行政院國家科學委員會專題研究計畫　期中進度報告

投資人行為與資產訂價(1/3)

計畫類別： 個別型計畫
計畫編號： NSC93-2416-H-002-031-
執行期間： 93 年 08 月 01 日至 94 年 07 月 31 日
執行單位： 國立臺灣大學國際企業學系暨研究所

計畫主持人： 洪茂蔚

報告類型： 精簡報告

處理方式： 本計畫可公開查詢

中　華　民　國 94 年 5 月 27 日

Based on Odean (1998), and Barberis and Thaler (2002), we build a disposition effect related theoretical model in this project that is based upon the irrational belief in mean reversion. This is motivated by how financial economists construct their theoretical models for the disposition effect based on the prospect theory; only few or even none put emphasis on irrational belief in mean reversion so far. Therefore, our main goal is to try to build a simple theoretical model based on irrational belief in mean reversion for better interpretation of the disposition effect. In addition to the mean reversion belief, the concept of cognitive reference price level from Grinblatt and Han (2001), and Odean (1998) is also employed into our setup of the mean reversion process. This helps investors judge whether they have cognitive paper losses or gains. Besides, similar to Dumas (1989), and Kogan and Uppal (2000, 2002) we incorporate heterogeneous agents into the economy. This completes our model and brings about various dynamic results.

The Disposition Effect Model

Following Dumas (1989), Odean (1998), Kogan and Uppal (2000), Grinblatt and Han (2001), Campbell and Viceira (2002), Chang and Hung (2002), and Wirjanto (2004), we provide a fundamental consumption-based theoretical model to elucidate the disposition effect in the asset market.

Investment Opportunity Set and the Economy

We assume that there are only two different assets in our economy. One is the risk-free asset in the bond market and the other is the risky asset in the stock market. We select the aggregate wealth share as the state variable and adopt the power utility over consumption with different constant relative risk aversion for two heterogeneous investors. We setup our model as follows:

For type I agent who is the disposition investor:

$$\mathrm{U}(C_1) = \frac{C_1^{1-\gamma_1}}{1-\gamma_1} \tag{1}$$

$$\frac{\mathrm{d}\,P_t}{P_t} = [\mu + \lambda_1 (1 - \frac{P_t}{R_t})]\,\mathrm{d}\,t + \sigma\,\mathrm{d}\,Z \tag{2}$$

For type II agent who is the rational investor:

$$\mathrm{U}(C_2) = \frac{C_2^{1-\gamma_2}}{1-\gamma_2} \tag{3}$$

$$\frac{\mathrm{d}\,P_t}{P_t} = \mu\,\mathrm{d}\,t + \sigma\,\mathrm{d}\,Z \tag{4}$$

and for both agents:

$$\frac{\mathrm{d}\,B_t}{B_t} = r_t\,\mathrm{d}\,t \tag{5}$$

$$ds = \bar{\kappa}(\bar{s} - s)dt + \sigma_s dZ_s \tag{6}$$

where $s = \dfrac{W_1}{W_1 + W_2}$ and subscripts denote the types.

Equation (1) and (2) are for the disposition investor while equation (3) and (4) are for the rational investor. Two agents are set to be heterogeneous not only in preference but also in expected price dynamics (in beliefs) outset. Equation (5) and (6) are the same for both agents. In equations (1) and (3), the power utility over consumption with different constant relative risk aversion, γ_1 and γ_2 are for the disposition investor and for the rational investor, respectively. In terms of the rational investor, the risky asset evolves according to equation (4) which follows a geometric Brownian motion; where the diffusion term, σ, presents the instantaneous volatility of the risky asset and the drift term, μ presents the instantaneous return of the risky asset. In contrast to the rational investor, the disposition investor measures (believes) the evolution of risky asset according to equation (2), which is almost the same as equation (4) with the exception of the drift term. In the drift term of equation (2), besides the constant return μ, there is one other excess premium component, $\lambda_1(1-\frac{P_t}{R_t})$; where $\lambda_1 \geq 0$, and P_t, R_t present the current risky asset price and the cognitive reference price level, respectively P_t varies with the stock price while R_t is fixed initially by the disposition investor.

The Optimal Consumption Policies and Portfolio Choices

We use an imaginary social planner to demonstrate the maximization problem of the whole economy. In our economy, the planner wants to maximize the following inter-temporal expected welfare utility and is subject to the inter-temporal budget constraint.

$$\max_{\{C_1, C_2, \alpha_1, \alpha_2\}} \mathrm{E}_0 \{ \int_0^\infty e^{-\beta t} [\theta \frac{C_1^{1-\gamma_1}}{1-\gamma_1} + (1-\theta) \frac{C_2^{1-\gamma_2}}{1-\gamma_2}] \mathrm{d}t \} \tag{7}$$

subject to

$$\mathrm{d}W = \mathrm{d}(W_1 + W_2) = \{[\alpha_1(\mu + \lambda_1(1 - \frac{P_t}{R_t}) - r_t) + r_t]W_1 - C_1\}\mathrm{d}t + \alpha_1 W_1 \sigma \mathrm{d}Z$$

$$+ \{[\alpha_2(\mu - r_t) + r_t]W_2 - C_2\}\mathrm{d}t + \alpha_2 W_2 \sigma \mathrm{d}Z \tag{8}$$

where C_1, C_2 and W_1, W_2 present the consumption and the wealth of the disposition investor and of rational investor, respectively. E is the expectation operator. β is the individual discount rate which is set to be constant over time. θ and $(1-\theta)$ are the planner's subjective weights of the disposition investor and of the rational investor, respectively. Finally, α_1 and α_2 denote the portfolio choices invested in the risky asset of the disposition investor and of the rational investor, respectively.

Following Campbell and Viceira (2002), it can be shown that the optimal consumption policies and portfolio choices for each investor are:

$$c_1 - w_1 = -(a_0 + a_1 s + a_2 s^2 + a_3 r + a_4 r^2 + a_5 rs) + \frac{1}{\gamma_1}\log\theta$$

$$c_2 - w_2 = -(b_0 + b_1 s + b_2 s^2 + b_3 r + b_4 r^2 + b_5 rs) + \frac{1}{\gamma_2}\log(1-\theta)$$

$$\alpha_1 = \frac{1}{\gamma_1}\frac{(\mu - r)}{\sigma^2} + \frac{1}{\gamma_1}\frac{\lambda_1(1-\frac{P}{R})}{\sigma^2} + (\frac{2A_2 - A_0\phi_1}{2A_2(A_1 + A_4)})[\frac{\sigma_s}{\sigma}\rho r + A_2(s-\bar{s}) - A_3]\frac{\sigma_s}{\sigma}\rho$$

$$= \alpha_1^M + \alpha_1^{IB} + \alpha_1^H$$

$$\alpha_2 = \frac{1}{\gamma_2}\frac{(\mu - r)}{\sigma^2} + (\frac{2B_2 - B_0\psi_1}{2B_2(B_1 + B_4)})[\frac{\sigma_s}{\sigma}\rho r + B_2(s-\bar{s}) - B_3]\frac{\sigma_s}{\sigma}\rho$$

$$= \alpha_2^M + \alpha_2^H$$

where

$$a_0 = (A_0\phi_1)^{-1}\{A_0(\phi_0 + \frac{\phi_1\log\theta}{\gamma_1}) - \frac{[\mu + \lambda_1(1-\frac{P}{R})]^2}{2\gamma_1\sigma^2} + \frac{\beta}{1-\gamma_1} + \frac{2A_2 - A_0\phi_1}{2A_2(A_1 + A_4)}[(A_2\bar{s} + A_3)^2(\frac{A_0\phi_1}{2A_2}) - A_2A_4]\}$$

$$a_1 = \frac{(2A_2 - A_0\phi_1)}{2A_2(A_1 + A_4)}(-A_2\bar{s} - A_3)$$

$$a_2 = \frac{2A_2 - A_0\phi_1}{4(A_1 + A_4)}$$

$$a_3 = (-A_0\phi_1)^{-1}\{[1 - \frac{\mu + \lambda_1(1-\frac{P}{R})}{\gamma_1\sigma^2}] + (\frac{\sigma_s}{\sigma}\rho)(\frac{2A_2 - A_0\phi_1}{2A_2(A_1 + A_4)})(A_2\bar{s} + A_3)(\frac{A_0\phi_1}{A_2})\}$$

$$a_4 = (-A_0\phi_1)^{-1}[\frac{1}{2\gamma_1\sigma^2} - (\frac{\sigma_s}{\sigma}\rho)^2(\frac{2A_2 - A_0\phi_1}{2A_2(A_1 + A_4)})(\frac{A_0\phi_1}{2A_2})].$$

$$a_5 = \frac{2A_2 - A_0\phi_1}{2A_2(A_1 + A_4)}\frac{\sigma_s}{\sigma}\rho$$

$$b_0 = (B_0\psi_1)^{-1}\{B_0[\psi_0 + \frac{\psi_1 \log(1-\theta)}{\gamma_2}] - \frac{\mu^2}{2\gamma_2\sigma^2} + \frac{\beta}{1-\gamma_2} + (\frac{2B_2 - B_0\psi_1}{2B_2(B_1 + B_4)})[(B_2\bar{s} + B_3)^2(\frac{B_0\psi_1}{2B_2}) - B_2B_4]\}$$

$$b_1 = \frac{(2B_2 - B_0\psi_1)}{2B_2(B_1 + B_4)}(-B_2\bar{s} - B_3)$$

$$b_2 = \frac{2B_2 - B_0\psi_1}{4(B_1 + B_4)}$$

$$b_3 = (-B_0\psi_1)^{-1}\{[1 - \frac{\mu}{\gamma_2\sigma^2}] + (\frac{\sigma_s}{\sigma}\rho)(\frac{2B_2 - B_0\psi_1}{2B_2(B_1 + B_4)})(B_2\bar{s} + B_3)(\frac{B_0\psi_1}{B_2})\}$$

$$b_4 = (-B_0\psi_1)^{-1}[\frac{1}{2\gamma_2\sigma^2} - (\frac{\sigma_s}{\sigma}\rho)^2(\frac{2B_2 - B_0\psi_1}{2B_2(B_1 + B_4)})(\frac{B_0\psi_1}{2B_2})]$$

$$b_5 = \frac{2B_2 - B_0\psi_1}{2B_2(B_1 + B_4)}\frac{\sigma_s}{\sigma}\rho$$

$$A_0 = \frac{-\gamma_1\theta^{\frac{1}{\gamma_1}}}{1-\gamma_1},\quad A_1 = \frac{\gamma_1\sigma_s^2(\rho^2 - 1)}{2},\quad A_2 = \frac{\gamma_1\bar{\kappa}}{1-\gamma_1},\quad A_3 = \frac{\sigma_s}{\sigma}\rho[\mu + \lambda_1(1 - \frac{P}{R})],\quad A_4 = \frac{\gamma_1\sigma_s^2}{2(1-\gamma_1)}$$

$$B_0 = \frac{-\gamma_2(1-\theta)^{\frac{1}{\gamma_2}}}{1-\gamma_2},\quad B_1 = \frac{\gamma_2\sigma_s^2(\rho^2 - 1)}{2},\quad B_2 = \frac{\gamma_2\bar{\kappa}}{1-\gamma_2},\quad B_3 = \frac{\sigma_s}{\sigma}\rho\mu, \text{ and } \quad B_4 = \frac{\gamma_2\sigma_s^2}{2(1-\gamma_2)}$$

and $\phi_0 = e^{\overline{c_1 - w_1}}(\theta^{\frac{-1}{\gamma_1}})[1 - \overline{c_1 - w_1}]$, $\phi_1 = \theta^{\frac{-1}{\gamma_1}} e^{\overline{c_1 - w_1}}$, $\psi_0 = e^{\overline{c_2 - w_2}}(1-\theta)^{\frac{-1}{\gamma_2}}[1 - \overline{c_2 - w_2}]$, $\psi_1 = (1-\theta)^{\frac{-1}{\gamma_2}} e^{\overline{c_2 - w_2}}$. The small letters of consumption and wealth present their corresponding log values.

Conclusion

We successfully model the higher portfolio choice and the disposition effect when the stock price is low. We also found out that higher cognitive reference price level, greater magnitude of irrational belief in mean reversion and less risk aversion attitude all strengthen the disposition effect.