行政院國家科學委員會補助專題研究計畫成果報告

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計畫類別：☑個別型計畫　　□整合型計畫
計畫編號：NSC 89－2415－H－002－006－
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計畫主持人：陳思寬副教授
共同主持人：陳昭南研究員

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行政院國家科學委員會專題研究計畫成果報告

聯繫匯率制度下貿易條件的內生波動：香港的實證

Endogenous Fluctuations of the Terms of Trade under Linked Exchange Rates: Evidence from Hong Kong

計畫編號：NSC 89-2415-H-002-006

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一、中文摘要

本計劃建立一個聯繫匯率(固定匯率)制度之小型開放經濟模型，藉以探討其貿易條件呈現內生性波動之可能。在借助"不確定性"文獻中之方法求得非確定動態均衡的存在條件之後，利用香港的貿易條件資料來驗證內生性波動之存在。

關鍵詞：貿易條件、內生波動

Abstract

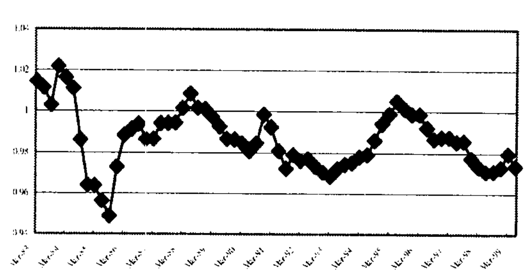
This research project aims to show the possibility of endogenous fluctuations of the terms of trade in a simple intertemporal optimizing model of a small open economy under linked (fixed) exchange rates. We derive the condition under which stochastic self-fulfilling expectations (sunspcts) may generate endogenous fluctuations in the terms of trade by following the line of research in the "indeterminacy" literature. We then look for evidence of this condition in the Hong Kong terms-of-trade data over the period of January 1984 to December 1998.

Keywords: terms of trade, endogenous fluctuations.

二、Introduction

There has been a lot of renew debates about the choices of the exchange rate regimes since the 1997 East Asia financial crises.[1] A number of analysts attributed the stable nominal exchange rate in Hong Kong to the existence of the fixed link to the U. S. dollar. In fact, the Hong Kong dollar is one of the two major currencies in the Asian region that have not been affected by the past wave of devaluations, the other being the renminbi (RMB) of the Mainland China.[2] The Hong Kong dollar has been linked to the US dollar at a fixed exchange rate since October 1983, and withstood abundance of events including several currency crisis during the 1990s.

Hong Kong's Quarterly Terms of Trade



But the terms of trade (real exchange rate) of Hong Kong still exhibited significant cyclical fluctuations after the Plaza Accord (September, 1985) as we can see from the attached figure. The purpose of the research project is to analyze the reasons

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[1] See Edwards (2000) and literature cited therein.

[2] See Yam (1998) for the succinct introduction of the currency board system in Hong Kong.

of the endogenous cyclical fluctuations of the terms of trade under the linked exchange rate systems.

The research into the endogenous fluctuations of the real exchange rates so far has been very preliminary and scarce, most of the papers were conducted by the authors of this project (eg. Chen (1999), Chen and Chen (1999, 1998)). But all the discussion is under the framework of the flexible exchange rate regime. In this research project we will focus on the fluctuations of the real exchange rate under the fixed (linked) exchange rate regime.

三、The Model

This research project purports an open economy macroeconomic model under fixed exchange rates by incorporating intertemporal optimization into the model in Chen and Lai (1989). This model is in fact an extension of Brock (1975) one-commodity model. We consider two commodities and also endogenize the money stock.

Consider an open economy which is small in that it faces a given price of foreign output, P*. Domestically produced goods differ from foreign goods. The supply of domestic goods is fixed at $\bar{y}$. It is assumed that the price of domestic output, P_t, is perfectly flexible. The central bank adopts a fixed exchange rate system, and the domestic money supply will change according to the changes of the balance of payments. For simplicity, let gold be the common currency of both the domestic and foreign money supply so that the nominal exchange rate between the two countries is equal to unity. Also for simplicity of analysis, let P*=1 so that the terms of trade q_t can be written as $q_t=P^*/P_t=1/P_t$.

The framework is similar to Brock's (1975), except in the treatment of the number of commodities consumed. The representative resident with rational expectations is assumed to maximize the expected value of the following additively separable utility function and is bound by the following budget constraint:

$$\text{Max} \sum_{t=0}^{\infty} \beta^t [U_1(C_{1t}) + U_2(C_{2t}) + V(\frac{M_t}{P_t})] \quad (1)$$

$$\text{s.t.} \quad \frac{M_t - M_{t-1}}{P_t} = \bar{y} - C_{1t} - \frac{C_{2t}}{P_t} \quad (2)$$

where β is the subjective time discount rate on future utility; U_1 is the one-period utility from domestic good consumption, C_{1t} ; U_2 is the one-period utility from import good consumption, C_{2t}, and V is the one-period utility from expected yields from domestic money holdings, M_t.

Let λ_t and λ_{t+1} be the multipliers associated with the budget constraint at time t and t+1. The first-order conditions for the constrained maximization are

$$\frac{\partial U_1(C_{1t})}{\partial C_{1t}} = \lambda_t, \quad (3)$$

$$\frac{\partial U_2(C_{2t})}{\partial C_{2t}} = \frac{\lambda_t}{P_t}, \quad (4)$$

$$\frac{\partial V(\frac{M_t}{P_t})}{\partial(\frac{M_t}{P_t})} = \lambda_t - \beta E_t(\lambda_{t+1}\frac{P_t}{P_{t+1}}). \quad (5)$$

An explicit solution is obtained most expeditiously by assuming that

(A1) The marginal utility of consumption of export goods, U_1 , is constant and accordingly λ_t is constant. By an appropriate choice of units, we set $\frac{\partial U_1(C_{1t})}{\partial C_{1t}} = \lambda_t = 1$, that is, $U_1(C_{1t}) = C_{1t}$, and accordingly $\lambda_t = \lambda_{t+1} = 1$.[3]

(A2) The utility function of import good consumption is of the form:

$$U_2(C_{2t}) = \begin{cases} C_{2t}^{1-a}/(1-a), & \text{if } a \neq 1, a > 0; \\ \ln C_{2t}, & \text{if } a = 1. \end{cases} \quad (6)$$

so that domestic demand for imports has a constant price elasticity of 1/a, and

and

(A4) The utility function of the real money balances is quadratic,

$$V(\frac{M_t}{P_t}) = \begin{cases} (\frac{M_t}{P_t})^{1-R}/(1-R), \text{if } R \neq 1, R > 0; \\ \ln(\frac{M_t}{P_t}), \text{if } R = 1. \end{cases} \quad (7)$$

so that the marginal utility is linear.

Then the first-order conditions can be rewritten as

$$\frac{\partial U_1(C_{1t})}{\partial C_{1t}} = 1 \quad (8)$$

$$\frac{\partial U_2(C_{2t})}{\partial C_{2t}} = C_{2t}^{-a} = \frac{1}{P_t} \quad (9)$$

$$\frac{\partial V(\frac{M_t}{P_t})}{\partial (\frac{M_t}{P_t})} = (\frac{M_t}{P_t})^{-R} = 1 - \beta \frac{P_t}{P_{t+1}} \quad (10)$$

From the second first-order condition (9) we obtain the demand for imports,

$$C_{2t} = P_t^{\frac{1}{a}}, \quad (11)$$

and from the third first-order condition (10) we can derive the money balances as

$$M_t = (P_t^{-R} - \beta P_t^{1-R} / P_{t+1})^{\frac{-1}{R}}. \quad (12)$$

Since by definition the supply of exports is the difference of domestic output and domestic consumption of export goods, the constancy of the marginal utility of consumption of export goods implies that the terms of trade is determined solely by the foreign demand for the home country's export goods and the home country's demand for import goods. The domestic demand for export goods is of no consequence here. The foreigners import whatever they want from the domestic economy at the given terms of trade; the domestic residents simply consume whatever is left. For symmetry, we write the foreign demand for imports (our exports), X_t^d, as

$$X_t^d = P_t^{-\frac{1}{b}}, \quad (13)$$

by assuming that the foreign residents have similar utility functions. It goes without saying that 1/b is the foreign country's price elasticity of import demands.

Combining (11) and (13), we can write the balance of trade in terms of import goods, or in terms of gold, T_t,

$$T_t = P_t X_t^d - C_{2t} = P_t^{1-\frac{1}{b}} - P_t^{\frac{1}{a}}. \quad (14)$$

From the budget constraint (2), we know that

$$\begin{aligned} M_t - M_{t-1} &= P_t(\bar{y} - C_{1t}) - C_{2t} \\ &= T_t = P_t^{1-\frac{1}{b}} - P_t^{\frac{1}{a}}, \end{aligned} \quad (15)$$

by also noting that $P_t X_t = P_t(\bar{y} - C_{1t})$. By lagging equation (12) for one period and substituting it and (12) into equation (15), we will get a nonlinear difference equation in P of second order,

$$\begin{aligned} &P_t(1 - \frac{\beta P_t}{P_{t+1}})^{\frac{-1}{R}} - P_{t-1}(1 - \frac{\beta P_{t-1}}{P_t})^{\frac{-1}{R}} - P_t^{1-\frac{1}{b}} \\ &+ P_t^{\frac{1}{a}} = F(P_{t+1}, P_t, P_{t-1}) = 0. \end{aligned} \quad (16)$$

四、Local Dynamic of the Equilibrium

To investigate the system's dynamic properties in a neighborhood of the stationary state, we linearize the nonlinear dynamic system around the stationary state value of P*. By substituting P* into equation (16), we find that the stationary equilibrium

[3] This simplifying assumption was also made by Boyer and Kingston (1987) in order to obtain a tractable model. Brock (1975) restricts his model to a one good closed economy case in which planned consumption must equal to the fixed flow of output in each period. Thus, the marginal utility of consumption is in effect made constant.

terms of trade P* takes the value of unity, unless it so happens that $\frac{1}{a}+\frac{1}{b}=1$. The resulting system of difference equat ons is

$$\begin{bmatrix} P_{t+1}-1 \\ P_t-1 \end{bmatrix} = \begin{bmatrix} \gamma & -\Delta \\ 1 & 0 \end{bmatrix} \begin{bmatrix} P_t-1 \\ P_{t-1}-1 \end{bmatrix}, \quad (17)$$

where

$$\gamma = 2 + R(\frac{1}{\beta}-1) + \frac{R}{\beta}(1-\beta)^{\frac{1+R}{R}}(\frac{1}{a}+\frac{1}{b}-1), \quad (18)$$

and

$$\Delta = 1 + R(\frac{1}{\beta}.-1) \quad (19)$$

The characteristic equation (17) is

$$\lambda^2 - \gamma\lambda + \Delta = 0. \quad (20)$$

We first state the following prcposition.

Proposition 1. The characteristic equation (20) with equation (18) and (19) has two roots satisfying $|\lambda|<1$ iff

(i) $-\frac{R}{\beta}(1-\beta)^{\frac{1+R}{R}}(\frac{1}{a}+\frac{1}{b}-1)>0$,

(ii) $4+2R(\frac{1}{\beta}-1)+\frac{R}{\beta}(1-\beta)^{\frac{1+R}{R}}(\frac{1}{a}+\frac{1}{b}-1) > 0$

(iii) $1+R(\frac{1}{\beta}-1)<1$, and it has two ıeal roots λ_1 and λ_2 with $|\lambda_1|>1>|\lambda_2|$ iff

(iv-a) $-\frac{R}{\beta}(1-\beta)^{\frac{1+R}{R}}(\frac{1}{a}+\frac{1}{b}-1)<0$,

(iv-b) $4+2R(\frac{1}{\beta}-1)+ \frac{R}{\beta}(1-\beta)^{\frac{1+R}{R}}(\frac{1}{a}+\frac{1}{b}-1)>0$

Proof. According to Grandmcnt and Laroque (1988), the characteristic equation (20) has two roots satisfying $|\lambda|<1$ iff

(i) $1+\Delta-\gamma>0$,

(ii) $1+\Delta+\gamma>0$,

(iii) $\Delta<1$, and it has two real roots λ_1 and λ_2, with $|\lambda_1|>1>|\lambda_2|$ iff

(iv) $|\gamma|-|1+\Delta|>0$.

By substituting γ and Δ into equations (18) and (19), and noting that, given $\gamma>0$, the condition $|\gamma|-|1+\Delta|>0$ is equivalent to the following two inequalities,

$1+\Delta-\gamma<0$,

$1+\Delta+\gamma>0$,

we immediately arrive at condition (i) through (iv-b) in Proposition 1.

We are now ready to consider the dynamic properties of the steady state. If the Marshall-Lerner condition is satisfied at this fixed point, that is, if $\frac{1}{a}+\frac{1}{b}-1>0$, it is immediately seen that condition (i) is violated and the stationary equilibrium point cannot be locally stable. On the other hand, condition (iv-a) and (iv-b) are apparently fulfilled, implying that the Jacobian matrix evaluated at P*=1 has one real eigenvalue greater, and the other smaller, than unity in absolute value. Woodford (1992, p.224) calls this the case of 'saddlepoint stability': there is a unique rational expectations equilibrium and no endogenous fluctuations are possible. We therefore arrive at

Corollary 2. The steady state exhibits saddle-point properties if the Marshall-Lerner condition is satisfied at the fixed point.

The dynamic properties of the stationary state when the Marshall-Lerner condition is not satisfied are more interesting in that there may exist periodic cycles in its neighborhood. The existence of such periodic equilibria may be established by applying the theory of Hopf bifurcation. A Hopf bifurcation refers to the development of periodic orbits ('self-oscillations') from a stable fixed point. It occurs at a critical value of the key parameter, say $\beta=\beta_0$, if the corresponding eigenvalues $\lambda_1(\beta)$ and $\lambda_2(\beta)$ are complex conjugates and cross the unit circle at β_0. We first state the following proposition:

Proposition 3. A Hopf bifurcation occurs at $\beta_0 = 1$, that is, periodic orbits of the terms of trade develop from a stable fixed point when the parameter β passes the critical value of β_0.

The above proposition is an application of the theorem on the existence of Hopf bifurcation (see Lorenz, 1989, pp. 96-97):

Let the mapping $P_{t+1} = H(P_t, \mu), P_t \in \Re^2, \mu \in \Re$, have a fixed point P*, at which the eigenvalues are complex conjugates. If there is a μ_0 such that

$(\operatorname{mod} \lambda(\mu_0) = 1 but \lambda^n(\mu_0) \neq \pm 1, n = 1,2,3,4,$

and

$$\frac{d(\operatorname{mod} \lambda(\mu))}{d\mu} \neq 0$$

then there is an invariant closed curve bifurcation from $\mu = \mu_0$.

In our model the eigenvalues are complex conjugates if

$$-2\sqrt{\frac{R+\beta-\beta R}{\beta}} < \frac{1}{\beta}(R-\beta R+2\beta)+ \frac{\frac{1}{a}+\frac{1}{b}-1}{\frac{\beta}{R}(1-\beta)^{\frac{-1-R}{R}}} < 2\sqrt{\frac{R+\beta-\beta R}{\beta}} \qquad (21)$$

Assume for the moment that the inequalities (21) hold.[4] A Hopf bifurcation occurs at

$$\beta_0 = 1,$$

because, at $\beta = \beta_0$,

$$\Delta = 1 .$$

The modulus crosses the unit circle and will have a value greater than unity, when the parameter β is changed to less than unity, a classic case of Hopf bifurcation which indicates the emergence of cycles. All perfect foresight equilibria beginning near the stationary state may diverge from the stationary state and be attracted to an invariant circle. In this case we would have indeterminacy in the sense that the equilibrium trajectories converge to the limit cycle (Woodford 1992, p.225).

Since the value of β can never exceed unity, the indeterminacy associated with the stable stationary state is impossible. However, following Rogoff (1992), if we introduce the government expenditure into the model and assume that it is financed by non-distortionary lump-sum taxes, so that the Ricardian equivalence holds, we can integrate the government's budget constraint into the individual's budget constraint. Using this specification, the representative resident's budget constraint is given by

$$\frac{M_t - M_{t-1}}{P_t} = \bar{y} - C_{1t} - \frac{C_{2t}}{P_t} - \frac{G_t}{P_t}, \qquad (22)$$

where G_t represents government consumption of import goods. It is assumed that government consumption does not affect the utility of private consumption. It is further assumed that there is an one period delay between actual purchase and decision: $G_t = fP_{t-1}{}^g$. Under these conditions, we will have $\Delta = 1 + R(\frac{1}{\beta} - 1) - fg < 1$ as $fg > R(\frac{1}{\beta} - 1)$. Thus, if

$$1 + \Delta - \gamma = (1 - \frac{1}{a} - \frac{1}{b})\frac{R}{\beta}(1-\beta)^{\frac{1+R}{R}} - fg > 0$$

holds, we will have $|\lambda| < 1$. Local indeterminacy associated with a stable stationary state therefore occurs.

五、Existence of Sunspot Equilibria

Because of indeterminacy of perfect-foresight equilibrium, households

[4] It can be easily seen that, when β is close to β_0, the inequalities are very likely to hold.

now face uncertainty about next period' price (the inverse of terms of trade). Based on the stochastic Euler equations (3)-(5), we can rewritten the system of difference equations (17) as

$$E_t P_{t+1} = (1+\Delta-\gamma)+(\gamma-\Delta)P_t-\gamma P_{t-1}+\gamma E_{t-1}P_t. \quad (23)$$

Define $\varepsilon_{t+1}=P_{t+1}-E_t P_{t+1}$ and $\varepsilon_t = P_t - E_{t-1}P_t$ and substitutes these expressions into (24). We then obtain

$$P_{t+1}=(1+\Delta-\gamma)+(-\Delta)P_t-\gamma P_{t+1}+\varepsilon_{t+1}-\gamma\varepsilon_t, \quad (24)$$

or in lag operator form,

$$[1+\Delta L+\lambda L^2)P_t=(1+\Delta-\gamma)+(1-\gamma L)\varepsilon_t,$$

where ε_t a mean-zero, serially uncorrelated random variable, if expectations are rational. It turns out that the price level in our model follows an ARMA(2,1) process. The AR(2) process is obtained because expectations about P_{t+1} are formed from a weighted sum of P_t and P_{t-1}. The MA(1) process arises because the equilibrium process of money holdings is disturbed by present and past transient shocks.

We now proceed to look for empirical evidence in terms of actual data of Hong Kong. Conceptually, the terms of trade, defined by the IMF countries as the unit value of exports divided by the unit value of imports, seem to be an ideal proxy variable for the inverse of P_t in equation (24). Following Benhabib and Farmer (1996, pp. 438-39) in assuming that sunspot or belief shocks are *i.i.d.* and are driven by the same stochastic process as the real shock, we estimate equation (24) using the IFS monthly data of the terms of trade. The results of estimation are reported in Table 1.

Table 1: Estimation of ARMA(2,1) Equation: Hong Kong's Terms of Trade, 1984:1 to 1998:12.

Variable	Coefficient	Std. Error	t-Statistic
Constant	0.9867	0.0017	595.4814
AR(1)	1.9085	0.0315	60.6561
AR(2)	-0.9209	0.0296	-31.1014
MA(1)	-0.9486	0.0400	-23.7191
$\bar{R}^2$	0.8921	σ	0.0142
SER	0.0047	$\chi^2(1)$	0.0900
μ	0.9857	$\chi^2(4)$	0.0752

Note that $\bar{R}^2$ denotes the adjusted coefficient of determinations; $\chi^2(1)$ and $\chi^2(4)$ denote the P-values of the Godfrey's nR^2 test statistic of serial correlation of the 1st order and up to the 4th order. μ denotes the mean and σ its standard deviation of dependent variables, and SER the standard error of the regression. The two inverted AR roots are complex conjugates: $0.95\pm0.10i$, with a modulus of 0.96. The autoregressive process is persistent but nonetheless stationary. The fact that our model is capable of generating complex roots explains why it can capture the cyclic fluctuations feature of data as displayed in the figure at page 2.

六、Concluding Remarks

Existing literature on exchange rate dynamics has proved unsuccessful in proving explanation based solely on economic fundamentals of short-terms real exchange rate movements. This research project has demonstrated the possibility of deterministic and stochastic endogenous fluctuation of the terms of trade in an open economy under linked exchange rates.

七、References

Benhabib, J. and R. Farmer (1996). "Indeterminacy and Sector-specific Externalities." *Journal of Monetary Economics* 37, 421-43.

Brock, W. A. (1975) "A Simple Perfect Foresight Monetary Model." *Journal of Monetary Economics* 1, 133-150.

Boyer, R. S. and G. H. Kingston. (1987) "Currency Substitution under Finance Constraints." *Journal of International Money and Finance*, Vol. 6, 225-25).

Chen, C. and S. Chen. (1999) "A Nonlinear Model of Real Exchange Rate Dynamics, with an Application to the Case of Taiwan." *Taiwan Economic Review*, Vol. 27, No. 4.

Chen, C. and S. Chen. (1998) "The Short Run Behavior of the Real Exchange Rate: Evidence from the G-Five Countries over the Recent Floating-Rate Period." *Discussion Papers* 9814, the Institute of Economics, Academia Sinica.

Chen, C., and C. Lai. (1989) "Money, Consumption, and Terms-of-trade Dynamics: A Note to Dornbusch and Mussa." *International Economic Review* 30, 1005-9.

Chen, S. (1999) "Complex Dynamics of the Real Exchange Rate in an Open Macroeconomic Model." *Journal of Macroeconomics*, Vol.21, No. 3.

Edwards, S. (2000) "Exchange Rate Regimes, Capital Flows and Crisis Prevention." NBER Conference on Economic and Financial Crises in Emerging Market Economies, Woodstock, VT Oct. 19, 2000.

Grandmont, J. M. and G. Laroque (1988). "Stability, Expectations and Predetermined Variables." Technical Report 522, Institute for Mathematical Studies in the Social Sciences, Stanford University.

Lorenz, H. W. (1989) Nonlinear Dynamical Economics and Chaotic Motion. Lecture Notes in Economics and Mathematical Systems 334. Springer-Verlag, New York.

Rogoff, K. S. (1992) "Traded Goods Consumption Smoothing and the Random Walk Behavior of the Real Exchange Rate." Bank of Japan *Monetary and Economic Studies* 10, 1-29.

Woodford, M. (1992) "Equilibrium Models of Endogenous Fluctuations: An Introduction." In *Macroeconomics: A Survey of Research Strategies*, edited by A. Vercelli and N. Dimitri. New York: Oxford University Press.

Yam, J. (1998) "The Hong Kong Dollar Link." Speeches of Financial Roadshow in Tokyo, Hong Kong Trade Development Council, March 1998.