A Minimum Cost Multicast Routing Algorithm with the Consideration of Dynamic User Membership

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Abstract. In this paper, we attempt to solve the problem of constructing a minimum cost multicast tree with the consideration of dynamic user membership. Unlike the other minimum cost multicast tree algorithms, this problem consists of one multicast group of fixed members and each destination member is dynamic and has a probability of being active as which was gathered by observation over some period of time. With the omission of node join/leave handling, this model is suitable for prediction and planning purpose than for online maintenance of multicast trees. We formally model this problem as an optimization problem and apply the Lagrangean relaxation method and the subgradient method to solve the problem. Computational experiments are performed on regular networks and random networks. According to the experiment results, the Lagrangean based heuristic can achieve up to 37.69% improvement compared to the simple heuristic.

1 Introduction

The power of Internet comes from its openness that interconnects computers around the world as long as they follow the protocols. After about one decade of continuous development, this global network has somehow revolutionized the way people communicate and the way businesses are done. However the application involving online audio and video require higher quality of transmission and may consume much more bandwidth over its transmission path, therefore it's worthwhile that we pay more attention to the problems that were aroused by such applications.

A very common scenario is that a source may try to send data to a specific group of destinations, for example a server of video streaming service sending its video stream to all of its service subscribers. Such traffic group communication is called multicast, as opposed to unicast and broadcast. The multicast traffic over IP often follows the route of a spanning tree over the existing network topology, called a multicast spanning tree, taking advantages of sharing common links over paths destined for different receivers. The efficiency of multicast is achieved at the cost of losing the service flexibility of unicast, because in unicast each destination can individually negotiate the service contract with the source. ¿From

C. Kim (Ed.): ICOIN 2005, LNCS 3391, pp. 254–263, 2005.

the viewpoint of network planning, each link in the network can be assigned with a cost, and the problem of constructing a multicast spanning tree with its cost minimized is called Steiner tree problem, which is known to be NP-complete. Reference [1] and [2] surveyed the heuristics of Steiner tree algorithms.

¿From the multicast protocols surveyed in [3], we can see that most complexity of these protocols comes from dealing with the changing of group members, that is, the joining and leaving of nodes. The motivation of this paper would be creating a mechanism for finding and evaluating the cost-efficiency of a multicast tree with a given network and fixed set of group members. Also the group members are dynamic in that they might shut-off for a while, and turn on later. Such probability may be acquired by observation of user behavior over a certain a period of time.

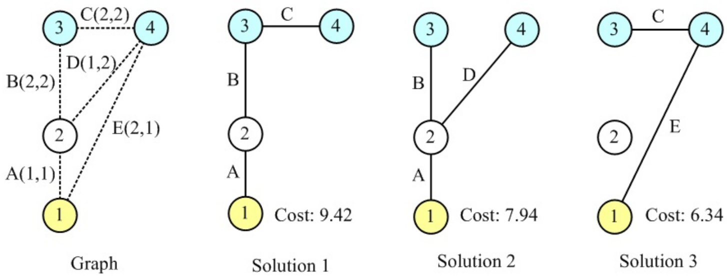

Fig. 1. Example network

Consider the network in Figure 1 with node 1 as the source and nodes 3 and 4 as the destinations which have active probabilities 0.7 and 0.8 respectively. The connection setup costs and transmission costs of the links are labeled in the parentheses beside the links. Figure 1 shows three possible solutions to construct the multicast tree. Consider the solution 1, because nodes 3 and 4 have active probabilities 0.7 and 0.8, the probability that links A and B have no traffic are 0.06. The probability that link C has traffic is 0.8. Consequently, the total cost of solution 1 is 9.42. The cost of solution 2 and 3 are 7.94 and 6.34 respectively. The details of the result are shown in Table 1.

In this paper, however, we do not deal with the complexity node joining and leaving in our heuristic, instead, the activity for a node in summarized as a probability. Therefore, the model proposed here tends to be of analytical and planning use. Still, the problem of multicasting has strong connection with the Steiner tree problem, which is a NP-complete problem, the approach of Lagrangean relaxation is taken to achieve accurate approximation with significantly reduced computation time.

The rest of this paper is organized as follows. In Section 2, we formally define the problem being studied, as well as a mathematical formulation of min-cost

Table 1. Total cost of example network

Cost	Link A	Link B	Link C	Link D	Link E	Total
Solution 1	1+1×(1-(0.3×0.2))	2+2×(1-(0.3×0.2))	2+2×(1-0.2)	x	x	9.42
Solution 2	1+1×(1-(0.3×0.2))	2+2×(1-0.3)	x	1+2×(1-0.2)	x	7.94
Solution 3	x	x	2+2×(1-0.3)	x	2+1×(1-(0.3×0.2))	6.34

optimization is proposed. Section 3 applies Lagrangean relaxation as a solution approach to the problem. Section 4, illustrates the computational experiments. Finally, in Section 5 we present our conclusions and the direction of future research.

2 Problem Formulation

2.1 Problem Description

In this paper we consider, for a network service provider, the problem of constructing a multicast spanning tree that sends traffic to receivers (destinations), while at the same time, the total cost resulted by the multicast tree is minimized. The network is modeled as a graph where the switches are depicted as nodes and the links are depicted as arcs. A user group is an application requesting transmission in this network, which has one source and one or more destinations. Given the network topology and bandwidth requirement of every destination, we want to determine the routing assignment (a tree for multicasting or a path for unicasting) of the user group.

By formulating the problem as a mathematical programming problem, we intend to solve the issue optimally by obtaining a network that will enable us to achieve our goal, i.e. one that ensures the network operator will spend the minimum cost on constructing and servicing the multicast tree. The notations used to model the problem are listed in Table 2.

Each destination $d \in D$ has a given probability Q_d that indicated the fraction of time that the destination is active, and thus the traffic is to be routed to that node. Such probability may be acquired by observation of user behavior over a certain a period of time. The cost associated with a link consists of two parts: 1) fixed cost of connection setup and 2) transmission cost proportional to link utilization. At the determination of the multicast tree, utilizations for all links may be computed, which are used to estimate the total cost.

2.2 Mathematical Formulation

According to the problem description in pervious section, the min-cost problem is formulated as a combinatorial optimization problem in which the objective

Table 2. Description of Notations

	Given Parameters
Notation	Description
D	The set of all destinations of multicast group
r	The source of multicast group
N	The set of all nodes in the network
L	The set of all links in the network
I_i	The set of all incoming links to node i
q_d	The probability that the destination d is active
a_l	The transmission cost associated with link l
b_l	The connection maintenance cost associated with link l
P_d	The set of all elementary paths from r to $d \in D$
δ_{pl}	The indicator function which is 1 if link is on path p
	Decision Variables
Notation	Descriptions
y_l	1 if link l is included in the multicast tree and 0 otherwise
x_p	1 if path p is included in the multicast tree and 0 otherwise
g_l	The fraction of time that the link l is active on the multicast tree
f_{dl}	1 if link l is used by destination $d \in D$ and 0 otherwise

is to minimize the total cost associated with the multicast tree, including the accumulated transmission costs (pay per time unit) and setup cost (pay per connection) on each link used.

Objective function (IP):

$$Z_{IP} = \min \sum_{l \in L} (b_l y_l)(a_l g_l) \quad . \tag{1}$$

subject to:

$$g_l \geq 1 - \prod_{d \in D} (1 - q_d f_{dl}) \qquad \forall l \in L \ . \tag{2}$$

$$\sum_{l \in l_i} y_l \leq 1 \qquad \forall i \in N - \{r\} \ . \tag{3}$$

$$\sum_{l \in l_r} y_l = 0 \ . \tag{4}$$

$$\sum_{p \in P_d} \delta_{pl} x_p \leq f_{dl} \qquad \forall l \in L, \forall d \in D \ . \tag{5}$$

$$\sum_{p \in P_d} x_p = 1 \qquad \forall d \in D \ . \tag{6}$$

$$f_{dl} \leq y_l \qquad \forall l \in L, \forall d \in D \ . \tag{7}$$

$$f_{dl} = 0 \text{ or } 1 \qquad \forall l \in L, \forall d \in D \ . \tag{8}$$

$$y_l = 0 \text{ or } 1 \qquad \forall l \in L \ . \tag{9}$$

$$x_p = 0 \text{ or } 1 \qquad \forall p \in P_d, \forall d \in D \ . \tag{10}$$

$$0 \leq g_l \leq 1 - \prod_{d \in D} (1 - q_d) \qquad \forall l \in L \ . \tag{11}$$

The objective function of (1) is to minimize the construction cost and total transmission cost of servicing the maximum bandwidth requirement destination through a specific link for the multicast group.

Constraint (2) is referred to as the utilization constraint, which defines the link utilization as a function of q_d and f_{dl}. Since the objective function is strictly an increasing function with g_1 and (1) is a minimization problem, each g_1 will equal the aggregate flow in an optimal solution. Constraints (3) and (4) are both tree constraints. Constraint (3) requires that the number of selected incoming links y_1 to node is less than 1 , while constraint (4) requires that there are no selected incoming links y_1 to the node that is the root of multicast group. Constraint (5) and (6) require that only one path is selected for each multicast source-destination pair. Constraint (7) requires that if link l is not included in the multicast tree, then it won't be used by any destination.

Furthermore, here is an example of many possible extensions that could be made to this problem but not discussed in this paper. Say the dependency among destinations, e.g., the members of the group can be further divided into subgroups such that the group members within each subgroup behave identically. The link utilization can be modeled as follows:

$$g_l = 1 - \prod_{m \in G} \left(1 - q_m \left(1 - \prod_{i \in M_m} (1 - f_{il})\right)\right) \quad . \qquad \text{(exp)}$$

Where G is the set of subgroups

As you may notice that the structure of this formula resembles the constraint for link utilization of constraint (1), with its f_{dl} replaced with $\left(1 - \prod_{i \in M_m} (1 - f_{il})\right)$.

3 Solution Approach

3.1 Lagrangean Relaxation

Lagrangean methods were used in both the scheduling and the general integer programming problems at first. However, it has become one of the best tools for optimization problems such as integer programming, linear programming combinatorial optimization, and non-linear programming [4] [5].

By using the Lagrangean Relaxation method, we can transform the primal problem (IP) into the following Lagrangean Relaxation problem (LR) where Constraints (2), (5) and (7) are relaxed.

Optimization problem (LR):

$$\begin{aligned} Z_d(\alpha, \beta, \theta) = \min & \sum_{l \in L} (b_l a_l + a_l g_l) \\ & + \sum_{l \in L} \alpha_l \left(\sum_{d \in D} \log(1 - q_d \cdot f_{dl}) - \log(1 - g_l) \right) \\ & + \sum_{l \in L} \sum_{d \in D} \beta_{dl} \left(\sum_{p \in P_d} \delta_{pl} \cdot x_p - f_{dl} \right) \\ & + \sum_{l \in L} \sum_{d \in D} \theta_{dl} (f_{dl} - y_l) \quad . \end{aligned} \qquad (12)$$

subject to: (3) (4) (6) (8) (9) (10) and (11).

Where $\alpha_l, \beta_{dl}, \theta_{dl}$ are Lagrangean multipliers and $\beta_{dl}, \theta_{dl} \geq 0$. To solve (12), we can decompose (12) into the following four independent and easily solvable optimization subproblems.

Subproblem 1: (related to decision variable x_p)

$$Z_{SUB1}(\beta) = \min \sum_{d \in D} \sum_{p \in P} \left(\sum_{l \in L} \beta_{dl} \cdot \delta_{pl} \right) \cdot x_p \ . \tag{13}$$

subject to: (6) (10).

Subproblem 1 can be further decomposed into $|D|$ independent shortest path problems with nonnegative arc weights β_{dl}. Each shortest path problem can be easily solved by Dijkstra's algorithm.

Subproblem 2: (related to decision variable y_l)

$$Z_{SUB2}(\theta) = \min \sum_{l \in L} \left(b_i - \sum_{d \in D} \theta_{dl} \right) \cdot y_l \ . \tag{14}$$

subject to: (3) (4) (9).

The algorithm to solve Subproblem 2 is:

Step 1 Compute the number of negative coefficients $\left(b_l - \sum_{d \in D} \theta_{dl} \right)$ for all links.

Step 2 Sort the links in ascending order according to the coefficient.

Step 3 According to the order and complying with constraints (3) and (4), if the coefficient is less than zero, assigns the corresponding negative coefficient of y_1 to 1 and 0 otherwise.

Subproblem 3: (related to decision variable g_l)

$$Z_{SUB3}(\alpha) = \min \sum_{l \in L} \left(a_l g_l - \alpha_l \cdot \log\left(1 - g_l\right) \right) \ . \tag{15}$$

subject to: (11).

This subproblem of minimization can be solved by substituting with its lower and upper bound because the minimum of this function appears at endpoints.

Subproblem 4: (related to decision variable f_{dl})

$$Z_{SUB4}(\alpha) = \min \sum_{l \in L} \sum_{d \in D} \left(\alpha_l \log\left(1 - q_d \cdot f_{dl}\right) + \left(\theta_{dl} - \beta_{dl}\right) f_{dl} \right) \ . \tag{16}$$

subject to: (8).

This subproblem of minimization can be solved by simply substitute f_{dl} with 0 and 1 and keep the one that yields the minimum.

According to the weak Lagrangean duality theorem [6], for any $\beta_{dl}, \theta_{dl} \geq 0$, $Z_D(\alpha_l, \beta_{dl}, \theta_{dl})$ is a lower bound on Z_{IP}. The following dual problem (D) is then constructed to calculate the tightest lower bound.

Dual Problem (D):

$$Z_D = \max Z_D\left(\alpha_l, \beta_{dl}, \theta_{dl}\right) \tag{17}$$

subject to:

$$\beta_{dl}, \theta_{dl} \geq 0$$

3.2 Getting Primal Feasible Solutions

During solving the dual problem, a simple algorithm is needed to provide an adequate initial upper bound of the primal problem Z_{IP}. Dijkstra algorithm is used to generate a minimum cost spanning tree over the given network, using the connection setup cost b_l as the arc weight of link l. The result yielded thereby is feasible and expected to give solution of better quality than a random guess. We also use the result of this simple heuristic to compare with Lagrangean relaxation based result in section 4 to prove our improvement.

To calculate the primal feasible solution of the minimum cost tree, the solutions to the Lagrangean Relaxation problems are considered. By solving the dual problem optimally we get a set of decision variables that may be appropriate for being the inputs of getting primal heuristics. However that solution might not be feasible and thus takes some more modifications. The set of g_l obtained by solving (15) may not be a valid solution to problem (IP) because the utilization constraint is relaxed. However, the utilization constraint may be a valid solution for some links. Also, the set of f_{dl} obtained by solving (16) may not be a valid solution because of the path and link constraints are relaxed and the union of y_l may not be a tree.

Here we propose a heuristics to obtain a primal feasible solution. While solving the Lagrangean relaxation dual problem, we may get some multipliers related to each links. According to the information, we can make our routing more efficient. Two of our getting primal heuristics are created by taking the LR multiplier β_{dl} as the source of arc weight in Dijkstra algorithm. We describe the Lagrangean based heuristic below.
[Lagrangean multiplier based heuristic]

Step 1 Calculate $\sum_{d \in D} \beta_{dl}$ as link l's arc weight.
Step 2 Use the arc weight obtained from step 1 and run the Dijkstra algorithm.

4 Computational Experiments

In this section, computational experiments on the Lagrangean relaxation based heuristic and simple primal heuristics are reported. The heuristics are tested on two kinds of networks- regular networks and random networks. Regular networks are characterized by low clustering and high network diameter, and random networks are characterized by low clustering and low diameter.

Two regular networks shown in Figure 2 are tested in our experiment. The first one is a grid network that contains 25 nodes and 40 links, and the second is a cellular network containing 19 nodes and 42 links. Random networks tested in this paper are generated randomly, each having 25 nodes. The candidate links between all node pairs are given a probability following the uniform distribution. In the experiments, we link the node pair with a probability smaller than 2%. If the generated network is not a connected network, we generate a new network.

For each testing network, several distinct cases, which have different predetermined parameters such as the number of nodes, are considered. The traffic

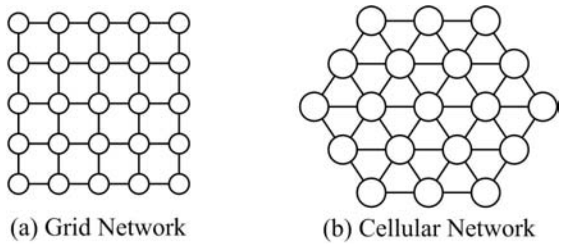

Fig. 2. Regular Networks

demand for multicast group is drawn from a random variable. The link connection maintenance costs and transmission cost are randomly generated between 1 and 5. The active probability of each destination is randomly generated between 0.1 and 1. The parameters used for all cases are listed in Table 3. The cost of the multicast tree is decided by multiplying the link transmission cost and the bandwidth requirement of the multicast group plus link maintenance costs. We conducted 200 experiments for each kind of network. For each experiment, the result was determined by the group destinations and link costs generated randomly. Table 4 summaries the selected results of the computational experiments.

Table 3. Parameters for Lagrangean Relaxation

Number of Iterations	1,000
Initial Multipliers	0
Improvement Counter	15
Delta Factor	2
Optimal Condition	Gap $<$ 0.001

For each testing network, the maximum improvement ratio between the simple heuristic and the Lagrangean based heuristic is 20.17%, 20.77%, and 37.69%, respectively. In general, the Lagrangean based heuristic performs well compared to the simple heuristic. There are two main reasons of which the Lagrangean based heuristic works better than the simple algorithm. First, the Lagrangean based heuristic makes use of the related Lagrangean multipliers which include the potential cost for routing on each link in the topology. Second, the Lagrangean based heuristic is iteration-based and is guaranteed to improve the solution quality iteration by iteration. Therefore, in a more complicated testing environment, the improvement ratio is higher. To summarize, by relaxing constraints in the primal problem and optimally solving dual problem, the set of LR multipliers revealed iteration by iteration became unique sources for improving our solutions in getting primal heuristics.

Table 4. Selected Results of Computational Experiments

CASE	Dest. #	SA[a]	UB[b]	LB	GAP[c]	Imp.[d]
		Grid Network			Max Imp. Ratio: 20.17 %	
A	5	27.34	26.05	25.99	0.22%	4.73%
B	5	40.10	32.01	31.54	1.49%	20.17%
C	10	66.40	54.78	53.83	1.76%	17.51%
D	10	72.24	66.75	63.83	4.57%	7.60%
E	15	53.42	48.01	47.04	2.06%	10.13%
F	15	103.34	98.02	92.56	5.90%	5.15%
G	20	164.34	145.43	144.61	0.57%	11.50%
H	20	156.61	132.82	113.68	16.84%	15.19%
		Cellular Network			Max Imp. Ratio: 20.77 %	
A	5	16.62	16.62	16.62	0.00%	0.00%
B	5	32.16	25.48	23.34	9.17%	20.77%
C	10	56.37	46.98	45.30	3.71%	16.66%
D	10	48.88	40.87	38.33	6.63%	16.39%
E	15	58.12	47.31	38.09	24.20%	18.60%
F	15	85.76	82.30	74.45	10.54%	4.03%
G	20	124.35	118.83	111.34	6.73%	4.44%
H	20	143.33	128.98	124.43	3.66%	10.01%
		Random Networks			Max Imp. Ratio: 37.69 %	
A	5	7.75	7.75	7.75	0.00%	0.00%
B	5	14.32	12.94	12.48	3.69%	9.64%
C	10	53.83	42.47	39.89	6.47%	21.10%
D	10	59.16	36.86	33.04	11.56%	37.69%
E	15	60.38	57.82	53.37	8.34%	4.24%
F	15	76.42	68.88	62.34	10.49%	9.87%
G	20	93.46	74.83	73.08	2.39%	19.93%
H	20	103.46	92.64	83.78	10.58%	10.46%

[a] SA: The result of the simple heuristic
[b] UB and LB: Upper and lower bounds of the Lagrangean based modified heuristic
[c] GAP: The error gap of the Lagrangean relaxation
[d] Imp.: The improvement ratio of the Lagrangean based heuristic

To claim optimality, we also depict the percentile of gap in Table 4. The results show that most of the cases have a gap of less than 20%. We also found that the simple heuristic performs well in many cases, such as the case A of Cellular network and case A of random network.

The contribution of this research would be quite academic, with the innovative idea of constructing a multicast tree that adapts to the activity of end users in a minimization problem, making the model itself aware of the phenomenon of dynamic user join and leave without all the fuss of dealing with it in our heuristic. For this reason, this model is ideal for network planning purpose. Still the computational results show that the structure of the problem is suitable for the methodology of Lagrangean relaxation. However this model is still in a simple form and interested researchers may come up with quite a few extensions to this simple model with ease.

5 Conclusions

In this paper, we attempt to solve the problem of min-cost multicast routing with the consideration of dynamic user membership. Our achievement of this paper can be expressed in terms of mathematical formulation and experiment performance. In terms of formulation, we propose a precise mathematical expression to model this problem well. In terms of performance, the proposed Lagrangean relaxation and subgradient based algorithms outperform the primal heuristics.

Some additional topics to this problem might be 1) Multiple groups of users and the behaviors of the members within one group are identical or somewhat correlated. 2) Multiple trees may be constructed over the network at the same time, with different data-rate demands. 3) Quality-of-service constraints may be added such as: link capacity, hop count and delay constraints. 4) Different getting primal feasible heuristics can be invented to produce solutions with better optimality.

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