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Abstract

An essential issue in designing, operating and managing modern networks is to assure end-to-end (both unicast and multicast modes) Quality-of-Service (QoS) from users' perspective, and in the meantime to optimize certain performance objectives from the system's perspective. In this second year final report, two near-optimal QoS-based unicast and multicast routing algorithms, respectively, developed by mathematical programming techniques are proposed for broadband Internet.

Contribution and Discussion

1. QoS-based Routing

The routing problem in virtual circuit networks has been a traditional research topic in computer networks and has attracted even more attention since the emergence of the Asynchronous Transfer Mode (ATM) technology. To ensure user-perceived end-to-end QoS requirement (e.g. mean packet delay, packet delay jitter and packet lost probability) and achieve good system-level performance measure (e.g. overall network utilization or average cross-network delay) are important to the user and the system operator. However, these two perspectives/ objectives may not be entirely agreeable with each other. This then places a major challenge to network managers and therefore calls for an integrated metho-dology to consider these two pers-pectives in a joint fashion.

Problem Formulation And Solution Approaches

The virtual circuit network is modeled as graph where the processors are depicted as nodes and the communication channels are depicted as arcs. We show the definition of the following notation.

V	$=\{1,2,\ldots,N\}$, the set of nodes in the graph
L	the set of communication links in the communication network
W	the set of Origin-Destination (O-D) pairs in the network
¥$_w$	(packets/sec): the arrival rate of new traffic for each O-D pair $w\in W$, which is modeled as a Poisson process for illustration purpose
C_l	(packets/sec), the capacity of each link $l\in L$
P_w	a given set of simple directed paths from the origin to the destination of O-

	D pair w
x_p	a routing decision variable which is 1 when path $p \in P_w$ is used to transmit the packets for O-D pair w and 0 otherwise
$¢_{pl}$	the indicator function which is 1 if link l is on path p and 0 otherwise
g_l	the aggregate flow over link l, which is $\sum_{p \in P_w} \sum_{w \in W} x_p ¥_w ¢_{pl}$
D_w	the maximum allowable end-to-end delay for O-D pair w

$$Z_{IP''} = \min \frac{1}{\sum_{w \in W} ¥_w} \sum_{l \in L} \frac{g_l}{C_l - g_l} \qquad \text{(IP”)}$$

subject to:

$$\sum_{l \in L} \sum_{p \in P_w} \frac{x_p ¢_{pl}}{C_l - g_l} \le D_w \qquad \forall w \in W \quad (1.1)$$

$$g_l = \sum_{p \in P_w} \sum_{w \in W} x_p ¥_w ¢_{pl} \le C_l \quad \forall l \in L \qquad (1.2)$$

$$\sum_{p \in P_w} x_p = 1 \qquad \forall w \in W \qquad (1.3)$$

$$x_p = 0 \text{ or } 1 \qquad \forall p \in P_w, w \in W. \qquad (1.4)$$

Constraint (1.1) requires that the end-to-end packet delay should be no larger than D_w for each O-D pair. Constraint (1.2) requires that the aggregate flow on each link should not exceed the link capacity. Constraints (1.3) and (1.4) require that the all the traffic for each O-D pair should be transmitted over exactly one path. The above formulation is a nonlinear multicommodity flow problem, since each O-D pair transmits different type of traffic over the network. And it is easy to show that (IP”) is a nonconvex programming problem by verifying the Hessian of $\sum_{l \in L} \sum_{p \in P_w} \frac{x_p ¢_{pl}}{C_l - g_l}$ with respect to x_p.

For the purpose of applying Lagrangean relaxation method, we transform the above problem formulation (IP”) into an equivalent formulation (IP). In (IP), two auxiliary variables are introduced: y_{wl} is defined as $\sum_{p \in P_w} x_p ¢_{pl}$ and f_l denotes the estimate of the aggregate flow.

$$Z_{IP} = \min \frac{1}{\sum_{w \in W} ¥_w} \sum_{l \in L} \frac{f_l}{C_l - f_l} \qquad \text{(IP)}$$

subject to:

$$\sum_{l \in L} \frac{y_{wl}}{C_l - f_l} \le D_w \qquad \forall w \in W \qquad (2.1)$$

$$\sum_{p \in P_w} x_p = 1 \qquad \forall w \in W \qquad (2.2)$$

$$x_p = 0 \text{ or } 1 \qquad \forall p \in P_w, w \in W \qquad (2.3)$$

$$\sum_{p \in P_w} x_p ¢_{pl} \le y_{wl} \qquad \forall w \in W, l \in L \quad (2.4)$$

$$y_{wl} = 0 \text{ or } 1 \qquad \forall w \in W, l \in L \qquad (2.5)$$

$$g_l \le f_l \qquad \forall l \in L \qquad (2.6)$$

$$0 \le f_l \le C_l \qquad \forall l \in L. \quad (2.7)$$

Redundant constraints associated with these auxiliary variables from (2.4) to (2.7) are added. Note that Constraints (2.4) and (2.6) should be equalities, and it is clear that the equality should hold at the optimal point. By introducing these auxiliary variables, the Lagrangean relaxation problem can be decomposed into independent and easily solvable subproblems.

The algorithm development is based upon Lagrangean relaxation. We dualize Constraints (2.1), (2.4) and (2.6) to obtain the following Lagrangean relaxation problem (LR).

$$Z_D(t,u,v) = \min[\frac{1}{\sum_{w\in W} \text{¥}_w} \sum_{l\in L} \frac{f_l}{C_l - f_l} + \sum_{w\in W} t_w$$

$$(\sum_{l\in L} \frac{y_{wl}}{C_l - f_l} - D_w) + \sum_{w\in W}\sum_{l\in L} v_{wl}(\sum_{p\in P_w} x_p \text{¢}_{pl} -$$

$$y_{wl}) + \sum_{l\in L} u_l(g_l - f_l)] \qquad \text{(LR)}$$

subject to:

$$\sum_{p\in P_w} x_p = 1 \qquad \forall w \in W \qquad (3.1)$$

$$x_p = 0 \text{ or } 1 \qquad \forall p\in P_w, w\in W \qquad (3.2)$$

$$y_{wl} = 0 \text{ or } 1 \qquad \forall w\in W, l\in L \qquad (3.3)$$

$$0 \le f_l \le C_l \qquad \forall l \in L. \qquad (3.4)$$

We can decompose (LR) into two independent subproblems.

Subproblem 1: for x_p

$$\min \sum_{w\in W}\sum_{p\in P_w}\sum_{l\in L} (v_{wl} + u_l \text{¥}_w) x_p \text{¢}_{pl} \text{ (SUB1)}$$

subject to (3.1) and (3.2).

Subproblem 2: for y_{wl} and f_l

$$\min[\sum_{l\in L}(\frac{1}{\sum_{w\in W}\text{¥}_w} \frac{f_l}{C_l - f_l} + \frac{\sum_{w\in W} t_w y_{wl}}{C_l - f_l} -$$

$$\sum_{w\in W} v_{wl} y_{wl} - u_l f_l) - D_w \sum_{w\in W} t_w] \text{ (SUB2)}$$

subject to (3.3) and (3.4).

(SUB1) can be further decomposed into $|W|$ independent shortest path problem with nonnegative arc weights. It can be easily solved by the Dijkstra's algorithm. The $-D_w \sum_{w\in W} t_w$ tem in the objective function of (SUB2) can be dropped first and added back to the objective value since it will not affect the optimal solution to (SUB2). Then (SUB2) can be decomposed into $|L|$ independent subproblems. For each link $l \in L$

$$\min[\frac{1}{\sum_{w\in W}\text{¥}_w} * \frac{f_l}{C_l - f_l} + \frac{\sum_{w\in W} t_w y_{wl}}{C_l - f_l}$$

$$- u_l f_l - \sum_{w\in W} v_{wl} y_{wl}] \qquad \text{(SUB2.1)}$$

subject to:

y_{wl}=0 or 1 $\forall w\in W$ and $0 \le f_l \le C_l$.

Problem (SUB2.1) is a complicated problem due to the coupling of y_{wl} and f_l. On the other hand, the $\frac{1}{\sum_{w\in W}\text{¥}_w} * \frac{f_l}{C_l - f_l}$ term in the objective function of (SUB2.1) is a nonnegative and monotonically increasing function with respect to f_l, and it will not affect the optimal value of the following terms in the (SUB2.1). Therefore, the algorithm developed in [4] can be used to solve (SUB2.1). Hence, the algorithm to solve (SUB2.1) is as follows:

Step 1. Solve $(\frac{t_w}{C_l - f_l} - v_{wl} = 0)$ for each O-D pair w, call them the break points of f_l.

Step 2. Sorting these break points and denoted as $f^1_l, f^2_l, \ldots, f^n_l$

Step 3. At each interval, $f_l^i \le f_l \le f_l^{i+1}$, $y_{wl}(f_l)$ is 1 if $\frac{t_w}{C_l - f_l} - v_{wl} \le 0$ and is 0 otherwise.

Step 4. Within the interval, $f_l^i \le f_l \le f_l^{i+1}$, let a_l be $\sum_{w \in W} t_w y_{wl}(f_l)$ and b_l be $\sum_{w \in W} v_{wl} y_{wl}(f_l)$, then the local minimal is either at the boundary point, f_l^i or f_l^{i+1}, or at point

$$f_l^* = C_l - \sqrt{\frac{a_l}{u_l}}.$$

Step 5. The global minimum point can be found by comparing these local minimum points.

According to the algorithms proposed above, we could successfully solve the Lagrangean relaxation problem optimally.

2. Multicast Routing

The rapid progress in World Wide Web and high bandwidth network technology have given rise to the new multicast traffic applications, e.g., video conferencing and electronic newspaper services. Here, we try to find a single minimum cost spanning tree to carry all the traffic for the multiple multicast groups rooted at the same source from the lower bound and upper bound approaches at the same time. This approach has good scalability nature when the number of member groups is becoming larger as compared to separate spanning tree for each multicast group.

Problem Formulation And Solution Approaches

The definition of the notation for revised multicast routing problem is shown below.

a_l	cost associated with link l
T	the set of all spanning trees rooted at the source node
r_g	traffic requirement of multicast group g
P_{gd}	the set of paths that destination d of multicast group g may use
G	the set of all multicast groups rooted at the common source node
h_g	the minimum number of hops to the farthest destination node in multicast group g
D_g	the set of destinations of multicast group g
¢_{pl}	the indicator function which is 1 if link l is on path p and 0 otherwise
$\pm_{tl}$	the indicator function which is 1 if link l is on tree t and 0 otherwise

And the decision variables for the revised multicast routing problem are denoted as follows.

y_{gl}	1 if link l is on the subtree adopted by multicast group g and 0 otherwise
x_{gpd}	1 if path p is selected for group g destined for destination d and 0 otherwise
z_t	1 if spanning tree t is selected to be shared by all the multicast groups and 0 otherwise

$$Z_{IP2} = \min \sum_{l \in L} \sum_{g \in G} r_g y_{gl} a_l \qquad \text{(IP2)}$$

subject to:

$$y_{gl} = 0 \text{ or } 1 \qquad \forall g \in G, l \in L \quad (4.1)$$

$$\sum_{l \in L} y_{gl} \geq \max\{h_g, |D_g|\} \quad \forall g \in G \quad (4.2)$$

$$z_t = 0 \text{ or } 1 \qquad \forall t \in T \quad (4.3)$$

$$\sum_{t \in T} z_t = 1 \quad (4.4)$$

$$\sum_{p \in P_{gd}} x_{gdp} \text{¢}_{pl} \leq y_{gl} \quad \forall g \in G, d \in D_g, l \in L \quad (4.5)$$

$$y_{gl} \leq \sum_{t \in T} \pm_{tl} z_t \qquad \forall g \in G, l \in L \quad (4.6)$$

$$\sum_{p \in P_{gd}} x_{gdp} = 1 \qquad \forall g \in G, d \in D_g \quad (4.7)$$

$$x_{gdp} = 0 \text{ or } 1 \ \forall g \in G, d \in D_g, p \in P_{gd}. \quad (4.8)$$

The objective function in IP2 is to minimize the routing cost for all the multicast groups. Constraints (4.1) and (4.2) require that the number of links on the multicast subtree adopted by the multicast group g be at least the maximum of h_g and the cardinality of D_g. The h_g and the cardinality of D_g are the legitimate lower bounds of the number of links on the multicast subtree adopted by the multicast group g. Apparently, Constraint (4.2) is a redundant constraint. From the computational experiments, the error gaps between the upper bound and the lower bound can be tighter after introducing Constraint (4.2).

Constraints (4.3) and (4.4) require that exactly one single shared tree be adopted by all multicast groups. Constraint (4.5) requires that if one path is selected for group g destined for destination d, it must also be on the subtree adopted by multicast group g. Constraint (4.6) requires that the subtree adopted by any multicast group must be a subset of the shared spanning tree. This spanning tree is selected to be shared by all multicast groups. Constraints (4.7) and (4.8) require that exactly only one path be selected for any group g destined for its destination d.

In (IP2), Constraints (3.5) and (3.6) are relaxed, which leads to the following formulation (LR2).

$$Z_D(u,v) = \min \sum_{l\in L}\sum_{g\in G} r_g y_{gl} a_l$$

$$+\sum_{g\in G}\sum_{d\in D_g}\sum_{l\in L} u_{gdl}\Big(\sum_{p\in P_{gd}} x_{gdp}\text{¢}_{pl} - y_{gl}\Big)$$

$$+\sum_{g\in G}\sum_{l\in L} v_{gl}\Big(y_{gl} - \sum_{t\in T} \pm_{tl} z_t\Big) \qquad \text{(LR2)}$$

subject to:

$$y_{gl} = 0 \text{ or } 1 \qquad \forall g\in G, l\in L \qquad (4.1)$$

$$\sum_{l\in L} y_{gl} \ge \max\{h_g, |D_g|\} \qquad \forall g\in G \qquad (4.2)$$

$$z_t = 0 \text{ or } 1 \qquad \forall t\in T \qquad (4.3)$$

$$\sum_{t\in T} z_t = 1 \qquad (4.4)$$

$$\sum_{p\in P_{gd}} x_{gdp} = 1 \qquad \forall g\in G, d\in D_g \qquad (4.5)$$

$$x_{gdp} = 0 \text{ or } 1 \; \forall g\in G, d\in D_g, p\in P_{gd}. \; (4.6)$$

We can decompose (LR2) into three independent subproblems.

Subproblem 3: for z_t

$$\min \; -\sum_{g\in G}\sum_{l\in L}\sum_{t\in T} v_{gl} \pm_{tl} z_t \qquad \text{(SUB3)}$$

subject to (4.3) and (4.4).

Subproblem 4: for y_{gl}

$$\min \sum_{l\in L}\sum_{g\in G} r_g y_{gl} a_l - \sum_{g\in G}\sum_{d\in D_g}\sum_{l\in L} u_{gdl} y_{gl}$$

$$+\sum_{g\in G}\sum_{l\in L} v_{gl} y_{gl} \qquad \text{(SUB4)}$$

subject to (4.1) and (4.2).

Subproblem 5: for x_{gdp}

$$\min \sum_{g\in G}\sum_{d\in D_g}\sum_{l\in L}\sum_{p\in P_{gd}} u_{gdl} x_{gdp} \text{¢}_{pl} \qquad \text{(SUB5)}$$

subject to (4.5) and (4.6).

(SUB3) can be easily solved by the minimum weight arborescences algorithm which can be found in [1, 2, 3]. (SUB4) can be decomposed into $|G|$ independent subproblems. For each multicast group g,

$$\min \sum_{l\in L}(r_g a_l + v_{gl} - \sum_{d\in D_g} u_{gdl}) y_{gl} \quad \text{(SUB4-1)}$$

subject to:

$$y_{gl} = 0 \text{ or } 1 \qquad \forall\, l \in L \quad (5.1)$$

$$\sum_{l\in L} y_{gl} \ge \max\{h_g, |D_g|\}. \qquad (5.2)$$

The algorithm to solve (SUB4-1) is stated as follows:

Step 1. Compute $\max\{h_g, |D_g|\}$ for multicast group g.

Step 2. Compute the number of negative coefficient $r_g a_l + v_{gl} - \sum_{d\in D_g} u_{gdl}$ for all links on multicast group g.

Step 3. If the number of negative coefficient is greater than $\max\{h_g, |D_g|\}$ for multicast group g, then assign the corresponding negative coefficient of y_{gl} to 1 and 0 otherwise.

Step 4. If the number of negative coefficient is no greater than $\max\{h_g, |D_g|\}$ for multicast group g, then assign the corresponding negative coefficient of y_{gl} to 1. Then, assign [$\max\{h_g, |D_g|\}$ – the number of negative coefficient of y_{gl}] numbers of smallest positive coefficient of y_{gl} to 1 and 0 otherwise.

(SUB5) can be further decomposed into $|G||D_g|$ independent shortest path problem with nonnegative arc weights. It can be easily solved by the Dijkstra's algorithm.

According to the algorithms proposed above, the Steiner tree problem no longer exists in this Lagrangean relaxation problem. And we could successfully solve the Lagrangean relaxation problem optimally.

By applying the optimization-based solution approaches that we propose, we successfully develop two effective and efficient algorithms to solve the QoS-based unicast and multicast routing problems. From an observation of the computational results, the proposed algorithms calculate solutions which are within a few percent of an optimal solution in minutes of CPU time for test networks of tens of nodes.

References:

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