行政院國家科學委員會專題研究計畫成果報告

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隨機折現因子下的擇時與選股能力模型

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計畫類別：個別型計畫

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計畫主持人：洪茂蔚

執行單位：國立台灣大學國際企業學系

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I. Introduction

There are two ways for managers to achieve abnormal returns. The first is security analysis, which is the ability of fund managers to identify the potential winning securities. The second is market timing, which is the ability of portfolio managers to time market cycles and react. Several market timing and selectivity models have been developed in two lines. Treynor and Mazuy (1966) develop the first quadratic market-timing model to examine market timing ability of fund managers. The intution behind the model is that a fund manager with market timing ability is on average to increase the stock portion of the managed portfolio when stock returns are high and reduce it when stock returns are low. A formal treatment of the quadratic market timing model is found in Jensen (1972), who developed theoretical structures for the evaluation of market timing performance of fund managers. Bhattacharya and Pfliederer (1983) Jensen model by minimizing the variance of the forecasting error. Admati, Bhattacharya, Pfliederer and Ross (1986) use the portfolio approach and factor approach to assess the market timing ability of fund managers.

The second market timing model was developed by Henriksson and Merton (1981) who use a free put option on the market portfolio with its exercise price equal to the risk free rate. In their model, the market timer forecasts either that equities outperform bonds or vise versa. This implied that the probability of receiving an up or a down signal in no way dependents upon how far the market will be up or down. Chang and Henriksson (1984) employ the Henriksson and Merton model in evaluating mutual fund performance and the empirical results did not support the hypothesis that fund managers are able to time the return on the market portfolio successfully. Jagannathan and Korajczyk (1986) offer explanations for apparent perverse timing involving possible option-like characteristics of fund returns.

In the next section we develop a new performance measure, which includes a variety of existing market timing models and is not limited by the benchmark portfolio. This method explores a convex stochastic discount factor to explain market timing effect and also do not use asset pricing as a benchmark to form a mutual fund evaluation.

II. A General Representation of Performance Measurement

This section presents a general representation of performance measure and discuss the assumption that are needed to derive the market timing ability of a fund manager. For convenience, consider an economy endowed with a probability space $\{\Omega, F, \Pr\}$. Let L^2 be the linear space of square-integrable random variables on the probability space. When equipped with the mean-square inner product, L^2 is known to be a Hilbert space.

For simplicity, portfolios are assumed to be purchased at time *t*, and the payoffs to the portfolios are received at time $t+1$. x_{t+1} is denoted as the $N \times 1$ vector of payoff under consideration. Uninformed investors with information set F_t are assumed to take passive strategies:

$$P_t = E[m_{t+1} x_{t+1} | F_t] \tag{1}$$

The return on any mutual fund portfolio which engage in passive strategies can be represented as

$$E[m_{t+1} R_{t+1} | F_t] = 1 \tag{2}$$

where m_{t+1} is the stochastic discount factor and $R_{t+1} = \frac{x_{t+1}}{P_t}$

Informed investors faces much better information G_t, which help them design profitable trading strategies.

$$E[m_{t+1} x_{t+1} | G_t] > P_t \tag{3}$$

The return of an informed investor with market timing ability can also be expressed as

$$E([m_{t+1} + C(m_{t+1})]R_{t+1}) = 1 \tag{4}$$

where $C(m_{t+1})$ is the convex function of stochastic discount factor.

Suppose that there is a riskless portfolio. This mean that there exists some portfolio θ with constant return, R^0, which is riskless return. After a bit of algebra, we can show that for any portfolio θ. We have

$$ER^d - R^0 = -\frac{Cov(R^d, m)}{Em + E[C(m)]} - \frac{Cov(R^d, C(m))}{Em + E[C(m)]} \quad (5)$$

Consequently, covariance with m and covariance with $C(m)$ have a negative effect on expected return.

There is always a portfolio θ^* solving the problem

$$\sup_{\theta} corr(\theta' X, m) \quad (6)$$

Assume that such a portfolio θ^* with a return R^*. If market are complete, then R^* is perfectly correlated with the stochastic discount factor, m, and $C(R^*)$ is also perfectly correlated with the convex function of stochastic discount factor, $C(m)$.

Theorem: *For any return R^d of informed investor with market timing,*

$$E(R^d) - R^0 = \alpha_\theta + \beta_\theta[E(R^*) - R^0] + \gamma_\theta\{E[C(R^*)] - R^0\} \quad (7)$$

where $\alpha_\theta = E(R^d) - R^0 - \beta_\theta[E(R^*) - R^0] - \gamma_\theta\{E[C(R^*)] - R^0\}$ and $\beta_\theta, \gamma_\theta$ are coefficients of multi-regression.

Equation (7) is a general form of market timing model. The first term in right hand side of (7) is the selectivity effect, which is the ability of managers to identify the potential winning securities. The second term is the component that results from the effect of state-price beta model. And the third term is the component that follows from the market timing effect. This is mean that an informed investor with market timing ability will create a little more expected return when return R^* deviates up or down from $E(R^*)$.

In addition, we can easily to see that if the return R^d of informed investor lies in the passive portfolio payoff space PS^P, then $E(m R^d) = 1$ and $E[C(m) R^d] = 0$. In other words, for every $R \in PS^P$, $E(m R) = 1$ and $E[C(m) R] = 0$.

Next, we seek to identify a new performance to measure abnormal return which include market timing and selectivity.

Corollary: *a new performance to measure abnormal return, which includes market timing and selectivity, can be represented as*

$$PM(R) = E[C(m)(R - R^0)] \tag{8}$$

where $E(m\,R) = 1$ *for any* $R \in PS^p$. *We can show that market-timing model of quadratic and option-like measures are special cases.*

Example 1: Quadratic regression

Treynor and Mazuy (1966), Jensen (1972), Kon and Jen (1979), Battacharya and Pfleiderer (1983), Admarti, Bhattacharya and Pfleiderer (1986), Lehmann and Modest (1987) and Lee and Rahman (1990) proposed a test for market timing ability based on the coefficient γ_p from the regression:

$$R^s - R^0 = \alpha_s + \beta_s * (R^m - R^0) + \gamma_s * (R^m - R^0)^2 + \varepsilon_s \tag{9}$$

and the performance measure with market timing and selectivity can be represented as

$$PM(R^s) = E(R^s) - R^0 - \beta_s [E(R^m) - R^0] \tag{10}$$

where β_s multi-regression coefficient of equation (9).

The convex function of stochastic discount factor implied by the quadratic regression is given by

$$C(m) = a + b\, R^m + c\,(R^m)^2 \tag{11}$$

where a, b, c are constants. Then it is straightforward to verify that

$$PM(R^s) = E[C(m)\, R^s] \tag{12}$$

Example 2: Option-like security

Henriksson and Merton (1981), Henriksson (1984), Chang and Lewellen (1984) and Jagannathan and Korajczyk (1986) assumed a option-like security model for market timing given by

$$R^s - R^0 = \alpha_s + \beta_s*(R^m - R^0) + \gamma_s*\max[0, -(R^m - R^0)] + \varepsilon_s \tag{13}$$

Performance measure of option-like model can be written as

$$PM(R^s) = E(R^s) - R^0 - \beta_s[E(R^m) - R^0] \tag{14}$$

where β_s multi-regression coefficient of equation (13).

We can show that the option-like security model is equivalent to having a convex function of stochastic discount factor representable as

$$C(m) = a' + b'\ R^m + c'\ \max(0, -R^m) \tag{15}$$

where a', b', c' are constants. Then it is straightforward to verify that

$$PM(R^s) = E[C(m)\,(R^s - R^0)] \tag{16}$$

Thus, the option-like market-timing model is also performance measure in our generalized performance measure.

Example 3: Period weighting measure

Grinblatt and Titman (1989) propose the period weighting measures for mutual fund performance of market timing given by

$$E[R^s - R^0] = \alpha_s + \beta_s*E[R^m - R^0] + E[\beta_{s,t}*(R^m - R^0)] \tag{17}$$

Performance measure of period weighting measures can be written as

$$PM(R^s) = E(R^s) - R^0 - \beta_s[E(R^m) - R^0] \tag{18}$$

where β_s multi-regression coefficient of equation (17).

Lemma 1: *If an investor has independent timing and selectivity information and non-increasing Rubinstein absolute risk aversion, then* R^s *is a convex function of* R^m.

We can show that the period weighting measure is equivalent to having a convex function of stochastic discount factor representable as

$$C(m) = f(R^m) \tag{19}$$

where f is a convex function of R^m. Then it is straightforward to verify that

$$PM(R^s) = E[C(m)(R^s - R^0)] \tag{20}$$

Thus, the period weighting measure is also performance measure in our generalized performance measure.

III. Property of Convex Performance Measure

After constructing a convex performance measure, there are several properties of this measure.

Theorem: *Let*

$$PM(R) = E[C(m)(R - R^0)]$$

where $E(m\,R) = 1 \quad \forall R \in PS^p$

Then

(a) $E[C(m)R] = 0$ *for any return in the passive portfolio payoff space. In other words, the performance of an uninformed investor is zero,* $PM(R) = 0$.

(b) *If there exist a convex performance measure, then the stock market admits law of one price.*

(c) *If there exist a positive convex performance measure,* $PM(R) > 0$, *then the stock market is free of arbitrage opportunity and* $C(m) > 0$ *when there exists selectivity or market timing.*

References

Admati, A.R., S. Bhattacharya, and Pfleiderer, P. (1986), "On timing and selectivity. " *Journal of Finance* 41, 715-730.

Bhattacharya, S., and Pfleiderer, P. (1983), "A note on performance evaluation. " Technical Report 714. Stanford, Calif,: Stanford University, Graduate School of Business.

Bollerslev, T.,J. M. Wooldridge (1988), "Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances." *Econometrics Reviews,* 11, 143-172.

Campbell, John Y. (1991), "A variance decomposition for stock returns." *Economic Journal*, 101, 157-179.

Campbell, John Y. (1993), "Intertemporal asset pricing without consumption data." *American Economic Review*, 83, 487-512.

Chang, E. C., and Lewellen, W. G. (1984), "Market timing and mutual fund investment performance." *Journal of Business* 57, 57-72.

Chen and Knez, (1996), *Review of Financial Studies.*

Cochrane, John, (1999), Lecture Notes.

Connor, G., and R. Korajczzyk. (1991), "Performance measure with the arbitrage pricing theory." *Journal of financial Economics* 15, 373-394.

Dybvig, P. ,and S. A. Ross, (1985), "Differencial informance and performance measurement using a security market line." *Journal of Finance* 40, 383-399.

Epstein, Larry G., and S. E. Zin (1991), "Substitution, risk aversion, and the temporal behavior of consumption and asset returns: An empirical analysis." *Journal of Political Economy,* 99, 263-286.

Fama, E. F. (1972), "Components of investment performance." *Journal of Finance* 27, 551-67.

Ferson & Schadt (1996), *Journal of Finance*.

Grinblatt and Titman, (1989), *Review of Financial Studies*.

Henriksson, R. D. (1984), "Market timing and mutual fund performance : An empirical investigation." *Journal of Business* 57, 73-96.

Henriksson, R. D., and Merton, R. C. (1981), "On market timing and investment performance II: Statistical procedures for evaluation forceasting skills." *Journal of Business* 54, 513-33.

Jagannathan, R. and R. A. Korajczyk, (1986), "Assessing the market timing performance of managed portfolios." *Journal of Business* 59, 217-236.

Jensen, M. C. (1972), "Optimal utilization of market forecasts and the evaluation of investment performance. " In G. P. Szego and K. Shell (eds), Mathematical Methods in Investment and Finance. Amsterdam: Elsevier.

Lee, C. F., and S. Rahman (1990), "Market timing, selectivity, and mutual fund performance: An empirical investigation." *Journal of Business,* 63, 261-78.

Lehmann, B. N., and D. M. Modest (1987), "Mutual fund performance evaluation: A comparison of benchmarks and benchmark comparisons." *Journal of Finance,* 42,233-265.

Treynor, J. L., and Mazuy, K. K. (1966), "Can mutual funds outguess the market ? " *Harvard Business Review,* 44, 131-36.