行政院國家科學委員會補助專題研究計畫成果報告

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※ 浮動與固定匯率制度下的實質匯率內生波動比較 ※

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行政院國家科學委員會專題研究計畫成果報告

浮動與固定匯率制度下的實質匯率內生波動比較

Endogenous Fluctuations of Real Exchange Rates under Different Exchange Rate Regimes

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一、中文摘要

本計劃提出一個世代交疊之開放經濟模型，用以討論在浮動匯率與固定匯率制度之下實質匯率可能發生的內生波動。我們運用非線性動態學中的理論(Hopf 與 flip 分歧理論)，來探討此經濟體系均衡點附近週期循環或定態太陽黑子均衡存在的條件。其中可能包含兩個非常重要的參數：主觀折扣率與跨期替代彈性。本計劃討論實質匯率的穩定與否與此二參數之間的密切關係，並比較在不同的匯率制度之下，造成實質匯率內生波動的條件異同，為 1997 年亞洲金融風暴之後又興起的匯率制度辯論，提供另一方向的分析。

關鍵詞：內生波動、實質匯率、週期循環

Abstract

This research plan purports to show that real exchange rate fluctuations may be generated endogenously through nonlinear business cycles in a simple overlapping generations economy which trades with the rest of the world under flexible or fixed exchange rates. The existence of periodic cycles or stationary sunspot equilibria near the steady state in this economy is established by applying the Hopf and flip bifurcation theorem and may involve two important parameters: the subjective rate of discount; and the elasticity of intertemporal substitution. We would like to find a close link between the non-explosiveness and stability of the system and the two parameters. We would also like to compare the different conditions for the endogenous fluctuations of the real exchange rate between the two exchange rate regimes and explore the policy implications.

Keywords: Endogenous fluctuations; Real exchange rate; periodic cycles

ENDOGENOUS FLUCTUATIONS OF REAL EXCHANGE RATES UNDER DIFFERENT EXCHANGE RATE REGIMES

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This research plan purports to show that real exchange rate fluctuations may be generated endogenously through nonlinear business cycles in a simple overlapping generations economy which trades with the rest of the world under flexible or fixed exchange rates. The existence of periodic cycles or stationary sunspot equilibria near the steady state in this economy is established by applying the Hopf and flip bifurcation theorems and may involve two important parameters: the subjective rate of discount; and the elasticity of intertemporal substitution. We find a close link between the non-explosiveness and stability of the system and the two parameters. We also compare the different conditions for the endogenous fluctuations of the real exchange rate between the two exchange rate regimes and explore the policy implications.

Keywords: Endogenous fluctuations, Periodic cycles.

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1 INTRODUCTION

In recent years there have been a number of studies looking into the possibility of endogenous business cycles in an overlapping generations economy.(e.g., Grandmont (1985), Medio (1992), and Farmer (1999).) They, however, all deal with a closed economy.[1] This paper purports to show that real exchange rate fluctuations may be generated endogenously through nonlinear business cycles in a simple overlapping generations economy which trades with the rest of the world under flexible or fixed exchange rates. Our primary purposes is to investigate whether the sensitivity of the equilibrium stability to the changes of the subjective rate of discount and the elasticity of intertemporal substitution is qualitatively the same in the closed-economy and open-economy versions of the overlapping generations models and to compare the differences between flexible and fixed exchange rate regimes. The model we use in this paper is a simple extension of the Samuelson overlapping generations model (Samuelson 1958) with money to an open economy. Individuals in this economy live for two periods, and all of them consume both export goods and import goods. There is a constant nominal quantity of money which is the only asset available to individuals as a store of value.

We are interested in the possible existence of periodic cycles near the steady state in such an open economy under flexible exchange rates. The existence of cycles is established using the flip or Hopf bifurcation theorems.[2] Two basic parameters are involved: the subjective rate of discount and the elasticity of intertemporal substitution. We derive a discrete dynamic system of this economy in which these two parameters play crucial roles. We find that a necessary condition for the system to be non-explosive is that the elasticity of intertemporal substitution be less than unity under the flexible exchange rate regime. This confirms the earlier results of the closed-economy overlapping generations literature.[3] In addition, we find that the larger the discounting, the more likely the stationary equilibrium will be stable. This result is in contrast with the findings in the

[1]One rare exception is Aloi, Dixon and Lloyd-Brage (2000) where they study a small open economy with a fixed exchange rate. The endogenous fluctuations in main macroeconomic aggregates were produced by incorporating increasing returns.

[2]For a succinct discussion of the theorem, see Lorenz (1989).

[3]Blanchard and Fisher (1989)[p.249].

closed economy models where a large discounting rate may be destabilizing. The reason is that in the open economy an increase in the gross rate of return on money has an additional effect: real income would decline because the terms of trade would deteriorate. On the contrary, we find that a necessary condition for the system to be non-explosive is that the elasticity of intertemporal substitution be larger than unity under the fixed exchange rate regime.

2 THE MODEL

We consider an open economy with a stationary population. Individuals live for two periods and are identical in each generation. An individual born at time t is young at time t and old at time $t+1$. Each individual is endowed with an exogenously fixed quantity of the export good, $\bar{y}$, when young, but receives no endowment when old. The export goods are non-storable but can be exchanged for money and for import goods which are also non-storable. The individual derives utility from consuming both export goods and import goods. The young individual's preference at time t is described by a separable utility function:[4]

$$\frac{(x_{1t}^{\alpha} z_{1t}^{1-\alpha})^{1-\gamma}}{1-\gamma} + \beta \frac{(x_{2t+1}^{\alpha} z_{2t+1}^{1-\alpha})^{1-\gamma}}{1-\gamma}, \gamma > 0, \gamma \neq 1, \tag{1}$$

where x_{1t} and z_{1t} denote, respectively, consumption of export goods and consumption of import goods at time t when the individual is young; and x_{2t+1} and z_{2t+1} denote, respectively, consumption of export goods and consumption of import goods at time $t+1$ when the individual is old. β is the discount factor and γ is the coefficient of relative risk aversion associated with the utility function. $\frac{1}{\gamma}$ is the elasticity of intertemporal substitution.

2.1 THE FLEXIBLE EXCHANGE RATE REGIME

There exists a fixed stock of fiat money, M, which is the only asset available to individuals as a store of value, and which is owned by the old generation at time $t = 0$ to be exchanged

[4]In order to make the analysis tractable, it is necessary to write the utility function in an additively separable form. As Zee (1987) pointed out this functional form possesses a convenient property that wealth elasticity of consumption is unity.

for export goods at price P_t and for import goods at price E_t in period t. To ensure a demand for domestic fiat money, the government enforces a legal prohibition on the use of foreign money. The economy is under flexible exchange rates and the foreign currency price of imports is assumed to be exogenously fixed at the level of unity. E_t is therefore also the exchange rate.

The maximization problem of the agent born at t is given by maximizing (1) subject to the current and future budget constraints,

$$\bar{y} - x_{1t} - q_t z_{1t} = \frac{M_t^d}{P_t}, \tag{2}$$

$$x_{2t+1} + q_{t+1} z_{2t+1} = \frac{M_t^d}{P_{t+1}}, \tag{3}$$

where $q_t = \frac{E_t}{P_t}$ is the real exchange rate or the terms of trade. The first-order conditions imply

$$\frac{\alpha z_{1t}}{(1-\alpha)x_{1t}} = \frac{1}{q_t}, \tag{4}$$

$$\frac{\alpha z_{2t+1}}{(1-\alpha)x_{2t+1}} = \frac{1}{q_{t+1}}, \tag{5}$$

$$\left(\frac{x_{1t}^{\alpha} z_{1t}^{1-\alpha}}{x_{2t+1}^{\alpha} z_{2t+1}^{1-\alpha}}\right)^{-\gamma} \frac{x_{1t}^{\alpha-1} z_{1t}^{1-\alpha}}{x_{2t+1}^{\alpha-1} z_{2t+1}^{1-\alpha}} = \beta \pi_{t+1}, \tag{6}$$

where $\pi_{t+1} = \frac{P_t}{P_{t+1}}$ is the inverse of one plus the inflation rate of domestic goods from period t to period $t+1$ and therefore is the gross rate of return on money. Equations (4) and (5) imply that both the young and the old choose to consume more of import goods and less of domestic goods the lower the terms of trade (the price of imports relative to domestic goods), q_t, and that their expenditure on import goods relative to that on domestic goods is of a fixed proportion of $\frac{\alpha}{1-\alpha}$. The fixed proportion of expenditure comes from the utility function being of the Cobb-Douglas type. Equation (6) specifies intertemporal substitution of consumption between two periods, t and $t+1$. Substituting (4) and (5) into (6) and rearranging terms, we obtain

$$\frac{x_{1t}}{x_{2t+1}} = (\beta \pi_{t+1})^{-\frac{1}{\gamma}} \left(\frac{q_t}{q_{t+1}}\right)^{(1-\alpha)(1-\frac{1}{\gamma})},$$

$$\frac{q_t z_{1t}}{q_{t+1} z_{2t+1}} = (\beta \pi_{t+1})^{-\frac{1}{\gamma}} \left(\frac{q_t}{q_{t+1}}\right)^{\alpha(\frac{1}{\gamma}-1)}.$$

From equations (2) - (6) we can derive the following four demand equations,

$$x_{1t} = \frac{\alpha \bar{y}}{1 + \beta^{\frac{1}{\gamma}} \pi_{t+1}^{\frac{1-\gamma}{\gamma}} (\frac{q_{t+1}}{q_t})^{\frac{(\gamma-1)(1-\alpha)}{\gamma}}}, \tag{7}$$

$$z_{1t} = \frac{(1-\alpha)\bar{y}}{q_t + \beta^{\frac{1}{\gamma}} q_{t+1} \pi_{t+1}^{\frac{1-\gamma}{\gamma}} (\frac{q_{t+1}}{q_t})^{\frac{\alpha(1-\gamma)-1}{\gamma}}}, \tag{8}$$

$$x_{2t+1} = \frac{\alpha \bar{y}}{\frac{1}{\pi_{t+1}} + \beta^{\frac{-1}{\gamma}} \pi_{t+1}^{\frac{-1}{\gamma}} (\frac{q_t}{q_{t+1}})^{\frac{(\alpha-1)(1-\gamma)}{\gamma}}}, \tag{9}$$

$$z_{2t+1} = \frac{(1-\alpha)\bar{y}}{\frac{q_{t+1}}{\pi_{t+1}} + \beta^{\frac{-1}{\gamma}} \pi_{t+1}^{\frac{-1}{\gamma}} q_t (\frac{q_t}{q_{t+1}})^{\frac{\alpha(1-\gamma)-1}{\gamma}}}. \tag{10}$$

It is easily recognized from equations (7) and (9) that x_{1t} is independent of q_t and x_{2t+1} is independent of q_{t+1}, and from equations (8) and (10) that $q_t z_{1t}$ is independent of q_t and $q_{t+1} z_{2t+1}$ is independent of q_{t+1}.

Market equilibrium requires that the export goods market clears

$$X_t^d = \bar{y} - x_{1t} - x_{2t}, \tag{11}$$

and that the balance of trade balances

$$\frac{X_t^d}{q_t} - z_{1t} - z_{2t} = 0. \tag{12}$$

X_t^d is the foreign demand for home exports, that is, the demand for imports in the foreign country. Assume that the foreign country has symmetric demand functions, then

$$X_t^d = z_{1t}^* + z_{2t}^* = \frac{(1-\alpha')\bar{y}'}{\frac{1}{q_t} + \frac{1}{q_{t+1}} \beta^{\frac{1}{\gamma'}} (\frac{q_t}{q_{t+1}})^{\frac{\alpha'(1-\gamma')-1}{\gamma'}}} + \frac{(1-\alpha')\bar{y}'}{\frac{1}{q_t} + \frac{1}{q_{t-1}} \beta^{-\frac{1}{\gamma'}} (\frac{q_t}{q_{t-1}})^{\frac{\alpha'(1-\gamma')-1}{\gamma'}}}, \tag{13}$$

which is derived from the foreign utility function

$$\frac{(x_{1t}^{*\alpha'} z_{1t}^{*1-\alpha'})^{1-\gamma'}}{1-\gamma'} + \beta \frac{(x_{2t+1}^{*\alpha'} z_{2t+1}^{*1-\alpha'})^{1-\gamma'}}{1-\gamma'}, \gamma' > 0,$$

where x_{1t}^* and z_{1t}^* denote, respectively, consumption of export goods and consumption of import goods at time t when the foreign resident is young; and x_{2t+1}^* and z_{2t+1}^* denote, respectively, consumption of export goods and consumption of import goods at time $t+1$ when the foreign resident is old. γ' is the coefficient of relative risk aversion; and β is the discount factor, assumed to be equal to the domestic discount factor. X_t^d is then a function of q_{t+1}, q_t and q_{t-1}. However, inasmuch as the rest of the world is "large" as compared with the small open economy in question, it seems proper to assume that foreign residents are risk neutral, that is, the elasticity of intertemporal substitution is unity, $\gamma' = \frac{1}{\gamma'} = 1$.[5] X_t^d then reduces to

$$X_t^d = \frac{2(1-\alpha')\bar{y}' q_t}{1+\beta} = X_t^d(q_t), \frac{\partial X_t^d(q_t)}{\partial q_t} > 0.$$

X_t^d is now a positive function of q_t only and is independent of q_{t-1} and q_{t+1}. We finally are able to rewrite equations (11) and (12) as

$$X_t^d - \bar{y} + x_{1t} + x_{2t} = g(\pi_{t+1}, \pi_t, q_{t+1}, q_t, q_{t-1}) = 0, \tag{14}$$

$$\frac{X_t^d}{q_t} - z_{1t} - z_{2t} = f(\pi_{t+1}, \pi_t, q_{t+1}, q_t, q_{t-1}) = 0, \tag{15}$$

which constitute the basis for analysis in this section of flexible exchange rate regime.

2.2 CHARACTERIZING EQUILIBRIA UNDER FLEXIBLE EXCHANGE RATES

In this section we will explore the dynamic properties of the stationary equilibrium. Stationary equilibria must obey the relationships,

$$\frac{X_t^d(\bar{q})}{\bar{q}} - z_{1t} - z_{2t} = f(1, 1, \bar{q}, \bar{q}, \bar{q}) = 0, \tag{16}$$

and

$$X_t^d(\bar{q}) - \bar{y} + x_{1t} + x_{2t} = g(1, 1, \bar{q}, \bar{q}, \bar{q}) = 0, \tag{17}$$

which are derived from equations (14) and (15) with $\pi_{t+1} = \pi_t = \bar{\pi} = 1$ and $q_{t+1} = q_t = q_{t-1} = \bar{q}$ in the steady state. From (16) and (17) we immediately know that the real

[5] This assumption enables us to insulate the influence of the foreign demand on the dynamic properties of the small open economy in question. If X_t^d is assumed to be a function of q_{t+1}, q_t and q_{t-1}, then the dynamic equation (20) below will take the following form

$$\lambda(\lambda - \frac{1+\beta^{\frac{1}{\gamma}}\gamma}{1-\gamma})(\frac{\partial X_t^d(\bar{q})}{\partial q_{t+1}}\lambda^2 + \frac{\partial X_t^d(\bar{q})}{\partial q_t}\lambda + \frac{\partial X_t^d(\bar{q})}{\partial q_{t-1}}) = 0$$

The home country's stability depends crucially on foreign demands.

exchange rate in the stationary state $\bar{q}$ has to satisfy

$$X(\bar{q}) = (1-\alpha)\bar{y}.$$

As long as the foreign demand function for domestic good is monotonically increasing, the stationary state of real exchange rate $\bar{q} = X^{-1}((1-\alpha)\bar{y})$ will be unique.

We first define the system (14) and (15) as

$$F(w_{t-1}, w_t, w_{t+1}) = 0 \in \mathcal{R}^2, \tag{18}$$

where $w_t = (q_t, \pi_t)'$. Without any further qualification, however, we cannot be sure that the equilibrium trajectories of (18) belong to $\mathcal{R}^2_+$ for any positive initial conditions of real exchange rates q_t and the gross rate of return on money π_{t+1}. Therefore, we can only examine the local properties of (18). We now proceed to analyze the properties of the linearized version of the system. The eigenvalues of the Jacobian matrix DF evaluated at the stationary state $\bar{q}$ and $\bar{\pi}$ are the solutions of the equation

$$|A_0\lambda^2 + B_0\lambda + B_1| = 0, \tag{19}$$

where

$$A_0 = \frac{\partial F}{\partial w_{t+1}} = \begin{bmatrix} \frac{\partial f}{\partial q_{t+1}} & \frac{\partial f}{\partial \pi_{t+1}} \\ \frac{\partial g}{\partial q_{t+1}} & \frac{\partial g}{\partial \pi_{t+1}} \end{bmatrix},$$

$$B_0 = \frac{\partial F}{\partial w_t} = \begin{bmatrix} \frac{\partial f}{\partial q_t} & \frac{\partial f}{\partial \pi_t} \\ \frac{\partial g}{\partial q_t} & \frac{\partial g}{\partial \pi_t} \end{bmatrix},$$

$$B_1 = \frac{\partial F}{\partial w_{t-1}} = \begin{bmatrix} \frac{\partial f}{\partial q_{t-1}} & \frac{\partial f}{\partial \pi_{t-1}} \\ \frac{\partial g}{\partial q_{t-1}} & \frac{\partial g}{\partial \pi_{t-1}} \end{bmatrix}.$$

After some computations, (19) finally reduces to a third-order equation,

$$\frac{\partial X_t^d(\bar{q})}{\partial q_t}\lambda^2(\lambda - \frac{1+\beta^{\frac{1}{\gamma}}\gamma}{1-\gamma}) = 0. \tag{20}$$

Aside from two roots being nil, there is a root with the value of

$$\lambda = \frac{1+\beta^{\frac{1}{\gamma}}\gamma}{1-\gamma}. \tag{21}$$

One can immediately see that if $0 < \gamma < 1$, then $\lambda > 1$, also if $1 < \gamma < 2$, then $\lambda < -1$. The system will be explosive if $\gamma < 2$. Given that $\gamma > 2$, the non-zero root will take a value of minus one when

$$\beta = \beta_0 = (1 - \frac{2}{\gamma})^\gamma. \tag{22}$$

The magnitude of γ must be substantially greater than unity (greater than 2) for β_0 to be non-trivial. We can also see that when β is less than β_0, the absolute value of the eigenvalue λ is less than unity so that the stationary state is stable; whereas when β is greater than β_0, the absolute value of the eigenvalue λ becomes greater than unity so that the stationary state is unstable. Algebraically, we have

$$\frac{\partial \lambda}{\partial \beta} = \frac{\partial(\frac{1+\beta^{\frac{1}{\gamma}}\gamma}{1-\gamma})}{\partial \beta} = \frac{\beta^{\frac{1-\gamma}{\gamma}}}{1-\gamma} < 0.$$

When β passes through β_0 from below, the dynamics of the system changes stability. β_0 is therefore the breaking point at which the qualitative behavior of the system changes and a flip or period-doubling bifurcation will emerge at this point. The flip bifurcation is one of the well-known local bifurcations of one-dimensional discrete dynamical systems, and we will use a simplified version in Lorenz (1989). He considers a continuous one-dimensional map of the form $x_{t+1} = f(x_t)$ indexed by a bifurcation parameter $\beta \in \mathcal{R}$, i.e., $x_{t+1} = f(x_t, \beta)$. The flip (or period-doubling) bifurcation theorem states that if $df(x, \beta)/dx = -1$ at $x = x^*$ and $\beta = \beta_0$, then a stable cycle of period 2 will bifurcate from the interior steady state x^* for β in a (right or left) neighborhood of β_0.[6] Period doubling is a quite common phenomenon in one-dimensional and more-dimensional discrete systems. In our case, when β varies only one root will cross the unit circle at -1, the other two roots will stay inside $(-1, 1)$ (zero in our example). The behavior of the system close to the the bifurcation point is essentially that of the one-dimensional system for the single variable corresponding to the nonzero root. Thus we may expect flip bifurcation in this case.[7] According to flip bifurcation theory, there will be an attracting period-2 cycle for

[6]Other conditions must also satisfied for the occurrence of bifurcations (see, for instance the bifurcation theorem stated in Grandmont(1989)). However, these other conditions are generically satisfied.

[7]This is essentially a simple application of centre manifold theory. By applying the centre manifold theorem, we can reduce the effective dimension of the system (18) but still have sufficient information on the dynamics. For more details, see Whitley (1983) and Carr (1981).

β slightly larger than β_0, and an attracting period-4 cycle if β increases a little further. If this continued, the dynamics would have an attracting period-2^n cycle and eventually become much more complex.[8]

2.3 THE FIXED EXCHANGE RATE REGIME

The agent when young can exchange export goods for import goods or money (gold). The old generations can use money (gold) to exchange for export goods and import goods. The maximization problem of the agent born at t is given by maximizing the utility function as above subject to the current and future budget constraints,

$$\bar{y} - x_{1t} - q_t z_{1t} = \frac{M_t^d}{P_t}, \tag{23}$$

$$x_{2t+1} + q_{t+1} z_{2t+1} = \frac{M_t^d}{P_{t+1}}, \tag{24}$$

where $q_t = \frac{E_t P_t^*}{P_t} = 1/P_t$. By normalizing the fixed nominal exchange rate E_t and the foreign price level P_t^* to unity, we can let the price level P_t to be equal to the reciprocal of the real exchange rate q_t.

The first-order conditions imply

$$\frac{\alpha z_{1t}}{(1-\alpha)x_{1t}} = \frac{1}{q_t}, \tag{25}$$

$$\frac{\alpha z_{2t+1}}{(1-\alpha)x_{2t+1}} = \frac{1}{q_{t+1}}, \tag{26}$$

$$\left(\frac{x_{1t}^{\alpha} z_{1t}^{1-\alpha}}{x_{2t+1}^{\alpha} z_{2t+1}^{1-\alpha}}\right)^{-\gamma} \frac{x_{1t}^{\alpha-1} z_{1t}^{1-\alpha}}{x_{2t+1}^{\alpha-1} z_{2t+1}^{1-\alpha}} = \beta \frac{q_{t+1}}{q_t}. \tag{27}$$

Substituting (25) and (26) into (27) and rearranging terms, we obtain

$$z_{2t+1} = \beta^{1/\gamma}\left(\frac{q_{t+1}}{q_t}\right)^{\frac{\alpha-\alpha\gamma}{\gamma}} z_{1t}, \tag{28}$$

$$x_{2t+1} = \frac{\alpha}{1-\alpha} q_{t+1} \beta^{1/\gamma}\left(\frac{q_{t+1}}{q_t}\right)^{\frac{\alpha-\alpha\gamma}{\gamma}} z_{1t}. \tag{29}$$

From equations (23) – (29) we can derive the following four demand equations,

$$x_{1t} = \frac{\alpha \bar{y}}{1 + \beta^{\frac{1}{\gamma}}\left(\frac{q_{t+1}}{q_t}\right)^{\frac{\alpha-\alpha\gamma}{\gamma}}}, \tag{30}$$

[8]For a lucid discussion of flip bifurcation, see Grandmont (1989) and Lorenz (1989)[pp.92–95].

$$z_{1t} = \frac{(1-\alpha)\bar{y}}{q_t[1+\beta^{\frac{1}{\gamma}}(\frac{q_{t+1}}{q_t})^{\frac{\alpha-\alpha\gamma}{\gamma}}]}, \tag{31}$$

$$x_{2t+1} = \frac{\alpha q_{t+1}\beta^{1/\gamma}(\frac{q_{t+1}}{q_t})^{\frac{\alpha-\alpha\gamma}{\gamma}}\bar{y}}{q_t[1+\beta^{\frac{1}{\gamma}}(\frac{q_{t+1}}{q_t})^{\frac{\alpha-\alpha\gamma}{\gamma}}]}, \tag{32}$$

$$z_{2t+1} = \frac{\alpha q_{t+1}\beta^{1/\gamma}(\frac{q_{t+1}}{q_t})^{\frac{\alpha-\alpha\gamma}{\gamma}}\bar{y}}{q_t[1+\beta^{\frac{1}{\gamma}}(\frac{q_{t+1}}{q_t})^{\frac{\alpha-\alpha\gamma}{\gamma}}]}. \tag{33}$$

Market equilibrium requires that the export goods market clears

$$X_t^d = \bar{y} - x_{1t} - x_{2t}, \tag{34}$$

and that the balance of trade balances

$$\frac{X_t^d}{q_t} - z_{1t} - z_{2t} = M_t - M_{t-1}, \tag{35}$$

where X_t^d is the foreign demand for home exports as assumed by the previous section. When money is the only asset, the balance of trade determines the dynamics of the money stock under the fixed exchange rate.

We are able to rewrite equations (34) and (35) as

$$X_t^d - q_t z_{1t} + \frac{\alpha}{1-\alpha} q_t z_{2t} - q_t M_t = g(M_t, M_{t-1}, q_{t+1}, q_t, q_{t-1}) = 0, \tag{36}$$

$$X_t^d - q_t z_{1t} - q_t z_{2t} - q_t M_t + q_t M_{t-1} = f(M_t, M_{t-1}, q_{t+1}, q_t, q_{t-1}) = 0, \tag{37}$$

which constitute the basis for analysis in this section of fixed exchange rates.

2.4 CHARACTERIZING EQUILIBRIA UNDER FIXED EXCHANGE RATES

In this section we will explore the dynamic properties of the stationary equilibrium. Stationary equilibria must obey the relationships,

$$\frac{X_t^d(\bar{q})}{\bar{q}} - z_{1t} - z_{2t} = f(\bar{M}, \bar{M}, \bar{q}, \bar{q}, \bar{q}) = 0, \tag{38}$$

and

$$X_t^d(\bar{q}) - \bar{q} z_{1t} + \frac{\alpha}{1-\alpha}\bar{q} z_{2t} - \bar{q}\bar{M} = g(\bar{M}, \bar{M}, \bar{q}, \bar{q}, \bar{q}) = 0, \tag{39}$$

which are derived from equations (36) and (37) with $M_t = M_{t-1} = \bar{M} = 1$ and $q_{t+1} = q_t = q_{t-1} = \bar{q}$ in the steady state. From (38) and (39) we immediately know that the real exchange rate in the stationary state $\bar{q}$ has to satisfy

$$X(\bar{q}) = (1-\alpha)\bar{M}.$$

As long as the foreign demand function for domestic good is monotonically increasing, the stationary state of real exchange rate $\bar{q} = X^{-1}((1-\alpha)\bar{M})$ will be unique.

We first define the system (36) and (37) as

$$G(w_{t-1}, w_t, w_{t+1}) = 0 \in \mathcal{R}^2, \tag{40}$$

where $w_t = (q_t, M_t)'$. Without any further qualification, however, we cannot be sure that the equilibrium trajectories of (40) belong to $\mathcal{R}^2_+$ for any positive initial conditions of real exchange rates q_t and the money supply Mt. Therefore, we can only examine the local properties of (40). We now proceed to analyze the properties of the linearized version of the system. The eigenvalues of the Jacobian matrix DF evaluated at the stationary state $\bar{q}$ and $\bar{M}$ are the solutions of the equation

$$|A_0\lambda^2 + B_0\lambda + B_1| = 0, \tag{41}$$

where

$$A_0 = \frac{\partial F}{\partial w_{t+1}} = \begin{bmatrix} \frac{\partial f}{\partial q_{t+1}} & \frac{\partial f}{\partial M_{t+1}} \\ \frac{\partial g}{\partial q_{t+1}} & \frac{\partial g}{\partial M_{t+1}} \end{bmatrix},$$

$$B_0 = \frac{\partial F}{\partial w_t} = \begin{bmatrix} \frac{\partial f}{\partial q_t} & \frac{\partial f}{\partial M_t} \\ \frac{\partial g}{\partial q_t} & \frac{\partial g}{\partial M_t} \end{bmatrix},$$

$$B_1 = \frac{\partial F}{\partial w_{t-1}} = \begin{bmatrix} \frac{\partial f}{\partial q_{t-1}} & \frac{\partial f}{\partial M_{t-1}} \\ \frac{\partial g}{\partial q_{t-1}} & \frac{\partial g}{\partial M_{t-1}} \end{bmatrix}.$$

After some computations, (41) finally reduces to a third-order equation,

$$\bar{q}(\lambda-1)[\lambda^2 + (\frac{\partial X_t^d}{\partial q_t}\frac{\gamma\bar{q}(1+\beta^{\frac{1}{\gamma}})^2}{-\alpha^2\bar{y}\beta^{\frac{1}{\gamma}}(1-\gamma)} - \frac{\gamma(\beta^{\frac{1}{\gamma}} + \frac{2\alpha-2\alpha\gamma+\gamma}{\gamma})}{\alpha(1-\gamma)})\lambda - \frac{\gamma(\frac{\alpha\gamma-\alpha-\gamma}{\gamma} - \beta^{\frac{1}{\gamma}})}{\alpha(1-\gamma)}] = 0. \tag{42}$$

Aside from one root being unity, there are two roots with the values of

$$T = \lambda_1 + \lambda_2 = \frac{\gamma\bar{q}(1+\beta^{\frac{1}{\gamma}})^2}{\alpha^2\bar{y}\beta^{\frac{1}{\gamma}}(1-\gamma)}\frac{\partial X_t^d}{\partial q_t} + \frac{\gamma^{\frac{1}{\gamma}} + 2\alpha - 2\alpha\gamma + \gamma}{\alpha(1-\gamma)}, \tag{43}$$

$$D = \lambda_1 \lambda_2 = \frac{\gamma^{\frac{1}{\gamma}} + \alpha - \alpha\gamma + \gamma}{\alpha(1-\gamma)}, \tag{44}$$

Next, we will use the graphic method introduced by Grandmont et al. (1998) to examine the local properties of the equilibrium dynamic system. The principle of this method is to describe in the plane how T and D in the associated characteristic polynomial (42) vary as functions of some relevant economic parameters. There is an eigenvalue equal to 1 when $D = 1 - T$, which is represented in the plane (T, D) by the line AC in Figure 1. Similarly, one eigenvalue is equal to -1 when (T, D) belongs to the line AB of equation $1 + D + T = 0$. The two local eigenvalues are complex conjugates of modulus 1 on the segment BC of the equation $D = 1$, $|T| \leq 2$ (both roots are nonreal in the open region above the parabola BOC of the equation $T^2 - 4D = 0$). Since both eigenvalues varnish when T and D are zero, the deterministic dynamic system is locally stable (the two eigenvalues have modulus less than 1) if an only if the point (T, D) lies in the interior of the triangle ABC defined by $1 + D - T > 0$, $1 + D + T > 0$, $|D| < 1$. The steady state is a saddle in the region of the plane in between the lines AB and AC defined by $|T| > |1 + D|$. It is a source in all other cases, i.e., when $|T| < |1 + D|$, $|D| > 1$.

The above graphical representation is convenient for studying not only local stability (or indeterminacy) but also local bifurcations, i.e., changes of stability of the steady state resulting from variations of the parameters (hence of T, D) defining the system. Combining the two equations (eq:sumfix) and (eq:productfix), we obtain

$$D = \Delta(T) = T - 1 - \frac{\bar{q}\gamma(1 + \beta^{\frac{1}{\gamma}})^2}{\bar{y}\alpha^2\beta^{\frac{1}{\gamma}}(1-\gamma)} \frac{\partial X_t^d}{\partial q_t},$$

which defines the locus $(T(\gamma), D(\gamma))$. If we fix β and make γ vary from 0 to ∞, we see in Figure 1 that the point (T, D) moves along the lines Δ that paralell $1 + D - T = 0$. A simple geometrical way to look at stability and bifurcations is to locate the position of the line Δ for a given value of γ. If $0 < \gamma < 1$, the lines Δ will be on the right side of the line $1 + D - T = 0$. If $\gamma > 1$, the lines Δ will be on the left side of the line $1 + D - T = 0$. Depending on the values of β, we can have either flip or hopf bifurcations.

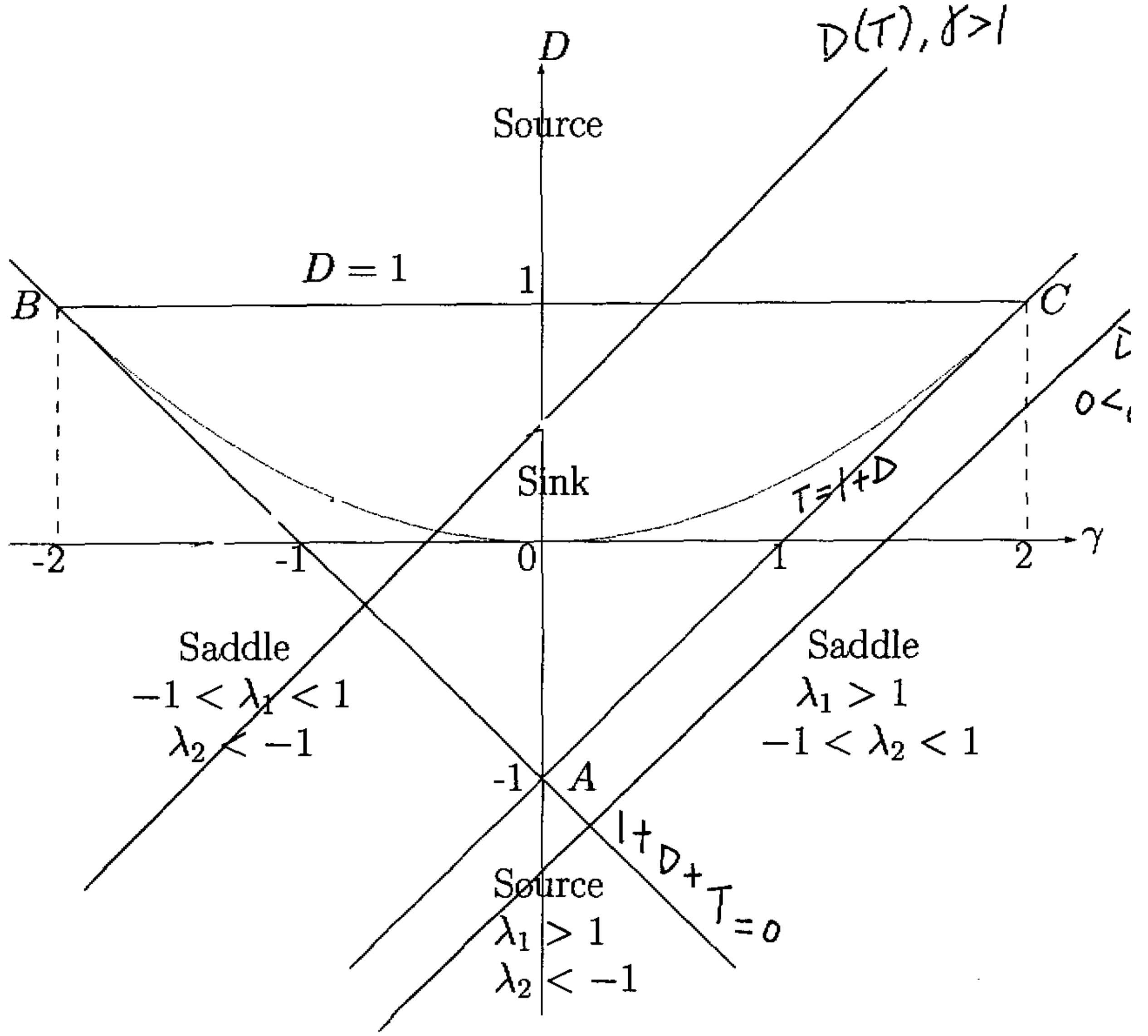

Figure 1: The line Δ

3 COMPARISON WITH THE CLOSED ECONOMY CASE

The structure of our model is most akin to Blanchard and Fisher (1989) (henceafter, B-F) which is a simplified version of Grandmont (1985) model. B-F found that for the system to be non-explosive, γ has to take a value greater than unity. The same condition is obtained in our open economy model. B-F also found that a larger value of β would help the system to converge. Even in the extreme case where utility is Leontif, i.e., $\gamma \to \infty$, $\beta > 1$ would be needed for the steady state equilibrium to be stable. This is in contrast to our finding that for the steady state equilibrium to be stable, the magnitude of β has to be relatively small, i.e., $\beta < \beta_0 < 1$.

In order to facilitate comparison, we write from (2) and (3)

$$m_t = \frac{M_t^d}{P_t} = \bar{y} - x_{1t} - q_t z_{1t},$$

$$m_{t+1} = \frac{M_t^d}{P_{t+1}} = x_{2t+1}^o + q_{t+1} z_{2t+1}.$$

Repeating the derivation of B-F, suppose that the gross rate of return on money π_{t+1} for some reason rises slightly. This will cause m_t to change by the amount of

$$\frac{dm_t}{d\pi_{t+1}} = \frac{\beta^{\frac{1}{\gamma}}(1-\gamma)\bar{y}}{\gamma(1+\beta^{\frac{1}{\gamma}})^2} \gtrless 0 \text{ as } \gamma \lesseqgtr 1, \tag{45}$$

and cause m_{t+1} to change by the amount of

$$\frac{dm_{t+1}}{d\pi_{t+1}} = \frac{\beta^{\frac{1}{\gamma}}(1+\beta^{\frac{1}{\gamma}}\gamma)\bar{y}}{\gamma(1+\beta^{\frac{1}{\gamma}})^2} > 0. \tag{46}$$

Comparison of (45) and (46) indicates that a slight increase of π_{t+1} from its steady state level of unity will always induce an increase in m_{t+1} regardless of $\gamma \gtrless 1$, but will induce an increase in m_t only if $\gamma < 1$. Thus under the condition $\gamma < 1$, both m_t and m_{t+1} will increase with the latter exceeding the former so that $m_{t+1} > m_t > \bar{m}$. $\bar{m}$ is the real balances in the steady state equilibrium. In order to make individuals willing to hold higher real balances in the next period, the gross rate of return on money should be higher than it is in the steady state level. Repeating this shows that real balances will keep on rising. Thus in both closed and open economy models the sequence of real balances will be explosive if $\gamma < 1$.[9]

On the other hand if $\gamma > 1$ it can be immediately seen that m_{t+1} increases as before but m_t decreases. It follows that $m_{t+1} > \bar{m} > m_t$. In order to make individuals willing to hold higher real balances in the next period, the gross rate of return on money should now be lower than its steady state level so that $m_{t+1} > \bar{m} > m_{t+2}$. Repeating this shows that real balances will fluctuate oscillatorily in the neighborhood of the steady state equilibrium, convergently or divergently. The oscillatory movements would be converging if the distance between m_{t+1} and $\bar{m}$ relative to that between m_t and $\bar{m}$ declines over time. This would happen if the rate of discount β is sufficiently small. Dividing (46) by (45), we obtain

$$\frac{dm_{t+1}}{dm_t} = \frac{1 + \beta^{\frac{1}{\gamma}}\gamma}{1 - \gamma}. \tag{47}$$

It is immediately seen that as β decreases, so will the absolute value of $\frac{dm_{t+1}}{dm_t}$. The opposite is true when β increases so that the oscillatory movements would be divergent when β is sufficiently large. The critical point of β can be obtained by equating (47) to minus unity,

$$\beta = \beta_0 = (1 - \frac{2}{\gamma})^{\gamma},$$

which is exactly the same as the bifurcation point β_0 in (22) as it should be. When β falls short of β_0, the oscillation will be damping and the steady state will be locally stable. If β exceeds β_0, the oscillation movements will be divergent and the steady state will be locally unstable. The cyclical equilibria may emerge when β passes through β_0. This is in

[9] In our model, by definition,

$$\frac{1}{\gamma} = -\frac{\frac{1}{\pi_{t+1}}}{m_{t+1}} \frac{dm_{t+1}}{d(\frac{1}{\pi_{t+1}})},$$

and by definition,

$$m_t = \frac{m_{t+1}}{\pi_{t+1}}.$$

Thus,

$$\begin{aligned} \frac{dm_t}{d\pi_{t+1}} &= -\frac{dm_t}{d(\frac{1}{\pi_{t+1}})} = -\frac{d(\frac{m_{t+1}}{\pi_{t+1}})}{d(\frac{1}{\pi_{t+1}})} \\ &= -[m_{t+1} + \frac{1}{\pi_{t+1}} \frac{dm_{t+1}}{d(\frac{1}{\pi_{t+1}})}] \\ &= m_{t+1}(\frac{1}{\gamma} - 1) \lesseqgtr 0 \quad \text{as} \quad \gamma \gtreqless 1. \end{aligned}$$

The condition holds for the open as well as closed economy.

sharp contrast to the findings in the closed-economy case. Setting $\alpha = 1$ and $c^y_{2t} = c^o_{2t} = 0$ for equations (1) – (10) in our model, we can easily derive

$$\frac{dm_{t+1}}{dm_t} = \frac{\frac{1}{\gamma} + \frac{1}{\beta}}{\frac{1}{\gamma} - 1}, \tag{48}$$

and setting (48) equal to minus unity to obtain the bifurcation point,

$$\beta_0 = \frac{\gamma}{\gamma - 2} \geq 1.$$

For the steady state equilibrium to be locally stable, the value of β has to exceed unity; "a preference for the future", in the words of Blanchard and Fischer(p.252). However, in a closed economy model as B-F, a large discounting rate may be destabilizing. Our diametrical opposite result regarding the stability condition (magnitude of β) in the open economy is due to the fact that in the open economy an increase in π_{t+1} has an additional effect which is absent in the closed economy, namely, that real income would decline because the terms of trade would deteriorate with an increase in π_{t+1}.

4 CONCLUDING REMARKS

In this paper we have extended the Samuelson overlapping generations model with money to an open economy under flexible and fixed exchange rates and showed that how cyclical behavior of the real exchange rate (and the domestic price) can emerge from a quite simple economic structure. Indeed, the result of this paper relies on the assumption that residents in the "large" foreign country are risk neutral. This strong assumption enables us to reduce the problem to a framework of a one-country model. Relaxation of this assumption requires a more complicated two-country model, which is beyond the scope of the present work but remains a challenging and interesting task for the future.

References

[1] M. Aloi, H. D. Dixon, and T. Llyod-Braga. Endogenous fluctuations in an open economy with increasing returns to scale. *Journal of Economic Dynamic and Control*, 24:97–125, 2000.

[2] Oiver J. Blanchard and Stanley Fisher. *Lectures on Macroeconomics*. The MIT Press, Cambridge, Massachusetts, 1989.

[3] J. Carr. *Applications of Centre Manifold Theory*, volume 35 of *Applied Mathematical Sciences*. Springer-Verlag, Berlin, 1981.

[4] R. E. A. Farmer. *Macroeconomis of Self-fulfilling Prophecies*. The MIT Press, Cambridge, MA., second edition, 1999.

[5] J. M. Grandmont. On endogenous competitive business cycles. *Econometrica*, 53:995–1045, September 1985.

[6] J. M. Grandmont. Nonlinear difference equations, bifurcations and chaos: An introduction. Lecture series, Institute for Mathematical Studies in the Social Sciences, Stanford University, 1989.

[7] H. W. Lorenz. *Nonlinear Dynamical Economics and Chaotic Motion*. Lecture Notes in Economics and Mathematical Systems 334. Springer-Verlag, New York, 1989.

[8] A. Medio. *Chaotic Dynamics*. Cambridge University Press, Cambridge, 1992.

[9] P. A. Samuelson. An exact consumption loan model of interest with or without the social contrivance of money. *Journal of Political Economy*, 66:1002–1011, 1958.

[10] D. Whitley. Discrete dynamical systems in dimensions one and two. *Bulletin of the London Mathematical Society*, 15:177–217, 1983.

[11] H. H. Zee. Government debt, capital accumulation, and the terms of trade in a model of interdependent economies. *Economic Inquiry*, 25:599–618, 1987.