行政院國家科學委員會專題研究計畫　期中進度報告

標的物不可交易下的選擇權評價(1/3)

計畫類別：個別型計畫
計畫編號：NSC92-2416-H-002-018-
執行期間：92 年 08 月 01 日至 93 年 07 月 31 日
執行單位：國立臺灣大學國際企業學系暨研究所

計畫主持人：洪茂蔚

報告類型：精簡報告

處理方式：本計畫可公開查詢

中　華　民　國 93 年 5 月 26 日

標的物不可交易下的選擇權評價 (1/3)
Valuation of Options When Underlying Assets are Non-tradable(1/3)

Abstract

One of the most difficult features of pricing weather derivatives is that the market is incomplete. This project presents a new method for pricing weather derivatives in an incomplete market. The proposed method has two advantages shown in only a few of the models discussed in other weather-related articles. First, the method for pricing assets in an incomplete market is used to overcome the non-tradable feature of the underlying assets of the weather derivatives in which the no-arbitrage method breaks down. Second, an efficient analytical method is incorporated into the proposed model to make the Asian-type payout of weather derivatives much easier to evaluate than by any other numerical methods.

Keywords : weather derivatives, non-traded asset, incomplete market.

1 Introduction

Weather derivatives represent probably one of the most rapidly growing fields in derivatives research. More researchers are now focusing on the field of weather derivatives and have presented various kinds of assumptions and methods for pricing weather derivatives. Cao and Wei (1999) used a Lucas equilibrium framework approach to evaluate weather derivatives and found that a zero market price of risk should be associated with weather derivatives. Pirrong and Jermakyan (1999) proposed a mesh-based method to integrate valuation and risk management for power and weather derivatives. Alaton (2000) constructed a model for pricing weather derivatives with a payout that depends on temperature, and uses historical data to establish a stochastic process that describes the evolution of temperature. Davis (2001) used “marginal substitution value” or the “shadow price” method of mathematical economics to investigate the pricing of weather derivatives.

Pricing weather derivatives raises two difficulties. First, the underlying assets of weather derivatives, such as temperature, humidity, rainfall and snowfall are non-tradable entities, and the non-tradability of these underlying assets prevent the creation of a risk-neutral portfolio through a delta hedging strategy. Methods constructed under the assumption of an incomplete market will be applied to compensate for this drawback. Second, weather derivative contracts, such as HDDs (Heating Degree Days) or CDDs (Cooling Degree Days), exhibit accumulative, geometric Brownian motion. The sum of geometric Brownian motions does not follow a geometric Brownian motion distribution. Pricing an option whose underlying asset follows the accumulation of geometric Brownian motions becomes a type of Asian option which is known to be more complex than ordinary European-type options, making such weather derivatives difficult to price. The literature includes several methods such as the closed-form expression of Geman and Yor (1993) and Milesky and Posner (1998), the numerical methods of Turnbull and Wakeman (1991), Levy and Turnbull (1992) and Vorst (1992, 1996), and the Monte Carlo simulation method.

The purpose of this project is to address these difficulties. In a recent article, Cochrane and Saá-Requejo (2000) proposed the "discount factor" asset pricing method to evaluate uncertain payoffs. This type of valuation lies between the rigidity of model-based pricing and the looseness of no-arbitrage pricing. This method is suitable when preference-free approaches break down, such as non-trade or thin trade circumstances. By additionally restricting the discount factor by both the positivity constraint and the volatility constraint, the boundaries of the asset price have a smaller range than the boundaries of the general arbitrage; such bounds are called "good-deal bounds." Since an exact solution for options that underlie non-tradable assets cannot be found, support is thus drawn from Cochrane and Saá-Requejo (2000) to find "good-deal bounds" for the weather derivatives. The idea of Geman and Yor (1993) is exploited because their formula is a more efficient analytical tool compared to that of other Asian option pricing methods, thus making the proposed weather derivative pricing formula a quasi-explicit one. The proposed formula has a closed form expression and can be used as easily as the Black and Scholes formula to price weather derivatives.

2 The Model

This section proposes a method to price call options, which is built on the assumption of market incompleteness and is applied in the weather market. However, before this method can be applied to call options used in the weather market, the option payoffs of HDD and CDD must be introduced.

The daily HDD and CDD can be modeled as follows:

$$hdd(t) = \max(\kappa - \Upsilon_t, 0) \tag{1}$$

$$cdd(t) = \max(\Upsilon_t - \kappa, 0) \tag{2}$$

where

1. κ is reference temperature, usually set at 65^o F (or 18^o C).
2. Υ_t is the temperature on day t and is defined as the average value of the maximum and the minimum temperatures of day t, $\Upsilon_t = \frac{\Upsilon_t^{(max)} + \Upsilon_t^{(min)}}{2}$

A call option contract used in the weather market is often the accumulation of daily HDDs or CDDs over a specific period. Other kinds of monthly contracts may be the accumulation of daily temperature minus the exercise price specified by the trading parties.

$$H_n = \sum_{t=1}^{n} hdd(t) \tag{3}$$

$$C_n = \sum_{t=1}^{n} cdd(t) \tag{4}$$

Consequently, the payoff of a call option is defined as follows:

$$HDD(T) = \xi \max(H_n - K, 0) \tag{5}$$

$$CDD(T) = \xi \max(C_n - K, 0) \tag{6}$$

where ξ is the nominal dollar amount.

Now that the basic payoff of a weather contract is defined, the call option can be priced using the proposed method. Let the temperature Υ_t follow a geometric Brownian motion. Given the original measure P, it can be expressed as follows:

$$\frac{d\Upsilon_t}{\Upsilon_t} = \mu_{\Upsilon} dt + \sigma_{\Upsilon B} dB_t^P + \sigma_{\Upsilon Z} dZ_t^P. \tag{7}$$

$d\Upsilon_t$ is expressed as the sum of two separate geometric Brownian motion processes that divide the non-traded basis asset Υ_t into one hedgeable part and one non-hedgable part. The hedgeable part is hedged using a proxy asset (S_t) to price it. All price information on the hedgeable part of $d\Upsilon_t$ can be found by pricing dS_t. This proxy asset can be defined as follows:

$$\frac{dS_t}{S_t} = \mu_s dt + \sigma_s dB_t^P. \tag{8}$$

To find a closed-form analytic solution for the call option used in the weather market, the accumulation of Υ_t is expressed in continuous time and the average of Υ_t between time t_0 and time t is defined as $W_n(t)$:

$$W_n(t) = \frac{1}{t} \int_{t_0}^{t} \Upsilon_s ds. \tag{9}$$

The concept of a pricing kernel, M_t, will be used to price weather derivatives. According to this pricing mechanism, the relationship between the price $C_{option,t}$ and the payoff $Max(W_n(T) - K, 0)$ for a call option will be described by the following equation.

$$C_{option,t} = \xi E_t^P[\frac{M_T}{M_t}\max(W_n(T) - K, 0)], \tag{10}$$

where $W_n(T)$ is either the accumulation of the daily HDDs as in Eq. (3) or daily CDDs as in Eq. (4) on the day of expiration. Restated, the exercise price K is subtracted from the terminal value of the accumulated daily index $W_n(T)$ and then discounted back by $\frac{M_T}{M_t}$ to derive the price of the call option. In Equation (10), ξ represents dollars per unit and it is always a constant in the proposed model. For simplicity in calculation, we set it to 1 throughout the remaining part of this article.

Imposing the volatility constraint and positivity constraints on the stochastic discount factor enables the lower bound of the call option to be derived by solving the following minimization problem:

$$\underline{C}_{option,t} = \min_M E_t\left[\frac{M_T}{M_t}\max(S_T - K, 0)\right], \tag{11}$$

$$st \;\; W_n(t) = E_t^P\left(\frac{M_u}{M_t}W_n(u)\right) \;\; ; \;\; M_u > 0 \;\; ; \;\; \frac{1}{dt}E_t^P\frac{dM^2}{M^2} \le A^2, \;\; t \le u \le T, \tag{12}$$

where A is the volatility constraint. Similarly, the upper bound of the call option can be derived by solving the corresponding maximization problem. These bounds are called the "good-deal" bounds for the call option.

The pricing kernel in this economy is

$$\frac{dM_t}{M_t} = -rdt - h_s dB_t^P \pm \sqrt{A^2 - h_s^2}dZ_t^P.$$

Hence, the lower bound of the call option that is priced when the market is incomplete can be represented as follows:

$$\underline{C}_{option,t} = E_t^P[\frac{M_T}{M_t}\max(W_n(T) - K, 0)] \tag{13}$$

$$= E_t^P[e^{-r(T-t)-\frac{1}{2}A^2(T-t)-h_s B_{T-t}^P - \sqrt{A^2-h_s^2}Z_{T-t}^P}\max(W_n(T) - K, 0)].$$

Equation (13) can be expressed as follows:

$$\underline{C}_{option,t} = E_t^Q[e^{-r(T-t)}\max(W_n(T) - K, 0)] \tag{14}$$

where $d\Upsilon_t$ after changing the measure, satisfies

$$\frac{d\Upsilon_t}{\Upsilon_t} = \mu_\Upsilon dt + \sigma_{\Upsilon B} dB_t^P + \sigma_{\Upsilon Z} dZ_t^P$$

$$= (\mu_\Upsilon - \sigma_{\Upsilon z} h_s - \sigma_{\Upsilon Z}\sqrt{A^2 - h_s^2})dt + \sigma_{\Upsilon B} dB_t^Q + \sigma_{\Upsilon Z} dZ_t^Q.$$

The payoff of Asian option $W_n(T)$ with strike price K, maturity T, and averaging period $[t_0,\ T]$ in Eq. (14) can be rewritten as follows:

$$\underline{C}_{option,t} = e^{-r(T-t)}E_t^Q[\max(W_n(T) - K, 0)] \tag{15}$$

where $W_n(T) = \frac{1}{T-t_0}\int_{t_0}^{T}\Upsilon_u du$.

It can be shown that the payoff of the Asian option can be expressed as

$$\underline{C}_{option,t} = E_t^Q[e^{-r(T-t)}\max(W_n(T) - K, 0)]$$

$$= \frac{e^{-r(T-t)}}{T - t_0}\frac{4\Upsilon_t}{\sigma_\Upsilon^2}c^{(\delta)}(h, q),$$

where

$$\sigma_{\Upsilon}^2 = \sigma_{\Upsilon B}^2 + \sigma_{\Upsilon Z}^2$$

$$\eta = \left[h_{\Upsilon} - h_s(\rho - a\sqrt{\frac{A^2}{h_s^2} - 1}\sqrt{1-\rho^2}) \right] \sigma_{\Upsilon}$$

$$\rho = corr\left(\frac{d\Upsilon}{\Upsilon}, \frac{dS}{S}\right) = \frac{\sigma_{\Upsilon B}}{\sigma_{\Upsilon}}$$

$$a = \begin{cases} +1 \text{ upper bound} \\ \\ \text{-1 lower bound} \end{cases}$$

$$\delta = \frac{2(r+\eta)}{\sigma_{\Upsilon}^2} - 1$$

$$h = \frac{\sigma_{\Upsilon}^2}{4}(T-t)$$

$$q = \frac{\sigma^2 \hat{K}}{4} = \frac{\sigma^2(\tilde{K} - \tilde{W}_n(t))}{4\Upsilon_t} = \frac{\sigma^2}{4\Upsilon_t}[K(T-t_0) - \int_{t_0}^{t} \Upsilon_u du].$$

When $q < 0$,

$$c^{(\delta)}(h,q) = \frac{1}{2(1+\delta)}[e^{2(1+\delta)h} - 1] - q.$$

When $q > 0$,

$$\int_0^\infty e^{-\lambda h} c^{(\delta)}(h,q)dh = \frac{\int_0^{\frac{1}{2q}} e^{-t} t^{\frac{\mu-\delta}{2}-2}(1-2qt)^{\frac{\mu+\delta}{2}+1}dt}{\lambda(\lambda-2-2\delta)\Gamma(\frac{\mu-\delta}{2}-1)},$$

where $\mu = \sqrt{2\lambda + \delta^2}$ and $\Gamma(\cdot)$ is the gamma function.

3 Conclusion

A new method is proposed to price weather derivatives. The proposed model explained in this article can function in an incomplete market and overcome the difficulty of determining the payoff functions of weather derivatives over a strike period. This feature, a characteristic of Asian-style derivatives, was once thought to raise a complex problem for any pricing model. The fact that the proposed model also has a closed form solution facilitating pricing, it is easier to use than other simulation pricing methods, such as the Monte Carlo method.