a noise trader buys or sells stocks to another noise trader, both in need of liquidity. Informed traders' superior information comes from private acquisition or market variables such as prices, public disclosures, and analysts' forecasts. It follows that, when pure noise trading volume is uncorrelated with observable market variables, it is independent of the trade between the noise and the informed traders. Consequently, pure poise trading volume is unrelated to the net trade between the two groups of traders since the net trade contains the possibility of some volume of the trade between the informed and the noise traders being cancelled out due to different signs.

If pure noise trading volume is uncorrelated with observable market variables, then only restrictions based on the speculative component can be estimated. That is, speculative trading volume represents the 'predictable' component of volume.14 This is because observable market variables, such as prices, public disclosures, and analysts' forecasts are independent of pure noise trading volume, and we cannot infer pure noise trading volume based on this information.<sup>15</sup> Therefore, a sufficient condition for pure noise trading volume being uncorrelated with market variables is that it is uncorrelated with the net trade between the noise and the informed traders.<sup>16</sup> However, in contrast to this case, in Section 3.2 we extend the model to consider pure noise trading being correlated with the net trade. Lang, Litzenberger, and Madrigal (1992) indicate that the empirical analysis based on the definition of speculative trading volume entails no loss of generality whether or not this correlation exists. In a market with a large number of informed traders, the change in informed trader *i*'s holdings at time 2 is<sup>17</sup>

$$\begin{split} \Delta \tilde{x}_{i2} &= \tilde{x}_{i2} - \tilde{x}_{i1} \\ &= \frac{\left(\tilde{f}_{i2} - \tilde{f}_{2}\right) + \left(\tilde{f}_{2} - \tilde{P}_{2}\right)}{r_{i}\sigma^{2}\left(\tilde{F}|I_{i2}\right)} - \frac{\left(\tilde{f}_{i1} - \tilde{f}_{1}\right) + \left(\tilde{f}_{1} - \tilde{P}_{1}\right)}{r_{i}\sigma^{2}\left(\tilde{P}_{2}|I_{i1}\right)} \end{split}$$

where  $\overline{f_t}$  is the average belief of all informed traders at time t, and  $\tilde{f}_{i2} = E[\tilde{F} | I_{i2}]$  and  $\tilde{f}_{i1} = E[\tilde{P}_2 | I_{i1}]$ is informed trader i's posterior belief of the future payoff of the risky asset at times 2 and 1, respectively. Note, in our setting,  $\tilde{f}_{it}$  (t =1,2) is also informed trader i's posterior expectation regarding the future accounting earnings. Accordingly, no trade occurs in the second period when net supply of the informed traders is constant in both periods. In other words, the informed traders' holding of units of the risky asset remain fixed across both periods of trade. Note, the zero informed trading volume implies that the trade among the informed traders and the trade between the informed and the noise traders are both zero. As for noise trading, unfortunately we cannot predict its pattern. These properties are formalised in Proposition 1.

**Proposition 1:** In the setting of pure noise trading volume being uncorrelated with observable variables, if there exists a linear non- $\overline{Y}$ -revealing and  $\overline{Z}$ -revealing rational expectations equilibrium with constant net supply, the informed traders achieve a Pareto-optimal allocation in the first period, and they do not trade in the second. This result shows that the informed traders ignore the opportunity to retrade in the second period, and the time 2 average private signal is redundant.

*Proof.* If net supply of the risky asset is the same across both periods, the informed trading volume for the risky asset at time 2 is (see Appendix 4 for details)

$$\tilde{V}_{2} = \frac{1}{2} \sum_{i} \frac{1}{r_{i}} \left| \frac{b_{2} \tilde{e}_{i}}{\sigma^{2} (\tilde{F} | I_{i2})} - \frac{a_{2} \tilde{e}_{i}}{\sigma^{2} (\tilde{P}_{2} | I_{i1})} \right|$$
(14)

From (3) and (13),  $\alpha_1/\alpha_2 = -a_2/\overline{r}\sigma^2 (\tilde{P}_2 | I_{i1})$ . Hence, (12) can be rewritten as

$$\tilde{x}_{i1}^* = \frac{\tilde{r}}{r_i} \left( \tilde{X} - \frac{\alpha_1}{\alpha_2} \tilde{e}_i \right)$$
(15)

Similarly, we can express  $\tilde{x}_{i2}^*$  as

$$\tilde{x}_{i2}^* = \frac{\bar{r}}{r_i} \left( \tilde{X} - \frac{\beta_1}{\beta_2} \tilde{e}_i \right)$$
(16)

From (15), (16), and  $\alpha_1/\alpha_2 = \beta_1/\beta_2$ , we obtain

<sup>&</sup>lt;sup>14</sup> This issue implies the result in Lang, Litzenberger, and Madrigal (1992).

<sup>&</sup>lt;sup>15</sup> When pure noise trading volume cannot be inferred, noise trading volume also cannot be estimated as the former is one component of the latter.

<sup>&</sup>lt;sup>16</sup> Lang, Litzenberger, and Madrigal (1992) indicate that this hypothesis is not directly testable because noise trading volume cannot be separately observed.

<sup>&</sup>lt;sup>17</sup> The change in trader *i*'s holdings at time 1 is  $\Delta \tilde{x}_{i1} = \tilde{x}_{i1} - \tilde{\xi}_{i2}$ .

### $\tilde{x}_{D}^{*} - \tilde{x}_{D}^{*} = 0. \blacksquare$

The contribution of Proposition 1 and its main difference from the previous articles are described as follows. Grundy and McNichols (1989) assume that traders acquire private signals at time 1 and observe a public signal at time 2. They show that no trade occurs in the second period if the sequence of prices cannot fully reveal time 1's average private signal and supply remains constant in both periods. On the contrary, in our model, the informed traders receive private signals in each period. Although Brown and Jennings (1989) also examine the setting of traders acquiring respective private signals in both periods, they only discuss the price change, but not the patterns of trading volume, nor whether private signals can be fully revealed by the price sequence. We show that the time 2 average private signal is fully revealed by the time 2 price, and thus the time 2 average private signal can be viewed as a public signal. Hence, our model is more general than the previous articles.

On the other hand, if the informed traders' holdings of the risky asset change across periods with random supply shocks, then the pricing relations in the first and second periods can be written as<sup>18</sup>

$$\tilde{P}_1 = \alpha_0 + \alpha_1 \overline{Y} + \alpha_2 \tilde{X}_1 \tag{17}$$

$$\tilde{P}_2 = \beta_0 + \beta_1 \overline{Y} + \beta_2 \tilde{X}_2 + \beta_3 \overline{Z}$$
(18)

where  $\tilde{X}_1$  and  $\tilde{X}_2$  denote the unobservable per capita supply of the informed traders in the first and second periods, respectively. Moreover, per capita supply of the informed traders at time 2,  $\tilde{X}_2$ , is induced by  $\tilde{X}_2 = \tilde{X}_1 + \Delta \tilde{X}$ , where  $\Delta \tilde{X}$  is uncorrelated with all other variables. Because  $\Delta \tilde{X}$  is unobservable,  $\overline{Z}$  cannot be fully revealed by the price sequence. That is, in this case,  $\overline{Z}$  is partially revealed by the price sequence even though  $\alpha_1/\alpha_2 = \beta_1/\beta_2$ . Hence, the information set is not  $\{\tilde{P}_2, \tilde{Y}_i, \bar{Z}\}$  but  $\{\tilde{P}_2, \tilde{Y}_i, \tilde{Z}_i\}$ . In the following, we first examine the change in the holding units of informed trader i at time 2. As for noise trading, we still can not predict its pattern. To grasp the empirical implications of a theoretical model of trading volume, the trading volume must be measured empirically. It is therefore useful to derive an expression of the total squared informed trading volume in the limit in order to remove the  $\Delta \tilde{x}_{it}$  term within the absolute sign shown in trading volume model  $\tilde{V}_{i}$ . It also allows us to understand what factors would influence informed trading volume. Proposition 2 states the results.

**Proposition 2:** In the setting of pure noise trading volume being uncorrelated with observable variables, if there exists a linear non- $\overline{Y}$ -revealing and partially- $\overline{Z}$ -revealing rational expectations equilibrium with random supply shocks, a Pareto-optimal allocation is not achieved for the informed traders at time 1. They have no concordant beliefs concerning the time 2 price and will trade in the second round. Thus, the time 2 private signal is valuable. Moreover, total trading volume arises from the market price of the risky asset and from all informed traders' individual expectations as well as the average expectation of its liquidating price in both periods.

*Proof.* Informed trader *i*'s optimal demand for the risky asset at each date is

$$\tilde{x}_{i1}^{\star} = \frac{\bar{r}}{r_i} \tilde{X}_1 + \frac{a_2}{r_i \sigma^2 \left(\tilde{P}_2 | I_{i1}\right)} \tilde{e}_i$$
$$\tilde{x}_{i2}^{\star} = \frac{\bar{r}}{r_i} \tilde{X}_2 + \frac{b_2}{r_i \sigma^2 \left(\tilde{F} | I_{i2}\right)} \tilde{e}_i + \frac{b_3}{r_i \sigma^2 \left(\tilde{F} | I_{i2}\right)} \tilde{\eta}_i$$

We can express  $\tilde{x}_{i1}^*$  and  $\tilde{x}_{i2}^*$  as

$$\begin{split} \tilde{x}_{i1}^{\star} &= \frac{\bar{r}}{r_i} \bigg( \tilde{X}_1 - \frac{\alpha_1}{\alpha_2} \tilde{e}_i \bigg) \\ \tilde{x}_{i2}^{\star} &= \frac{\bar{r}}{r_i} \bigg( \tilde{X}_2 - \frac{\beta_1}{\beta_2} \tilde{e}_i - \frac{\beta_3}{\beta_2} \tilde{\eta}_i \bigg) \end{split}$$

The change in the *i*th trader's demands is

$$\tilde{x}_{i2}^* - \tilde{x}_{i1}^* = \frac{\bar{r}}{r_i} \left( \Delta \tilde{X} - \frac{\beta_3}{\beta_2} \tilde{\eta}_i \right)$$

Thus, the informed traders do not ignore the opportunity to retrade in the second period. Because the noise trading volume cannot be predicted, and only the informed trading volume is predicable, without loss of generality, the trading volume of the risky asset at time 2 is

$$\tilde{V}_{2} = \frac{1}{2} \sum_{i} \frac{1}{r_{i}} \left| \bar{r} \left( \tilde{X}_{2} - \tilde{X}_{1} \right) + q_{0} \tilde{\eta}_{i} \right|$$
(19)

where  $q_0 = b_3/\sigma^2 (\tilde{F} | I_{i2})$ . As  $N \rightarrow \infty$ , the squared trading volume can be expressed as the following function, which can be tested empirically. (See Appendix 5 for details)

$$\tilde{V}_{2}^{2} = q_{1} Var \left( \frac{\left( \tilde{f}_{i2} - \bar{f}_{2} \right)}{\sigma^{2} \left( \tilde{F} | I_{i2} \right)} - \frac{\left( \tilde{f}_{i1} - \bar{f}_{1} \right)}{\sigma^{2} \left( \tilde{F} | I_{i1} \right)} \right) + q_{2} \bar{r}^{2} \left( g_{0} + g_{1} P_{2} + g_{2} \bar{f}_{2} + g_{3} P_{1} + g_{4} \bar{f}_{1} \right)^{2}$$

<sup>&</sup>lt;sup>18</sup> Contrast to Section 3.2, because noise trading volume is uncorrelated with observed variables and noise trading cannot be estimated, this section only considers random shocks arises from the noise traders.

$$+q_{3}\bar{r}\left(g_{0}+g_{1}P_{2}+g_{2}\bar{f}_{2}+g_{3}P_{1}+g_{4}\bar{f}_{1}\right) = (20)$$

Equation (20) shows that volume arises from the market price of the risky asset and from informed traders' individual expectations as well as the average expectation of the liquidating value of the risky asset in both periods. Specifically, the variance of the change in the difference between individual and average beliefs at time 2 is correlated with trading volume.

Our findings are significant and different from the previous papers. Brown and Jennings (1989) demonstrate that change in the value of technical analysis is caused by change in the variance of the supply increments, that is, change in noise. They do not examine how information revealed by the price sequence affects trading volume. Demski and Feltham (1994) highlight the variance of price changes of the risky asset between the two trading rounds, and the expected trading volume at the public announcement date. Like Grundy and McNichols (1989), they assume that the private signals at the first date provide information about the public report at the second date. However, in fact, informed traders often acquire various private signals about public information before its disclosure. Hence, our setting is more consistent with economic intuition than these previous models.

The empirical intuition behind (20) is as follows. A trader's posterior expectation of the future accounting earnings,  $\tilde{f}_{ii}$  (t=1,2), can be proxied by individual analyst's forecasts of firms' earnings. The average of all analysts' earnings forecasts can be used as a proxy for the average of traders' posterior expectations of the future accounting earnings,  $\tilde{f}_i$  (t=1,2). According to (20), total trading volume at time 2 can be measured by analysts' individual and average earnings forecasts as well as the asset's prices in both periods.<sup>19</sup> Hence, (20) can reveal the contribution of our model with respect to empirical implications.

# 3.2. Pure noise trading volume correlated with observable variables

Section 3.1. examines the setting in which pure noise trading volume is uncorrelated with the net trade between the noise and the informed traders.

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In the case of supply shocks, we can only estimate the informed trading volume, which includes two components. One is the trade between the noise and the informed traders; the other, the trade among the informed traders. As for noise trading volume, it cannot be predicted. In this section, we turn to consider the situation where pure noise trading volume is correlated with observable variables. In the case of no supply shocks, the result would be the same as that in Section 3.1, where each informed trader has no incentive to trade at time 2. Hence, we do not discuss its characteristics again and focus in this section on the equilibrium with random supply shocks. We now show how to derive analogues of Proposition 2 when pure noise trading volume is correlated with the net trade between the noise and the speculative traders. This characterisation has the following empirical implications. Although, in most cases, noise traders trade due to liquidity needs, it is possible that their trading behaviour is influenced by that of the informed traders, particularly when the informed trader's buy or sell orders are large enough to send precise and strong signals. As mentioned in Section 3.1, informed traders trade based on their analysis of market variables as well as private acquisition. Consequently, when noise traders' trading decisions are influenced by informed traders, pure noise trading is correlated with observable market variables and its volume is therefore predictable. Since speculative trading volume does not constitute the entire predictable component of volume, we need to induce a model of total trading volume for the informed and the noise traders.

Because there are two types of traders in this setting, the informed and the noise traders, the total market supply of the risky asset, S, is the sum of the net supply of all informed traders and the aggregate supply of noise traders. Recall that the net supply of informed traders is

$$\sum_{i} \tilde{x}_{ii} = S - \tilde{\Theta}_{i}$$

where  $\tilde{\Theta}_i$  denotes the aggregate supply of the noise traders, t=1,2.<sup>20</sup> In the case of no supply noise, the holding units of the informed trader *i* and the aggregate holdings of noise traders will remain unchanged in both periods. This implies that  $\tilde{X}_1 = \tilde{X}_2$ and  $\tilde{\Theta}_1 = \tilde{\Theta}_2$ , and thus we denote both periods' per capita supply of informed traders by  $\tilde{X}$  and the aggregate supply of noise traders by  $\tilde{\Theta}$ . However, if total supply remains constant in both periods and net supply contains random shock,<sup>21</sup> then

$$\sum_{i} (\tilde{x}_{i2} - \tilde{x}_{i1}) = -(\tilde{\Theta}_{2} - \tilde{\Theta}_{1})$$
(21)

Equation (21) can be expressed as

$$\sum_{i} \Delta \tilde{x}_{i2} = -\Delta \tilde{\Theta}_2 \tag{22}$$

<sup>&</sup>lt;sup>19</sup> For this purpose, researchers can use Institutional Brokers Estimate System (IBES) provided by the Lynch, Jones and Ryan Company in the US. This database reports detailed survey data on analysts' forecasts of firms' annual earnings.

<sup>&</sup>lt;sup>20</sup> In equilibrium, the net supply of all informed traders is equal to the aggregate demand of informed traders. Hence, we also use  $\Sigma_t \tilde{x}_u$  as the aggregate demand of informed traders. Similarly,  $\tilde{\Theta}_t$  can represent the aggregate demand of noise traders.

<sup>&</sup>lt;sup>21</sup> Because  $\Sigma_i \tilde{x}_{it} = S - \tilde{\Theta}_i$ ,  $\Sigma_i (\tilde{x}_{i2} - \tilde{x}_{i1}) = (S_2 - S_1) - (\tilde{\Theta}_2 - \tilde{\Theta}_1)$ . Also, total supply remains constant in both periods (i.e.,  $S_2 = S_1$ ) so that  $\Sigma_i (\tilde{x}_{i2} - \tilde{x}_{i1}) = -(\tilde{\Theta}_2 - \tilde{\Theta}_1)$ .

This implies that the sum of the change in the holding units of the informed traders at time 2,  $\sum_i \Delta \tilde{x}_{i2}$ , is equal to the change in the aggregate holding units of the noise traders at time 2,  $\Delta \tilde{\Theta}_2$ , but with different signs. From (22),  $\sum_i \Delta \tilde{x}_{i2}$  represents the net trade, so does  $\Delta \tilde{\Theta}_2$ . Note,  $\sum_i \Delta \tilde{x}_{i2}$  contains the possibility of the trade between the informed and the noise traders being cancelled out due to different signs.

Recall that the definition of informed trading volume at time 2 is

$$\tilde{V}_2 = \frac{1}{2} \sum_{i} \left| \Delta \tilde{x}_{i2} \right| > 0$$

and it consists of the trade among the informed traders as well as half of the trade between the informed and the noise traders. In this section, we examine the case where pure noise trading volume is correlated with observable market variables. Being a primitive quantity, noise trading volume at time 2,  $\tilde{V}_{n2}$ , represents the trade among the noise traders and half of the trade between the informed and the noise traders. Because the trades of unmodelled noise traders are exogenously specified, our paper incorporates this variable to inject randomness in the supply of the risky asset. The reason that we add such exogenous variables  $\tilde{G}_{i}$  and  $\tilde{H}_{i}$ in the model is to demonstrate the possibility of estimating trading volume between different parties empirically. Our analysis shows that in the case of noisy supply the total trading volume of the risky asset at time 2,  $\tilde{V}_{total2}$ , arises from the noise and the informed traders, that is,  $\tilde{V}_{total2} = \tilde{V}_2 + \tilde{V}_{n2}$ .<sup>22</sup> The following proposition summarises the results.

**Proposition 3:** In the setting of pure noise trading volume being correlated with observable variables, if there exists a linear non- $\overline{Y}$ -revealing and partially- $\overline{Z}$ -revealing rational expectations equilibrium with random supply shocks, the informed and the noise traders will retrade at time 2. Total trading volume arises from the informed and the noise traders and these volumes are predictable. In addition, total trading volume is increasing with the relationship between noise trading and the absolute value of the net trade.

*Proof.* Our analysis suppose that pure noise trading volume  $\tilde{G}_i$  is correlated with the net trade between the informed and the noise traders,  $\Delta \tilde{\Theta}_i$ , that is,  $Cov(\tilde{G}_i, \Delta \tilde{\Theta}_i) \neq 0$ . From (22), it is reasonable to assume that pure noise trading volume at time 2,

 $\tilde{G}_{i}$ , is positively related to  $|\sum_{i}\Delta \tilde{x}_{i2}|$ . Consequently, the pure noise trading volume at time 2 can be written as

$$\tilde{G}_2 = k \left| \sum_i \Delta \tilde{x}_{i2} \right| \tag{23}$$

where k is constant, representing the relationship between pure noise trading volume and the absolute value of the net trade between the informed and the noise traders. Similarly, half of the trade between the informed and the noise traders,  $\tilde{H}_2$ , can be expressed as

$$\tilde{H}_2 = l \left| \sum_i \Delta \tilde{x}_{i2} \right| \tag{24}$$

where *l* represents the relationship between  $\tilde{H}_2$  and the absolute value of the net trade. Because  $\tilde{V}_{n_2} = \tilde{G}_2 + \tilde{H}_2$ ,

$$\widetilde{V}_{n2} = k \left| \sum_{i} \Delta \widetilde{x}_{i2} \right| + l \left| \sum_{i} \Delta \widetilde{x}_{i2} \right|$$
$$\equiv h \left| \sum_{i} \Delta \widetilde{x}_{i2} \right|$$
(25)

where *h* represents the relationship between noise trading volume and the absolute value of the net trade. Because  $\tilde{V}_{total2}^2 = (\tilde{V}_2 + \tilde{V}_{n2})$ ,<sup>2</sup> we also obtain that factors influencing total trading volume include the market price of the risky asset and the informed traders' individual expectations as well as the average expectation of its liquidating price in both periods. In addition, total trading volume is increasing with the relationship between  $\tilde{V}_{n2}$  and  $|\sum_i \Delta \tilde{x}_{i2}|$  (see Appendix 6 for details).

Proposition 3 shows that in the setting of noise trading volume being correlated with the net trade, total trading volume results from the trade among the informed traders, the trade among the noise traders, and the trade between the informed and noise traders. Each of the three components can be predicted. Our results differ from the recent related papers, for example, Kim and Verrecchia (1994), McNichols and Trueman (1994), Demski and Feltham (1994), Kim and Verrecchia (1991), Grundy and McNichols (1989), and Brown and Jennings (1989). The first two papers assume informed traders are risk neutral and the latter four, like our model, that they are risk averse. But these papers do not investigate, in the framework of individual trader observing private signals at each trading date, whether the price sequence can fully reveal the average private signal in both periods and cause informed traders to revise their posterior beliefs, and thus induce trade. In addition to the derivation of the trading volume model, we examine how this model can be applied to empirical analysis. Note, including the noise trading volume in the model in Section 3, our paper attempts to formulate a trading volume model as a basis for

<sup>&</sup>lt;sup>22</sup> From the definition of  $\tilde{V}_{i}$ , it seems that noise trading volume at time t,  $\tilde{V}_{ni}$ , can also be written as  $\tilde{V}_{ni} = (1/2)\Sigma_n |\Delta \tilde{x}_{nt}| = \tilde{G}_i + \tilde{H}_i$ , where  $\Delta \tilde{x}_{nt}$  is the change in the holding units of noise trader n at time t. However, noise trader n's holding units at times 1 and 2,  $\tilde{x}_{n1}$  and  $\tilde{x}_{n2}$ , are not modeled in this paper, and hence are exogenous. Assuming there is some relationship between  $\tilde{V}_{ni}$  and the net trade, we can use this relationship to estimate  $\tilde{V}_{ni}$ , and hence get an estimate of  $\tilde{V}_{total t}$ .

further empirical research.

In this case, total trading volume is influenced by the same factors as (20). But different from (20), its magnitude is increasing with h. That is, the greater the relationship between noise trading and the absolute value of the net trade, the more the total trading volume. Therefore, concerning factors influencing trading volume, our results are similar to Lin, Wang and Tsai (1995). Nevertheless, we incorporate noise traders in our model while Lin, Wang and Tsai (1995) only consider informed traders.

### 4. Conclusions

We have developed a two-period, three-date noisy rational expectations model in which we assume traders acquire prior to trade a private signal about the time 3 accounting earnings at both time 1 and time 2. At time 3 consumption occurs, as does the public announcement of the accounting earnings, which is also the cash flow. The two private signals about two accounting earnings forecasts, at two different dates, are useful for forming expectations about the firm's cash flows, but there is no correlation in their noise terms. The acquisition of private signals is very common in the real world. Dealers' trading with their security analysts' forecasts and employees' trading with their superior information of the company's performance are typical examples. The traders either trade to a Pareto optimum at time 1 or they have no concordant beliefs concerning future prices, depending on which equilibrium characterises the sequence of dates.

We showed the following results. First, there exists a rational expectations equilibrium price of and demand for the risky asset when the time 1 average private signal is not revealed by price sequence but the time 2 average private signal is. Secondly, we investigate the setting where pure noise trading volume is uncorrelated with observable market variables. When the risky asset has constant net supply, the informed traders would ignore the opportunity to retrade and a Paretooptimal allocation is achieved at time 1, and the time 2 private signal is redundant. In contrast, when supply is with random shocks, the informed traders would trade in the second round, and the time 2 private signal is valuable. Unfortunately, in this case, noise trading volume is not predictable. Finally, under the setting of pure noise trading volume being correlated with observable market variables, when supply shocks across periods of trade are random, the supply shocks would cause the noise and the informed traders to trade in the second period. Since speculative trading volume does not constitute the entire predictable component of volume, we induce a model of total trading volume for the informed and the noise traders. On the other hand, without supply shocks, the holdings of the risky asset by the informed traders at times 1 and 2 remain unchanged, and no trade occurs at date 2.

In sum, trading volume and price reactions to information, consistent with the empirical implications, depend on the information environment as a whole. In the case of random supply shocks and pure noise trading being uncorrelated with the net trade, our model shows that total trading volume arises from the market price of the risky asset and informed traders' individual expectations as well as the average expectation of the liquidating value of the risky asset in both periods. However, if pure noise trading is correlated with observable variables, total trading volume is also influenced by the relationship between the absolute value of the net trade and the noise trading, and the greater the relationship, the more the total trading volume. Our models have implications for researchers interested in examining the price and trading volume responses to private signals since they indicate many factors related to trading volume reactions. Thus, the model may provide a useful structure for those interested in empirically examining the determinants of trading volume reactions to information releases.

Further studies can examine whether there exists positive autocorrelation between the trading volume at times 1 and 2 and how acquisition cost is affected by the precision of information. Wang (1994) examines how trading volume is related to the absolute change in prices and dividends and how these relations are affected by information asymmetry. Demski and Feltham (1994) show that the three components of trading volume are the change in the holdings of the unmodeled noise traders, the informed traders and the uninformed traders, when the trade of informed and uninformed traders are derived from an analysis of optimal strategies given their preferences and information. These also provide a direction for further discussions of our model developed above.

## Appendix

**Appendix 1:** *Equilibrium characterisation of the two-period rational expectations model* The ratios  $\alpha_1/\alpha_2$  and  $\beta_1/\beta_2$  can be solved as follows.

$$\frac{\alpha_{1}}{\alpha_{2}} = \frac{-\sigma_{x}^{2}}{\bar{r} \left(\sigma_{w}^{2} \sigma_{x}^{2} + \frac{\alpha_{1}^{2}}{\alpha_{2}^{2}} \sigma_{w}^{2} \sigma_{e}^{2} + \sigma_{x}^{2} \sigma_{e}^{2}\right)}$$
Also,  $\frac{\beta_{1}}{\beta_{2}} = \frac{\theta}{\psi}$ , where
$$\theta = \beta_{1} \beta_{3} \sigma_{F}^{4} \sigma_{\eta}^{2} - \beta_{1} \beta_{3} \sigma_{w}^{2} \sigma_{F}^{2} \sigma_{\eta}^{2} + \beta_{3}^{2} \sigma_{F}^{4} \sigma_{\eta}^{2} + \beta_{3}^{2} \sigma_{\eta}^{2} \sigma_{v}^{2} \sigma_{F}^{2} + \beta_{2}^{2} \sigma_{F}^{2} \sigma_{\eta}^{2} \sigma_{x}^{2}$$

$$+ \beta_{2}^{2} \sigma_{v}^{2} \sigma_{x}^{2} \sigma_{F}^{2} + 3\beta_{3} \beta_{1} \sigma_{F}^{6} + 3\beta_{3} \beta_{1} \sigma_{F}^{4} \sigma_{v}^{2} - \beta_{1}^{2} \sigma_{F}^{4} \sigma_{w}^{2} - \beta_{1}^{2} \sigma_{F}^{6}$$

$$\psi = \beta_{2}^{2} \sigma_{x}^{2} \sigma_{v}^{2} \sigma_{v}^{2} \sigma_{F}^{2} + 4\beta_{1} \beta_{3} \sigma_{F}^{4} \sigma_{v}^{2} \sigma_{v}^{2} - 2\beta_{1} \beta_{3} \sigma_{F}^{6} \sigma_{v}^{2} + \beta_{1}^{2} \sigma_{w}^{2} \sigma_{e}^{2} \sigma_{v}^{2} \sigma_{F}^{2} + \beta_{2}^{2} \sigma_{x}^{2} \sigma_{v}^{2} \sigma_{F}^{2}$$

$$+ 2\beta_{1} \beta_{3} \sigma_{F}^{4} \sigma_{v}^{2} \sigma_{e}^{2} + \beta_{2}^{2} \sigma_{x}^{2} \sigma_{w}^{2} \sigma_{F}^{2} - \beta_{3}^{2} \sigma_{F}^{6} \sigma_{\eta}^{2} + \beta_{3}^{2} \sigma_{v}^{2} \sigma_{w}^{2} \sigma_{F}^{2} + 2\beta_{1} \beta_{3} \sigma_{F}^{4} \sigma_{w}^{2} \sigma_{\eta}^{2}$$

$$+ \beta_{1}^{2} \sigma_{w}^{2} \sigma_{\eta}^{2} \sigma_{e}^{2} - \beta_{1}^{2} \sigma_{w}^{2} \sigma_{F}^{6} - 2\beta_{3}^{2} \sigma_{F}^{8} - 3\beta_{3}^{2} \sigma_{v}^{2} \sigma_{F}^{6} - \beta_{1}^{2} \sigma_{F}^{6} \sigma_{e}^{2} + \beta_{2}^{2} \sigma_{x}^{2} \sigma_{\eta}^{2} \sigma_{F}^{2}$$

$$+ \beta_{3}^{2} \sigma_{v}^{2} \sigma_{e}^{2} \sigma_{\eta}^{2} \sigma_{F}^{2}$$

Under the specified conditions, we have at least one real root for both ratios  $\alpha_1/\alpha_2$  and  $\beta_1/\beta_2$ . Therefore, we can obtain the conditions of  $\alpha_1/\alpha_2 = \beta_1/\beta_2$ . In this case, the pricing relations (3) and (4) are linearly dependent and the sequence of prices does not reveal  $\overline{Y}$ , but they can reveal  $\overline{Z}$ .

Appendix 2: Proof of optimal demand at time 2

$$\begin{split} & \max_{\bar{x}_{l_{2}}} \mathbf{X} E \bigg[ -e^{-r_{l} \big[ (\bar{W}_{l_{2}} - \bar{x}_{l_{2}}\bar{P}_{2} - K_{2}) + \bar{x}_{l_{2}}\bar{F} \big]} |I_{l_{2}} \bigg] \\ &= \max_{\bar{x}_{l_{2}}} \mathbf{X} \Big( -e^{-r_{l}\bar{W}_{l_{2}}} \Big) \Big( e^{r_{l}\bar{x}_{l_{2}}\bar{P}_{2}} \Big) \Big( e^{r_{l}K_{2}} \Big) \bigg( e^{-r_{l}\bar{x}_{l_{2}}E(\bar{F} |I_{l_{2}}) + \frac{1}{2}r_{l}^{2}\bar{x}_{l_{2}}^{2}\sigma(\bar{F} |I_{l_{2}})} \bigg) \end{split}$$

Differentiating the exponent with respect to  $\tilde{x}_{i2}$  and setting it equal to zero yields the optimal demand of Equation (6).

Appendix 3: Proof of optimal demand at time 1

$$\begin{split} \mathbf{M}_{\bar{x}_{i_{1}}}^{\mathbf{A}} X E \Bigg[ -e^{-r_{i} \left[ \left( \tilde{W}_{i_{1}} - \tilde{x}_{i_{1}} \tilde{P}_{1} - K_{1} \right) + x_{i_{1}} \tilde{P}_{2} \right]} |I_{i_{1}} \Bigg] e^{-\frac{1}{2} r_{i}^{2} x_{i_{2}}^{2} \sigma^{2} \left( \tilde{F} | I_{i_{2}} \right)} \\ & \equiv \mathbf{M}_{\bar{x}_{i_{1}}}^{\mathbf{A}} X E \Bigg[ -e^{-r_{i} \left( \tilde{W}_{i_{1}} - \tilde{x}_{i_{1}} \tilde{P}_{1} - K_{1} + \tilde{x}_{i_{1}} \tilde{P}_{2} \right)} |I_{i_{1}} \Bigg] \\ & = \mathbf{M}_{\bar{x}_{i_{1}}}^{\mathbf{A}} X \left( -e^{-r_{i} \tilde{W}_{i_{1}}} \right) \cdot \left( e^{r_{i} \tilde{x}_{i_{1}} \tilde{P}_{1}} \right) \cdot \left( e^{-r_{i} \tilde{x}_{i_{1}} E \left[ \tilde{P}_{2} |I_{i_{1}} \right] + \frac{1}{2} r_{i}^{2} \tilde{x}_{i_{1}}^{2} \sigma^{2} \left( \tilde{P}_{2} |I_{i_{1}} \right)} \right] \end{split}$$

Differentiating the exponent with respect to  $\tilde{x}_{i1}$  and setting it equal to zero yields the optimal demand of Equation (10).

# Appendix 4: Proof of Proposition 1

By the projection theorem, informed trader i's posterior belief is given by

$$f_{i2} = E\left[\tilde{F} \middle| \tilde{P}_2, \tilde{Y}_i, \overline{Z} \right] = b_0 + b_1 \tilde{P}_2 + b_2 \tilde{Y}_i + b_3 \overline{Z}$$

As  $N \rightarrow \infty$ , we have

$$\bar{f}_2 = \frac{\sum_i \left( b_0 + b_1 \tilde{P}_2 + b_2 \tilde{Y}_i + b_3 \overline{Z} \right)}{N} \rightarrow b_0 + b_1 \tilde{P}_2 + b_2 \overline{Y} + b_3 \overline{Z}$$

Thus in a market with a large number of traders,  $\tilde{f}_{i2} - \bar{f}_2 = b_2 \tilde{e}_i$ . We can then write  $p \lim(\bar{f}_2 - \tilde{P}_2)$  as  $\bar{r}\tilde{X}\sigma^2(\tilde{F}|I_{i2})$ . Similarly, as  $N \to \infty$ , we can obtain  $\tilde{f}_{i1} - \bar{f}_1 = a_2 \tilde{e}_i$ , and

$$p \lim \left(\bar{f} - \tilde{P}_{1}\right) = a_{0} + a_{1}\tilde{P}_{1} + a_{2}\bar{Y} - \tilde{P}_{1}$$
(A1)

According to (11), we can rewrite (A1) as  $\overline{r} X \sigma^2 (\tilde{P}_2 | I_{i1})$ .

#### **Appendix 5:** *Proof of Proposition 2* The squared informed trading volume is

$$\tilde{V}_{2}^{2} = \frac{1}{4} \sum_{i} \frac{1}{r_{i}^{2}} \Big[ q_{0} \tilde{\eta}_{i} + \tilde{r} \Big( \tilde{X}_{2} - \tilde{X}_{1} \Big) \Big]^{2} + \frac{1}{4} \sum_{i \neq j} \frac{1}{r_{i} r_{j}} \Big| q_{0} \tilde{\eta}_{i} + \tilde{r} \Big( \tilde{X}_{2} - \tilde{X}_{1} \Big) \Big| q_{0} \tilde{\eta}_{j} + \tilde{r} \Big( \tilde{X}_{2} - \tilde{X}_{1} \Big) \Big|$$
(A2)

where  $\tilde{X}_2 \sim N(\mu_x, \sigma_{x_2}^2)$ ,  $\tilde{X}_1 \sim N(\mu_x, \sigma_{x_1}^2)$ . If traders' risk aversion coefficients are ex ante independent of  $\tilde{e}_i$ ,  $\tilde{\eta}_i$ , and  $\tilde{F}$ , then, taking probability limit (*p*lim), the second term of (A2) without the constant 1/4 can be written as

$$p \lim \sum_{i \neq j} \frac{1}{r_i r_j} \left| q_0 \tilde{\eta}_i + \bar{r} \left( \tilde{X}_2 - \tilde{X}_1 \right) \left\| q_0 \tilde{\eta}_j + \bar{r} \left( \tilde{X}_2 - \tilde{X}_1 \right) \right\|$$
(A3)

Since  $r_i$ ,  $\tilde{X}_i$ , and  $\tilde{\eta}_i$  are independent, then the expectation of the product of  $\frac{1}{r_i r_j}$  and  $\left| q_0 \tilde{\eta}_i + \bar{r} \left( \tilde{X}_2 - \tilde{X}_1 \right) \right|$ 

 $\left|q_0\tilde{\eta}_j + \bar{r}(\tilde{X}_2 - \tilde{X}_1)\right|$  is equal to the product of the expectation of  $\frac{1}{r_ir_i}$  and the expectation of  $\left|q_0\tilde{\eta}_i + \bar{r}(\tilde{X}_2 - \tilde{X}_1)\right|$ 

$$\left| q_0 \tilde{\eta}_j + \bar{r} \left( \tilde{X}_2 - \tilde{X}_1 \right) \right|. \text{ Also, } \sum_{i \neq j} E \left[ \frac{1}{r_i r_j} \left| q_0 \tilde{\eta}_i + \bar{r} \left( \tilde{X}_2 - \tilde{X}_1 \right) \right| \left| q_0 \tilde{\eta}_j + \bar{r} \left( \tilde{X}_2 - \tilde{X}_1 \right) \right| \right] \text{ has } N(N-1) \text{ identical terms, but}$$

$$\sum_{i \neq j} E \left[ \frac{1}{r_i r_j} \right] \cdot \sum_{i \neq j} E \left[ \left| q_0 \tilde{\eta}_i + \bar{r} \left( \tilde{X}_2 - \tilde{X}_1 \right) \right| \left| q_0 \tilde{\eta}_j + \bar{r} \left( \tilde{X}_2 - \tilde{X}_1 \right) \right| \right] \text{ has } N^2 (N-1)^2 \text{ identical terms. Therefore, the expectation}$$

should be written as

$$E\left[\sum_{i\neq j}\frac{1}{r_ir_j}\Big|q_0\tilde{\eta}_i+\bar{r}\left(\tilde{X}_2-\tilde{X}_1\right)\Big|\Big|q_0\tilde{\eta}_j+\bar{r}\left(\tilde{X}_2-\tilde{X}_1\right)\Big|\right]$$
$$=\frac{1}{N(N-1)}E\left[\sum_{i\neq j}\frac{1}{r_ir_j}\right]\cdot E\left[\sum_{i\neq j}\Big|q_0\tilde{\eta}_i+\bar{r}\left(\tilde{X}_2-\tilde{X}_1\right)\Big|\Big|q_0\tilde{\eta}_j+\bar{r}\left(\tilde{X}_2-\tilde{X}_1\right)\Big|\right]$$

Because the expectation's relationship holds, the plim's relationship must hold too. Accordingly, (A3) can be written as

$$p \lim \sum_{i \neq j} \frac{1}{r_i r_j} \sum_{i \neq j} \frac{\left| q_0 \tilde{\eta}_i + \bar{r} \left( \tilde{X}_2 - \tilde{X}_1 \right) \right\| q_0 \tilde{\eta}_j + \bar{r} \left( \tilde{X}_2 - \tilde{X}_1 \right) \right|}{N(N-1)}$$
(A4)

Moreover, for large samples  $(N \rightarrow \infty)$ , given the realisation  $\tilde{X}_2 - \tilde{X}_1$ , we would expect

$$\sum_{i\neq j} \left| q_0 \tilde{\eta}_i + \bar{r} \left( \tilde{X}_2 - \tilde{X}_1 \right) \right\| q_0 \tilde{\eta}_j + \bar{r} \left( \tilde{X}_2 - \tilde{X}_1 \right) \right|$$

#### Appendix 5 (continued)

to behave approximately as the sum of independent random variables so that

$$p \lim_{i \neq j} \sum_{i \neq j} \frac{|q_0 \tilde{\eta}_i + \bar{r}(X_2 - X_1)| |q_0 \tilde{\eta}_j + \bar{r}(X_2 - X_1)|}{N(N-1)}$$
  
=  $p \lim_i \sum_i \frac{|q_0 \tilde{\eta}_i + \bar{r}(X_2 - X_1)|}{N} p \lim_j \sum_j \frac{|q_0 \tilde{\eta}_j + \bar{r}(X_2 - X_1)|}{(N-1)}$  (from Slutsky's theorem)  
=  $E(|B_1| |\Delta X) E(|B_2| |\Delta X)$ 

where, given the realisation of  $\Delta X = X_2 - X_1$ ,  $B_i$  is the normally distributed with mean  $\overline{r}\Delta X$  and variance  $q_0^2 \sigma_{\eta}^2$ . Note that, according to Slutsky's theorem, plimAB = plimA plimB (See Judge et al. 1988, and Durrett 1991). From Leone, Nelson, and Nottingham (1961), if Y is the normally distributed,  $Y \sim N(\mu, \sigma^2)$ , then |Y| has a folded normal distribution and its mean is

$$E|Y| = \sqrt{2/\pi}\sigma e^{-\mu^2/2\sigma^2} + \mu [1 - 2F(-\mu/\sigma)]$$

Similarly, because  $B_i | \Delta X$  is the normally distributed, then  $E(|B_i| | \Delta X)$  can be written as

$$E\left(|B_i||\Delta X\right) = \sqrt{\frac{2}{\pi}q_0^2\sigma_\eta^2} \exp\left[-\frac{1}{2}\left(\frac{\bar{r}\Delta X}{2q_0\sigma_\eta}\right)^2\right] + \bar{r}\Delta X\left[1 - \Phi\left(-\frac{\bar{r}\Delta X}{\sqrt{2}q_0\sigma_\eta}\right)\right]$$

where  $\Phi$  is the cumulative density of the normal distribution. Hence, for large  $\sigma_{\eta}^2$ , that is, if investors' period 2 private signal contains large variance of idiosyncratic error,  $E(|B_i||\Delta X)$  can be rewritten as

$$E\left(\left|B_{i}\right|\left|\Delta X\right)\approx\tau_{1}\sqrt{\frac{2}{\pi}}q_{0}\sigma_{\eta}+\tau_{2}\bar{r}\Delta X$$

Hence, as  $N \rightarrow \infty$ , from Khinchine's theorem (See Judge et al. 1988, and Gnedenko 1997),  $p \lim(\Sigma_i \tilde{\eta}_i^2 / N) = \sigma_{\eta_i}^2$ ,  $p \lim(\Sigma_i \tilde{\eta}_i / N) = 0$ , and given the realisation  $\tilde{X}_2 - \tilde{X}_1 (=\Delta X)$ , we obtain

$$p \lim \bar{V}_{2}^{2}$$

$$= \frac{1}{4} \sum_{i} \frac{1}{r_{i}^{2}} \Big[ q_{0}^{2} \sigma_{\eta}^{2} + \bar{r}^{2} (X_{2} - X_{1})^{2} \Big] + \frac{1}{4} \sum_{i \neq j} \frac{1}{r_{i} r_{j}} \bigg( \tau_{1} \sqrt{\frac{2}{\pi}} q_{0} \sigma_{\eta} + \tau_{2} \bar{r} (X_{2} - X_{1}) \bigg)^{2}$$

$$\equiv \bigg( \frac{J_{1}}{4} + \frac{J_{2} \tau_{1}^{2}}{2\pi} \bigg) q_{0}^{2} \sigma_{\eta}^{2} + (\frac{J_{1} \bar{r}^{2}}{4} + \frac{J_{2} \tau_{2}^{2} \bar{r}^{2}}{4}) (X_{2} - X_{1})^{2} + \frac{J_{2}}{\sqrt{2\pi}} \tau_{1} \tau_{2} \bar{r} q_{0} \sigma_{\eta} (X_{2} - X_{1})$$

$$\equiv A_{1} + A_{2} + A_{3}$$

where

$$\begin{split} A_{1} &\equiv q_{1}q_{0}^{2}\sigma_{\eta}^{2} = q_{1}Var \Biggl( \frac{\left(\tilde{f}_{i2} - \bar{f}_{2}\right)}{\sigma^{2}\left(\tilde{F}|I_{i2}\right)} - \frac{\left(\tilde{f}_{i1} - \bar{f}_{1}\right)}{\sigma^{2}\left(\tilde{P}|I_{i1}\right)} \Biggr) \\ A_{2} &\equiv q_{2}\bar{r}^{2} (X_{2} - X_{1})^{2} \\ &= q_{2}\bar{r}^{2} \Biggl[ \Biggl( \frac{\alpha_{0}}{\alpha_{2}} - \frac{\beta_{0}}{\beta_{2}} - \frac{\alpha_{1}a_{0}}{\alpha_{2}a_{2}} + \frac{\beta_{1}a_{0}}{\beta_{2}a_{2}} + \frac{\beta_{3}b_{0}}{\beta_{2}b_{3}} - \frac{\beta_{3}b_{2}a_{0}}{\beta_{2}b_{3}a_{2}} \Biggr) + \Biggl( \frac{1}{\beta_{2}} + \frac{\beta_{3}b_{1}}{\beta_{2}b_{3}} \Biggr) P_{2} + \Biggl( - \frac{\beta_{3}}{\beta_{2}b_{3}} \Biggr) \bar{f}_{2} \\ &+ \Biggl( - \frac{1}{\alpha_{2}} - \frac{\alpha_{1}a_{1}}{\alpha_{2}a_{2}} + \frac{\beta_{1}a_{1}}{\beta_{2}a_{2}} - \frac{\beta_{3}b_{2}a_{1}}{\beta_{3}b_{3}a_{2}} \Biggr) P_{1} + \Biggl( \frac{\alpha_{1}}{\alpha_{2}a_{2}} - \frac{\beta_{1}}{\beta_{2}a_{2}} + \frac{\beta_{3}b_{2}}{\beta_{2}b_{3}a_{2}} \Biggr) \bar{f}_{1} \Biggr]^{2} \end{split}$$

(A5)

**Appendix 5** (continued)

$$= q_2 \bar{r}^2 (g_0 + g_1 P_2 + g_2 \bar{f}_2 + g_3 P_1 + g_4 \bar{f}_1)^2$$
$$A_3 \equiv q_3 \bar{r} (X_2 - X_1)$$
$$= q_3 \bar{r} (g_0 + g_1 P_2 + g_2 \bar{f}_2 + g_3 P_1 + g_4 \bar{f}_1)$$

(A5) can be written as Equation (20).

**Appendix 6:** *Proof of Proposition 3* From (25)

$$\begin{split} \tilde{V}_{n2}^{2} &= h^{2} N^{2} \left( \tilde{X}_{2} - \tilde{X}_{1} \right)^{2} \\ & 2 \tilde{V}_{2} \tilde{V}_{n2} = 2 \left\{ \frac{1}{2} \sum_{i} \frac{1}{r_{i}} \left| \vec{r} \left( \tilde{X}_{2} - \tilde{X}_{1} \right) + q_{0} \tilde{\eta}_{i} \right| \right\} \left\{ h N \left| \tilde{X}_{2} - \tilde{X}_{1} \right| \\ &= h N \sum_{i} \frac{1}{r_{i}} \sum_{i} \left| \vec{r} \left( \tilde{X}_{2} - \tilde{X}_{1} \right) + q_{0} \tilde{\eta}_{i} \right\| \tilde{X}_{2} - \tilde{X}_{1} \right| \end{split}$$

As  $N \rightarrow \infty$ ,  $2\tilde{V}_2 \tilde{V}_{n2}$  can be rewritten as

$$hN\sum \frac{1}{r_i} \left(\tau_1 \sqrt{\frac{2}{\pi}} q_0 \sigma_\eta + \tau_2 \overline{r} \Delta X\right) \left| \tilde{X}_2 - \tilde{X}_1 \right|$$

and its value is positive. Because  $\tilde{V}_{total2}^2 = (\tilde{V}_2 + \tilde{V}_{n2})^2$ , we also obtain that factors influencing total trading volume include the market price of the risky asset and the informed traders' individual expectations as well as the average expectation of its liquidating price in both periods. In addition, total trading volume is increasing with h.

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