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The goal of this project is to propose a new compound option model constructed in the incomplete market to suit the non-tradeable characteristics of R&D projects. The incorporation of the compound option is used to capture the multistage nature of the activity. The value of an R&D investment is not primarily determined by the cash flows coming from the initial investment but by the future investment opportunities provided by the original investment. Therefore, using the compound option to handle this compound effect is more appropriate than general Black-Scholes methodology. To capture the non-tradeability of R&D projects, we draw support from Cochrane and Saá-Requejo (2000) to solve this incomplete market problem.

The Model

This section develops a method to evaluate a compound option with a non-tradeable underlying asset. Development of this model is motivated by the fact that almost all kinds of R&D projects include multiple-stage investments, each installment of which is an option on the subsequent stage. Few projects can be finished in a single stage, and investment cost can be determined in the beginning of the project. Hence, before we construct our model, some features about compound options should be introduced.

A. Compound Option

A compound option proposed by Geske in 1979 is one kind of derivative, whose underlying asset is another option. A call on call compound option has two strike prices, K_V and K_S and two exercise times t and T. Geske established the model by treating the firm's stock as an option on the value of the firm, and then the option on the stock became a compound option with respect to firm value. Let CC be the compound option, S be the stock of firm and V be the value of the firm. Then, the partial differential equation of a compound option can be expressed as follows:

$$-rCC - rV\frac{\partial CC}{\partial V} - \frac{1}{2}\sigma_V^2 V^2 \frac{\partial^2 CC}{\partial V^2} = 0. \tag{1}$$

At the expiration date of this option, t, the value of the call is either zero if the stock price S_t is less than or equal to the exercise price, K_s, or is equal to the difference between the stock price and the exercise price if the stock price is greater than the exercise price. Hence, the boundary condition of the first option at time t can be expressed as follows:

$$CC_t = \max(0, S_t - K_S, 0). \quad (2)$$

Since the stochastic variable determining the option's value in Equation (1) is not the stock price, S, which enters the boundary condition, but the firm value V, we can not find the option value from Equation (2) directly. Because the stock is the option on the firm value, V, by Ito's lemma we can find the partial differential equation between S and V as follows:

$$-rS - rV\frac{\partial S}{\partial V} - \frac{1}{2}\partial_V^2 V^2 \frac{\partial^2 S}{\partial V^2} = 0. \quad (3)$$

The boundary condition of Equation (4) can be expressed as follows:

$$S_T = \max(0, V_T - K_V, 0). \quad (4)$$

Solving Equation (1) and Equation (3), the value of a compound option can be expressed as follows:

$$CC = VN_2(x, y; \rho) - K_v e^{-rT} N_2(x - \sigma\sqrt{t}, y - \partial\sqrt{T}; \rho) - K_S e^{-rt} N(x - \sigma\sqrt{t})$$

Where

$$x = \frac{\ln\frac{V}{\overline{V}} + (r - \frac{1}{2}\sigma_V^2)t}{\sigma_V\sqrt{t}}$$

$$y = \frac{\ln\frac{V}{K_V} + (r - \frac{1}{2}\sigma_V^2)T}{\sigma_V\sqrt{T}}$$

$\overline{V}$ is the value of V such that

$$S_{T-t} - K_S = VN(x) - K_V e^{-r(T-t)} N(y - \sigma_V\sqrt{T-t}) - K_S = 0$$

B. Compound Option Under Incomplete Market

Since the underlying asset (as the R&D project in this case) is non-tradable, perfect hedging is impossible; hence, the above "no arbitrage" methods that were employed in the Black-Scholes formula are not applicable. The idea behind "approximate hedging" can, however, be used to price options whose underlying asset is non-tradable. Following Cochrane and Saá-Requejo (2000), weak restrictions are imposed on the discount factor to rule out arbitrage opportunities and high Sharpe ratios. Imposing the volatility constraint and positivity constraints on the stochastic discount factor enables the lower bound of the compound option to be derived by solving the following minimization problem.

$$CC = \min_{\Lambda} E_0^P \left[\frac{\Lambda_t}{\Lambda_0} \max\,(S_t - K_S, 0) \right],$$

$$s.t.\ \ S_t = E_t^P \left(\frac{\Lambda_T}{\Lambda_t} \max\,(V_T - K_V,\, 0) \right);\ \Lambda_t > 0;\ \frac{1}{dt} E_t^P \frac{d\Lambda^2}{\Lambda^2} \le A^2;\ 0 \le t \le T, \qquad (5)$$

Where A is the volatility constraint, K_S is the strike price of compound option with expiration t, K_V is the strike price of the underlying option with expiration T. Similarly, the upper bound for the price of the call option can be derived by solving the corresponding maximization problem. Those bounds are called the "good-deal" bounds for the compound option.

By treating the first option as the underlying asset, we can find the value of the R&D project by using the following compound option model:

$$CC = Ve^{\eta T} N_2(x, y; \rho) - K_v e^{-rT} N_2(x - \sigma\sqrt{t}, y - \sigma\sqrt{T}; \rho) - K_s e^{-rt} N(x - \sigma\sqrt{t}) \qquad (6)$$

where

$$x = \frac{\ln \frac{V}{\overline{V}} + (r + \eta - \frac{1}{2}\sigma_V^2)t}{\sigma_V \sqrt{t}}$$

$$y = \frac{\ln \frac{V}{K_V} + (r + \eta - \frac{1}{2}\sigma_V^2)T}{\sigma_V \sqrt{T}}$$

$$\sigma_V^2 \equiv E_t \frac{dV_T^2}{V_t^2} = \sigma_{vz}^2 + \sigma_{vw}^2$$

$$\eta \equiv \left[h_v - h_s \left(\rho_1 - a\sqrt{\frac{A^2}{h_s^2} - 1}\sqrt{1-\rho_1^2} \right) \right] \sigma_V ,$$

$$h_\lambda \equiv \frac{\mu_\lambda - r}{\sigma_\lambda};\ h_v \equiv \frac{\mu_v - r}{\sigma_v}$$

$$\rho = \sqrt{\frac{t}{T}}$$

$$\rho_1 \equiv corr\left(\frac{dV}{V}, \frac{d\lambda}{\lambda} \right) = \frac{\sigma_{vz}}{\sigma_v},$$

$$a = \begin{cases} +1 & upper\ bound \\ -1 & lower\ bound. \end{cases}$$

Conclusion

Using the real option pricing model to price R&D projects is becoming more popular than traditional methodologies (DCF, DTA, etc.). Since R&D projects are considered as something new, they are usually non-traded assets. Although, numerous articles mention this feature of R&D projects, few articles solve this difficulty by concretely proposing a suitable methodology. We propose a new compound option model that is constructed in the incomplete market to suit the non-tradeable feature of R&D projects. There are two advantages of our model. First, we incorporate Geske's model to capture the multistage characteristic of R&D projects. Second, we set up a model in an incomplete market to capture another characteristic of R&D projects as non-tradeable assets.