Application of Combined Adaptive Fourier Filtering Technique and Fault Detector to Fast Distance Protection

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***Abstract*—This paper presents the application of a combined adaptive Fourier filtering technique and fault detector to fast distance protection of transmission lines. The filtering technique is extended from the Fourier filters and can be applied under arbitrary data window length. The proposed filtering technique possesses the advantage of recursive computing, and a decaying dc offset component is removed from fault signals by using an adaptive compensation method. A variable data window scheme is embedded in the technique to adaptively speed up its transient response under various system and fault conditions. A fault detector is developed to initiate the process of the technique. For the sake of speed and security, two phasor estimation methods based on the proposed technique are designed to achieve fast distance protection. An algorithm consisting of the two methods is used to detect internal faults by "OR" logic. Extensive simulation studies show that the algorithm significantly reduces tripping time of a distance relay and provides better protection performance than that of the conventional filters with fixed data windows.**

***Index Terms*—Adaptive filtering, computer relaying, decaying dc offset, digital distance protection, discrete Fourier transform (DFT).**

I. Introduction

THE growth of power systems in both size and complexity promotes the requirements of high-speed relays to protect major equipment and maintain system stability. Distance relays are commonly used in the protection of transmission lines. With the advancements of digital signal processors (DSPs), many distance relay commercial products are of microprocessor-based systems. Moreover, distance relays can easily perform various tasks by multiple processors using parallel processing techniques. Therefore, it is possible to implement more complex algorithms to achieve high-speed distance protection and better performance of additional functions [1], [2].

The accuracy and speed of digital filtering algorithms are vital for phasor-based digital distance relays. When a fault occurs, the voltage and current signals are severely distorted. In addition to the fundamental frequency signals, these signals may contain harmonics and decaying dc components [1], [2]. This makes the fundamental phasors very difficult to be fast and accurately estimated and affects the performance of a distance relay.

A number of phasor estimation algorithms suitable for transmission-line protection have been proposed [3]–[13]. However, the discrete Fourier transform (DFT)-based filter is the most popular algorithm and has become standard in the industry [1], [2]. The computational cost of recursive DFT-based filter is very low and good harmonic immunity can be achieved. However, its performance can be adversely affected by decaying dc components, leading to erroneous estimates [9]–[11]. Consequently, distance relays have a tendency to over-reach or under-reach in the presence of the decaying dc offset. For a high-performance digital relay, such a large error cannot be tolerated. A digital mimic filter is proposed to suppress the dc offset in current waveforms [9]. This filter achieves the best performance when the time constant of the dc offset is equal to the time constant of the mimic filter. Recently, two DFT-based dc offset removal algorithms, using fixed full-cycle or half-cycle data windows, were proposed in [10] and [11]. However, the phasor estimation cannot be very fast due to the long data window length of the filters.

The filtering capability of the DFT-based filter depends on its window length. A short data window will give a fast response but unstable output. A long one gives stable output but the response will be delayed. The most suitable window length depends on various factors, such as fault locations, fault types, and fault resistance, etc. This means that a compromise between the filter's delay and its noise suppression capabilities is required. It is possible to select a suitable filtering algorithm and a data window at different stages of a fault to complete fast transmission-line protection [1], [2]. For example, Sidhu *et al.* proposed the use of least-error-square (LES) filters of different data window lengths for computing voltage and current phasors and providing faster tripping of a distance relay [12]. The adaptive data window approaches overcome the speed and accuracy problems, which cannot be solved by a fixed data window algorithm.

This paper describes the design and testing of an algorithm based on the new adaptive filtering technique for fast distance protection. Theoretical basis, analysis, and derivation of the filtering technique are described. The technique is computationally efficient and no statistical information concerning the signals is required. The proposed algorithm provides capability for fast tripping decisions with taking accuracy and security into account. Extensive simulation tests and comparative studies of the algorithm with the conventional full-cycle and half-cycle DFT filters plus digital mimic filters are reported and discussed.

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II. Proposed Fault Detector and Adaptive Fourier Filtering Technique

A. Fault Detector

In [15], the authors have proposed a robust fault detection index using two-terminal synchronized phasors and a distributed line model. Using Clarke transformation to decouple the interphase quantities, the fault detection index can be expressed as

$$\begin{aligned} M_m &= \frac{1}{2}\exp[-\Gamma_m L][V_{Sm} + Z_{Cm}I_{Sm}] \\ &\quad - \frac{1}{2}[V_{Rm} + Z_{Cm}I_{Rm}] \\ &= E_m - B_m \quad m = 0, \alpha, \beta \end{aligned} \tag{1}$$

where L is total length of the protected line. Γ_m and Z_{Cm} are modal propagation constant and surge impedance, respectively. V_{Rm}, V_{Sm}, I_{Rm}, and I_{Sm} are the synchronized m-modal phasors of receiving-end/sending-end voltages and currents, respectively.

The fault detection index $|M_m|$ (the absolute value of index M_m) will be held at zero under healthy conditions [15]. It means E_m is equal to B_m. However, the detection index will rise with a very large slope when a fault occurs. In this paper, since only single-end data are obtained, E_m is taken as the fault detector and it is shown as follows:

$$E_m = \frac{1}{2}\exp[-\Gamma_m L][V_{Sm} + Z_{Cm}I_{Sm}] \quad m = 0, \alpha, \beta. \tag{2}$$

The E_m introduces a bias value under healthy conditions. To remove the bias value, it is further modified as

$$\left|\bar{E}_m\right| = \mathrm{abs}(|E_m| - |E_m|_{\mathrm{pre}}) \tag{3}$$

where $\mathrm{abs}(\cdot)$ denotes the absolute value, and $|E_m|_{\mathrm{pre}}$ is the value of one cycle ahead of $|E_m|$.

Therefore, the value of the index will be equal to zero for every moving data window under nonfault conditions. The fault detection index, composed of voltage and current signals, provides fast and robust ability for fault detection. Taking measurement and calculation errors into account, a predefined threshold (**Th1**) is needed to avoid misoperation. It is desired that the proposed fault detector has a fast transient response. Thus, a short data window of a one-quarter cycle is adopted to detect faults and then initiate the process of the proposed adaptive Fourier filtering algorithm.

B. Fourier Filtering Technique With a Full-Cycle Window Length

The basic principles of the discrete Fourier filter are reviewed and analyzed in Appendix A. A DFT-based decaying dc offset removal scheme with a full-cycle data window that accurately estimates the phasor from a fault current signal has been proposed [11]. The main components of a fault current signal can be expressed by

$$x(t) = X\cos(\omega t + \phi_1) + X_d\exp(-\alpha t) \tag{4}$$

where

X	amplitude of the current signal;
$\omega = 2\pi \cdot 60$	fundamental angular frequency of the current signal;
ϕ_1	phase angle of the current signal;
$X_d\exp(-\alpha t)$	decaying dc offset;
$1/\alpha = \tau$	time constant of the current signal.

The signal is sampled with N samples per cycle to produce the sample set

$$\begin{aligned} x_n &= X\cos(\omega n/60N + \phi_1) \\ &\quad + X_d\exp(-\alpha n/60N) \quad n = 0, 1, 2, \ldots, N-1. \end{aligned} \tag{5}$$

The fundamental component calculated using the full-cycle DFT filter is given by

$$\hat{x}_r = \frac{2}{N}\sum_{n=0}^{N-1} x_{n+r}e^{-jn\theta} \tag{6}$$

where r denotes the index of the moving data window.

Combining (5) and (6), the fundamental current phasor is expressed as

$$\hat{x}_r = A_r + B_r \tag{7}$$

where

$$A_r = Xe^{j\phi_1}e^{jr\theta} \tag{8}$$

$$B_r = \frac{2}{N}X_d e^{-\alpha r/60N}\sum_{n=0}^{N-1} e^{-\alpha n/60N}e^{-jn\theta}. \tag{9}$$

Since the conventional DFT filter does not take the decaying dc offset into consideration, it incurs errors in estimating the phasor when the decaying dc offset is presented in a fault signal. If we want to get the exact solution, we must take B_r into consideration. Thus, we define

$$a = e^{j2\pi/N} = e^{j\theta}, \quad d = e^{-\alpha/60N}. \tag{10}$$

After some algebraic manipulations, it yields

$$d = \frac{\hat{x}_{r+2} - a\hat{x}_{r+1}}{\hat{x}_{r+1} - a\hat{x}_r} \tag{11}$$

and

$$A_r = \frac{\hat{x}_{r+1} - d\hat{x}_r}{(a-d)}. \tag{12}$$

Then, the accurate fundamental current phasor can be obtained as

$$X = \mathrm{abs}(A_r) \tag{13}$$

$$\phi_1 = \mathrm{angle}\left(A_r e^{-j2\pi(r-1)/N}\right). \tag{14}$$

Thus, from (11)–(14), we can remove the decaying dc offset from the fault current signal using the adaptive compensation term "d." The phasor is accurately computed from the three consecutive DFT estimates $\hat{x}_r, \hat{x}_{r+1}$, and $\hat{x}_{r+2}$ by using recursive computing, and so the technique is suitable for real-time implementation. Since the full-cycle DFT is used to estimate phasors, the technique needs approximately one-cycle delay time to remove the dc offset.

C. Fourier Filtering Technique With Arbitrary Window Length

Similarly, substituting sampled values x_n in (5) into the right-hand side (RHS) of (A.2), we can obtain

$$\begin{aligned}\sum_{n=0}^{K-1} x_{n+r}\cos(n\theta) &= \frac{X}{2}e^{j\phi_1}e^{jn\theta}\sum_{n=0}^{K-1}e^{jn\theta}\cos(n\theta) \\ &\quad + \frac{X}{2}e^{-j\phi_1}e^{-jr\theta}\sum_{n=0}^{K-1}e^{-jn\theta}\cos(n\theta) \\ &\quad + X_d e^{-\alpha r/60N}\sum_{n=0}^{K-1}e^{-\alpha n/60N}\cos(n\theta) \qquad (15)\end{aligned}$$

$$\begin{aligned}\sum_{n=0}^{K-1} x_{n+r}\sin(n\theta) &= \frac{X}{2}e^{j\phi_1}e^{jn\theta}\sum_{n=0}^{K-1}e^{jn\theta}\sin(n\theta) \\ &\quad + \frac{X}{2}e^{-j\phi_1}e^{-jr\theta}\sum_{n=0}^{K-1}e^{-jn\theta}\sin(n\theta) \\ &\quad + X_d e^{-\alpha r/60N}\sum_{n=0}^{K-1}e^{-\alpha n/60N}\sin(n\theta). \qquad (16)\end{aligned}$$

Combining (A.2), (15), and (16), we then derive the complex phasor

$$\begin{aligned}\hat{x}_r &= X_{c,r} - jX_{s,r} \\ &= \frac{X}{2}e^{j\phi_1}e^{jr\theta}Z_1 + \frac{X}{2}e^{-j\phi_1}e^{-jr\theta}Z_2 \\ &\quad + X_d e^{-\alpha r/60N}e^{jr\theta}Z_3 \qquad (17)\end{aligned}$$

where

$$\begin{aligned}Z_1 &= M_1\sum_{n=0}^{K-1}e^{jn\theta}\cos(n\theta) - jM_3\sum_{n=0}^{K-1}e^{jn\theta}\sin(n\theta) \\ &\quad - jM_2\sum_{n=0}^{K-1}e^{jn\theta}[\cos(n\theta)+j\sin(n\theta)] \qquad (18\text{-}1)\end{aligned}$$

$$\begin{aligned}Z_2 &= M_1\sum_{n=0}^{K-1}e^{-jn\theta}\cos(n\theta) - jM_3\sum_{n=0}^{K-1}e^{-jn\theta}\sin(n\theta) \\ &\quad - jM_2\sum_{n=0}^{K-1}e^{-jn\theta}[\cos(n\theta)+j\sin(n\theta)] \qquad (18\text{-}2)\end{aligned}$$

$$\begin{aligned}Z_3 &= M_1\sum_{n=0}^{K-1}e^{-\alpha n/60N}\cos(n\theta) \\ &\quad - jM_3\sum_{n=0}^{K-1}e^{-\alpha n/60N}\sin(n\theta) \\ &\quad - jM_2\sum_{n=0}^{K-1}e^{-\alpha n/60N}[\cos(n\theta)+j\sin(n\theta)]. \qquad (18\text{-}3)\end{aligned}$$

We can easily prove that $Z_1 = 2$ and $Z_2 = 0$. The proof is shown in Appendix B. Therefore, (17) can be further rewritten as the following equations:

$$\begin{aligned}\hat{x}_r &= Xe^{j\phi_1}e^{jr\theta} + X_d e^{-\alpha r/60N}e^{jr\theta}Z_3 \\ &= A_r + B_r \qquad (19)\end{aligned}$$

where

$$A_r = Xe^{j\phi_1}e^{jr\theta} \qquad (20\text{-}1)$$

$$B_r = X_d e^{-\alpha r/60N}e^{jr\theta}Z_3. \qquad (20\text{-}2)$$

Using the skills in the above section, we also define $a = e^{j2\pi/N}$ and $d = e^{-\alpha/60N}$, and then (11)–(14) still can be used to compute phasors. So the accurate phasor is estimated from three consecutive estimates of the Fourier filter in (A.2). It should be noted that the three estimates are computed using a fixed data window length K and a moving data window scheme. Therefore, the dc offset removal scheme can estimate a phasor using an arbitrary data window length K. To save the computational burden, the three estimates can be calculated using recursive formula in (A.8)–(A.10).

D. Fourier Filtering Technique With Adaptive Window Length

This paper uses the variable data window technique to adaptively speed up the response of the proposed filter mentioned in the above section. The technique requires the proposed fault detector to initiate the process of the filter. Figs. 1 and 2 show the overall flowchart and diagram of the proposed filter with an adaptive data window for phasor estimation, respectively. The procedure is described as follows.

Stage 1) Once a fault is detected, the filter starts with an initial data window **win_init** to calculate one estimate by using (A.8)–(A.10). After three consecutive estimates with the same data window, such as W1, W2, and W3 shown in Fig. 2, (11)–(14) are utilized to estimate the accurate phasor. After passing **win_init** samples, prefault samples are completely removed from the data window since a fault has detected at least two data samples.

Stage 2) Then, the phasor is computed using a variable data window technique. After each estimation process, the filter progressively increases its data window length to estimate the phasors. The noise immunity is adaptively varied with the data window length. The convergent speed of the technique is adapted to system and fault conditions. Moreover, the decaying dc offset is adaptively removed from the fault signal.

Stage 3) When the data window length reaches the full-cycle length (**win_full**), it will be fixed and does not change again. Then, the filter proceeds with the phasor estimation using a moving data window approach.

III. Application to Transmission-Line Distance Protection

The distance relay uses fault impedance estimation for making trip decisions. According to the calculated impedance, the fault is identified as an internal or external fault with respect to the protected zone. There are distinct types of possible faults,

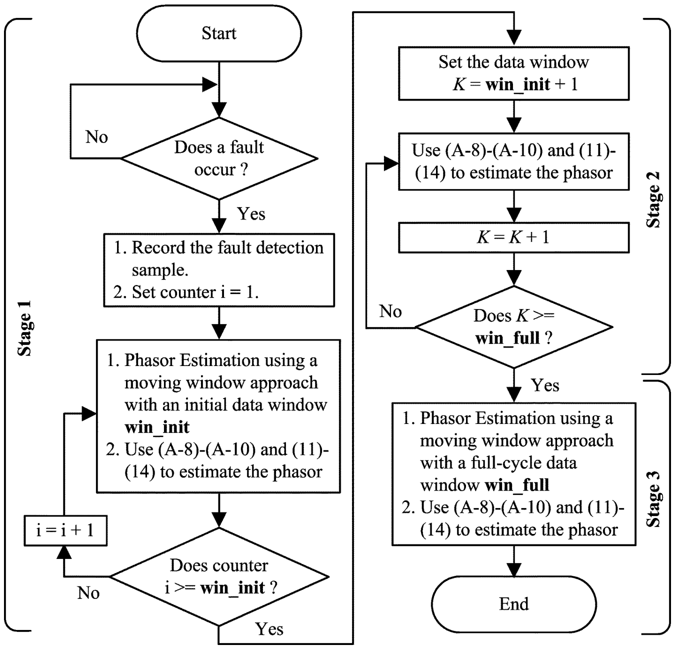

Fig. 1. Flowchart of the proposed adaptive Fourier filtering technique.

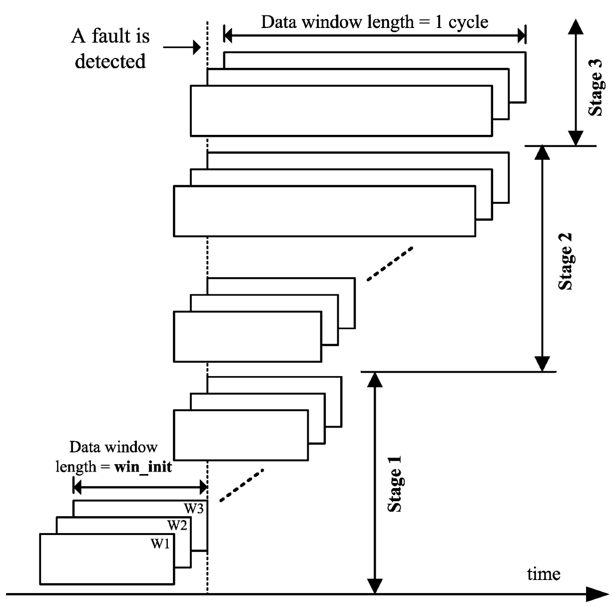

Fig. 2. Diagram of the adaptive filtering technique.

and the equations that govern the relationship between voltages and currents are different for each fault. These estimation equations of apparent impedance are listed in Table I. The six elements are evaluated continuously by the distance relay to measure the apparent impedance to the fault point. In this paper, the impedance protection used for studies is based on an Mho distance characteristic.

The phasors utilized in fast distance protection can be accomplished by the adaptive filtering techniques proposed in Section II. The initial calculations using the short data window may introduce enormous errors in the current and voltage phasors. These errors may cause transient over-reach and decrease security. By increasing the tripping counter, we can increase the security of the distance relay. However, it will introduce time delay for the tripping decision. Assume that the first protection

TABLE I
APPARENT IMPEDANCE CALCULATION EQUATIONS

Relay Element	Fault Type	Equation
Ground Relay	A-G	$V_A/(I_A + 3k_0 \cdot I_0)$
	B-G	$V_B/(I_B + 3k_0 \cdot I_0)$
	C-G	$V_C/(I_C + 3k_0 \cdot I_0)$
Phase Relay	A-B	$(V_A - V_B)/(I_A - I_B)$
	B-C	$(V_B - V_C)/(I_B - I_C)$
	C-A	$(V_C - V_A)/(I_C - I_A)$

where letters A, B, and C denote phase quantities. Letter G denotes ground fault. $k_0 = (Z_0 - Z_1)/Z_1$ and I_0 is the zero sequence current. Z_0 and Z_1 are the zero and positive sequence line impedances.

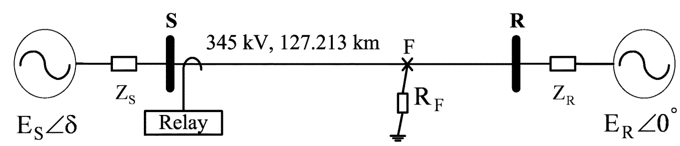

Fig. 3. One-line diagram of the simulated transmission system.

zone setting of the protected line is 80%. We develop two filtering methods to satisfy the requirements of both tripping decision time and security. The two methods are shown as follows.

Method 1) The adaptive filtering technique with an initial data window **win_init** $= 1/2$ cycle incorporated with a reach setting of 80%.

Method 2) The adaptive filtering technique with an initial data window **win_init** $= 1/4$ cycle incorporated with a reduced reach setting 65% of the protected line.

When method 1) is used to estimate phasors and to make the relay trip decision, extensive simulation studies verify that method 1) provides a faster trip time of a relay than that of the conventional DFT filter with a fixed full-cycle data window and it will not cause misoperation. When faults occur in the range of 0%–65% of the protected line, method 2) provides a faster relay response time than method 1) since it uses a shorter initial data window. However, method 2) cannot detect faults occurring in the range of 65%–80% of the line. Using the parallel processing technique, we can combine the two methods to detect internal faults by "OR" logic (i.e., one of them detects an internal fault and then the fault is confirmed). Therefore, the original reach setting of 80% can be preserved. This combined algorithm can guarantee a fast trip time for near faults and high security for remote faults close to the reach boundary. This algorithm provides faster-relay response and higher security than that of the conventional methods with a fixed data window such as full-cycle DFT (FCDFT) and half-cycle DFT (HCDFT) plus digital mimic filters.

IV. PERFORMANCE EVALUATION

Fig. 3 depicts the simulated transmission system encountered in Taiwan. The EMTP/ATP simulator [16] is used to model the system and generate fault data to test the performance of the proposed algorithm. The power angle δ between E_S and E_R is set as 15°. The related parameters of the simulated system are given in Appendix C. The three-phase voltage and current signals at the relay location, which is at bus "S," are sampled at

960 Hz (16 points per cycle). These six signals are then filtered using a second-order low-pass Butterworth filter with a cutoff frequency of 300 Hz. The phase lag at the fundamental signal is about 10.4°, which corresponds to a time delay of about 0.48 ms. The phase delay produced by this filter is about one-half the sampling period. In order to choose the setting of **Th1**, we performed extensive simulation studies under various system and fault conditions. The thresholds **Th1** for fault detection are set as 3000. The simulation studies show that the setting value works well under various conditions. To demonstrate the performance of the proposed algorithm that can meet the requirement for fast distance protection, extensive simulation studies for various systems and fault conditions have been performed. Some results are presented and discussed below.

A. Examples and Comparative Studies

This paper compares the new adaptive algorithm against the conventional FCDFT and HCDFT plus digital mimic filters. Digital mimic filters with a time constant of two cycles are assumed. Fig. 4 shows the performance of the fault detector, phasor estimation, and convergence locus of the estimated fault impedance for a phase-A to ground fault. The fault position is set at 90 km away from bus S. The fault inception angle is 0° with respect to phase-A voltage waveform to produce a large decaying dc offset, and the fault resistance is 0.1 Ω. As shown in Fig. 4(a), the proposed fault detector quickly detects the fault at 3.125 ms after fault inception. Fig. 4(b)–(d) depicts the comparative results of the phase A voltage and current phasor estimation using various filters. From these figures, we observe the following.

- The proposed filtering methods with the removal of dc offset still work well for voltage phasor estimation as shown in Fig. 4(b).
- Since the fault current contains a large decaying dc component, the computed results using the FCDFT and HCDFT filters produce erroneous estimates, especially for the HCDFT filter as shown in Fig. 4(c).
- The mimic filter cannot completely remove the decaying dc offset component. As shown in Fig. 4(d), the FCDFT and HCDFT with mimic filters can only reduce the estimation errors. The FCDFT+mimic filter takes a longer time to estimate the current phasor. The HCDFT+mimic filter has fast transient response, but its filtering capability is affected due to its short data window. Therefore, the estimation using the HCDFT+ mimic filter introduces oscillations in the current phasor and the estimation errors degrade the security and dependability of a distance relay.
- From Fig. 4(b) and (d), it is obviously seen that the proposed filtering methods provide fast and accurate voltage and current phasor estimation. The decaying dc offset is effectively removed from the fault current signal. The noise immunity is adaptively varied with the data window length and the effects of transient components, which are not filtered by low-pass filters, are adaptively suppressed.

Fig. 4(e) and (f) depicts the convergence locus of the estimated impedance. For the sake of security, a trip command is issued only if two successive computed fault impedances lie inside the reach setting of the relay. For the internal fault, the trip time of the FCDFT+mimic, HCDFT+mimic, and the proposed algorithm are 19.792, 14.583, and 12.5 ms, respectively. As shown in Fig. 4(f), method 2) works well since a reduced reach setting of 65% solves the overestimate errors of the current phasor.

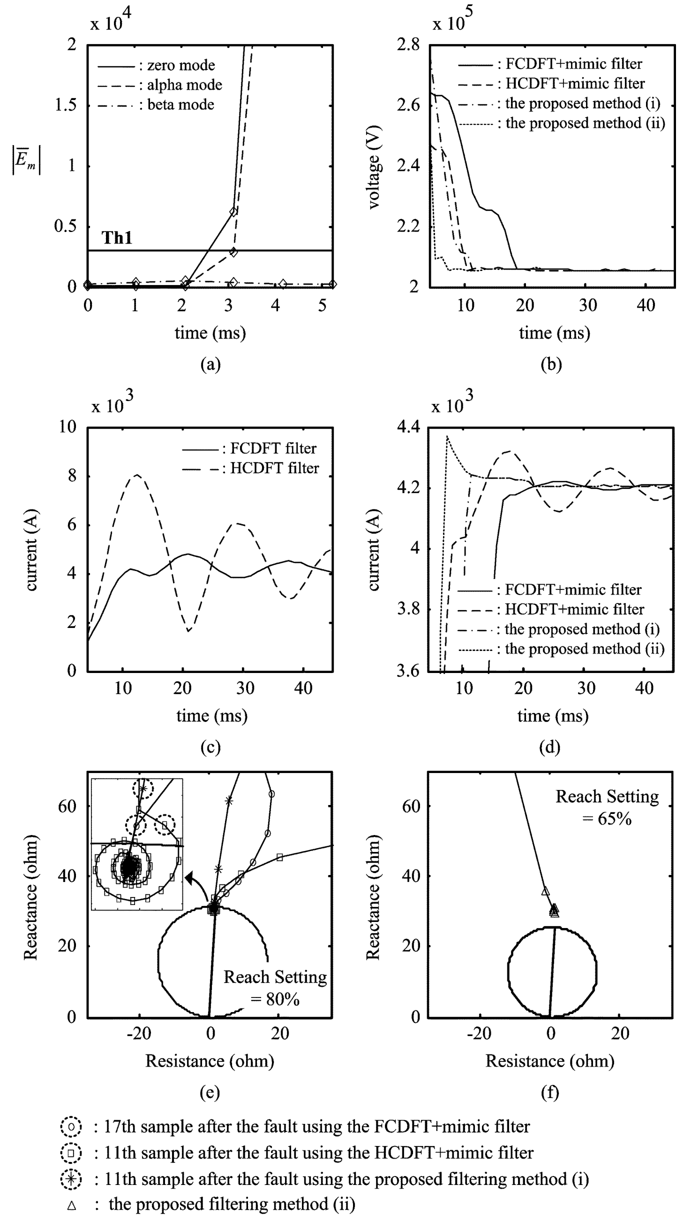

Fig. 4. Postfault results of phasor estimation and impedance convergence locus for a phase-A to ground fault.

Fig. 5 depicts the relay response for a phase A-B short fault. The fault is located at 40 km away from bus S. The trip time of the FCDFT+mimic, HCDFT+mimic, and the proposed algorithm are 13.542, 7.292, and 6.25 ms, respectively. The trip time of the proposed algorithm is determined by the method 2) with a reduced reach setting as shown in Fig. 5(b).

The above two studies are internal fault cases. The response of an external fault, phase A–B to ground fault, is presented in Fig. 6. The fault is set at 110 km and the fault resistance is 5 ω. As shown in this figure, the proposed algorithm provides a correct relay response. The simulation results verify that the relay adopting the proposed algorithm has faster response and higher

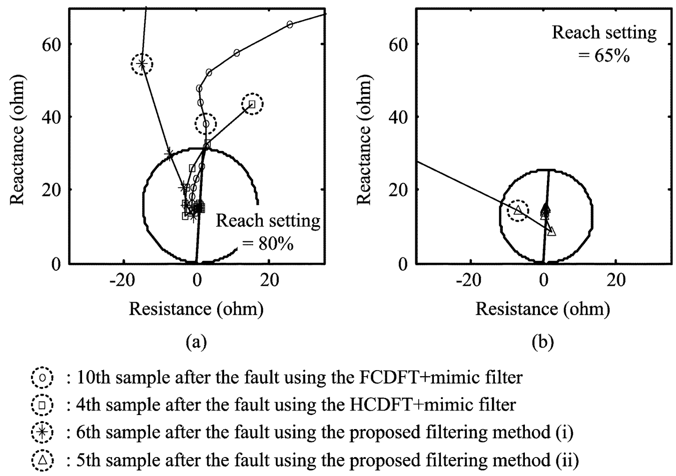

Fig. 5. Relay response of the various filters for a phase A-B short fault.

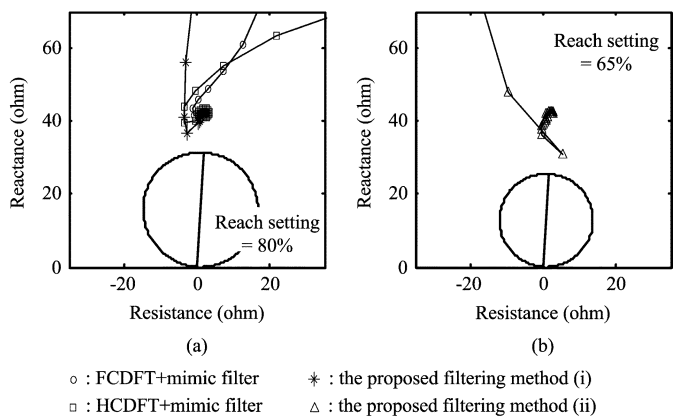

Fig. 6. Relay response of the various filters for an external fault.

security than that of the conventional FCDFT and HCDFT plus digital mimic filters.

V. Statistical Analysis

To verify the robustness of the proposed algorithm, extensive fault simulations under various system and fault conditions were performed. The fault cases are described as follows:

- fault location 10, 20, 30,. . . , 90, 110, and 120 km;
- fault types: A-G, A-B, A-B-G, and A-B-C-G;
- fault resistance: 0.1, 1, 5, and 10 Ω;
- fault inception angle: 0°, 45°, and 90° with respect to A-phase voltage waveform at bus S;
- prefault load flow: power angle δ is set as 10°, 15°, and 20°, respectively.
- system impedance: $Z_{\rm S}$ and $Z_{\rm R}$ are simultaneously increased and decreased by a multiple of 5, respectively.

Fig. 7 shows the statistical results of the tripping decision time for internal fault cases. It is seen that the tripping decision time of the proposed algorithm is faster than that of the FCDFT and HCDFT plus mimic filters with a fixed data window, respectively. Although the HCDFT+mimic filter has a fast response, it sometimes may cause misoperation. Considering the tripping decision time and security, the proposed algorithm provides the best performance. For external faults, the proposed algorithm never causes misoperation. The proposed filtering technique presents a new approach to fast transmission-line protection with adaptive features. It can also be used to achieve a fast accurate fault location estimation.

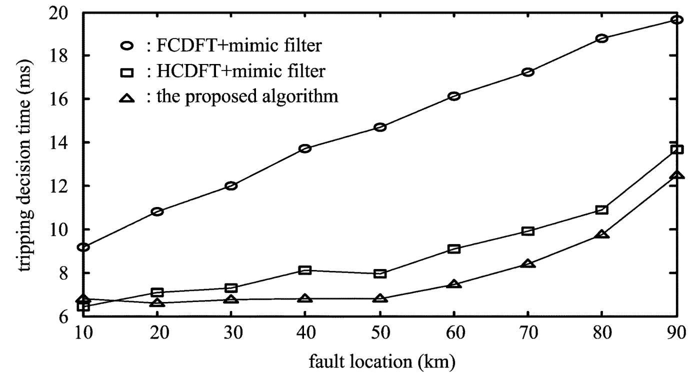

Fig. 7. Statistical results of tripping decision time under various system and fault conditions.

VI. Conclusion

An algorithm based on a fault detector and a new adaptive Fourier filtering technique for distance protection is presented in this paper. A robust fault detector is developed to initiate the process of the filtering technique. The basic principles and detail formulation with proofs are given. The filtering technique combined with an adaptive window length for fast and accurate phasor estimation is described.

Digital technology provides the chance for diverse implementations of the distance protection function. The algorithm based on the adaptive filtering technique has been presented to provide fast distance protection. The simulation studies have shown both faster time response and better steady-state accuracy of the proposed algorithm. The comparative studies indicate that the relay trip time of the proposed algorithm is faster than that of conventional DFT filters with fixed data windows. With the availability of fast and cheap signal processors, the proposed algorithm can be easily implemented. The algorithm shows great promise in fast digital distance relaying for transmission lines.

Appendix A

Consider a voltage or current signal of fundamental frequency ω given by

$$x(t) = X_c \cos(\omega t) + X_s \sin(\omega t) + e(t) \qquad \text{(A.1)}$$

where $e(t)$ is an error signal, which models all nonfundamental frequency signals.

The signal $x(t)$ is sampled N times per cycle and the resulting sample values are denoted by x_n, where $n = 0, 1, 2, \ldots, N-1$. The parameters X_c and X_s can be estimated from the K sampled values $x_n (n = 0, 1, 2, \ldots, K-1)$ by the least squares technique and the formula is shown as follows [2]:

$$\begin{bmatrix} X_c \\ X_s \end{bmatrix} = \begin{bmatrix} M_1 & M_2 \\ M_2 & M_3 \end{bmatrix} \begin{bmatrix} \sum_{n=0}^{K-1} x_n \cos(n\theta) \\ \sum_{n=0}^{K-1} x_n \sin(n\theta) \end{bmatrix}$$
$$\theta = \frac{2\pi}{N} \qquad \text{(A.2)}$$

where

$$M_1 = \frac{1}{\Delta} \sum_{n=0}^{K-1} \sin^2(n\theta) \qquad \text{(A.3-1)}$$

$$M_2 = -\frac{1}{\Delta}\sum_{n=0}^{K-1}\cos(n\theta)\sin(n\theta) \tag{A.3-2}$$

$$M_3 = \frac{1}{\Delta}\sum_{n=0}^{K-1}\cos^2(n\theta) \tag{A.3-3}$$

$$\Delta = \sum_{n=0}^{K-1}\cos^2(n\theta)\sum_{n=0}^{K-1}\sin^2(n\theta) - \left[\sum_{n=0}^{K-1}\cos(n\theta)\sin(n\theta)\right]^2. \tag{A.3-4}$$

Then, the complex phasor of the signal $x(t)$ is expressed by

$$\hat{x} = X_c - jX_s. \tag{A.4}$$

From (A.2)–(A.4), we can estimate the fundamental phasor using the arbitrary data window length K. In particular, if the data window length equals one cycle or a half cycle, we obtain $M_1 = M_3 = 2/K$, and $M_2 = 0$. When $K = N$, (A.2) can be rewritten as

$$X_c = \frac{2}{N}\sum_{n=0}^{N-1}x_n\cos(n\theta) \tag{A.5-1}$$

$$X_s = \frac{2}{N}\sum_{n=0}^{N-1}x_n\sin(n\theta). \tag{A.5-2}$$

Thus, the conventional full-cycle DFT filter is given by

$$\hat{x} = X_c - jX_s = \frac{2}{N}\sum_{k=0}^{N-1}x_n e^{-jn\theta}. \tag{A.6}$$

Similarly, when $K = N/2$, and then we derive half-cycle DFT shown as follows:

$$\hat{x} = \frac{4}{N}\sum_{k=0}^{N/2-1}x_n e^{-jn\theta}. \tag{A.7}$$

Conventionally, a moving data window technique is utilized to estimate phasors [2]. A more popular approach for the implementation of the Fourier filters is to use a recursive estimation procedure [2], [3]. From (A.2), we can estimate the real and imaginary parts of a complex phasor by the following equation:

$$\begin{bmatrix} X_{c,r} \\ X_{s,r} \end{bmatrix} = \begin{bmatrix} M_1 & M_2 \\ M_2 & M_3 \end{bmatrix}\begin{bmatrix} Y_{c,r} \\ Y_{s,r} \end{bmatrix}. \tag{A.8}$$

The right-hand part of (A.8) can be calculated by a formula, which is given by

$$Y_{c,r} - jY_{s,r} = [(Y_{c,r-1} - jY_{s,r-1}) + x_r \cdot e^{-jK\theta} - x_{r-K}] \cdot e^{j\theta} \tag{A.9}$$

where r denotes the index of the moving data window. Thus, the complex phasor can be expressed by

$$\hat{x}_r = X_{c,r} - jX_{s,r}. \tag{A.10}$$

From (A.8)–(A.10), we can estimate a phasor using a recursive formula, which is valid for the arbitrary data window length K. The elements M_1, M_2, and M_3 in (A.8) are held to be constant when the data window length is fixed. These elements can be computed offline and stored *a priori* to save the amount of time required to perform online calculations.

Appendix B

1) The proof of $Z_1 = 2$.
We know that

$$e^{jn\theta} = \cos(n\theta) + j\sin(n\theta). \tag{A.11}$$

Substituting (A.3) and (A.11) into (18-1), we can get the following equation:

$$\begin{aligned} Z_1 = &\frac{\sum_{n=0}^{K-1}\sin^2(n\theta)}{\Delta} \cdot \left\{\sum_{n=0}^{K-1}[\cos(n\theta) + j\sin(n\theta)]\cdot\cos(n\theta)\right\} \\ &- j\frac{\sum_{n=0}^{K-1}\cos^2(n\theta)}{\Delta} \cdot \left\{\sum_{n=0}^{K-1}[\cos(n\theta) + j\sin(n\theta)]\cdot\sin(n\theta)\right\} \\ &+ j\frac{\sum_{n=0}^{K-1}\cos(n\theta)\sin(n\theta)}{\Delta}\cdot\sum_{n=0}^{K-1}[\cos(n\theta) + j\sin(n\theta)]^2 \end{aligned} \tag{A.12}$$

Then, (A.12) can be further reduced as

$$Z_1 = \frac{1}{\Delta}\left\{2\cdot\sum_{n=0}^{K-1}\cos^2(n\theta)\sum_{n=0}^{K-1}\sin^2(n\theta)^2 - 2\cdot\left[\sum_{n=0}^{K-1}\cos(n\theta)\sin(n\theta)\right]\right\}. \tag{A.13}$$

Therefore

$$Z_1 = \frac{1}{\Delta}\cdot 2\Delta = 2. \tag{A.14}$$

2) The proof of $Z_2 = 0$.
Similarly, substituting (A.3) and (A.11) into (18-2), we obtain

$$\begin{aligned} Z_2 = &\frac{\sum_{n=0}^{K-1}\sin^2(n\theta)}{\Delta} \cdot \left\{\sum_{n=0}^{K-1}[\cos(n\theta) - j\sin(n\theta)]\cdot\cos(n\theta)\right\} \\ &- j\frac{\sum_{n=0}^{K-1}\cos^2(n\theta)}{\Delta} \cdot \left\{\sum_{n=0}^{K-1}[\cos(n\theta) - j\sin(n\theta)]\cdot\sin(n\theta)\right\} \\ &+ j\frac{\sum_{n=0}^{K-1}\cos(n\theta)\sin(n\theta)}{\Delta}\cdot K. \end{aligned} \tag{A.15}$$

Then, (A.15) can be rearranged as follows:

$$Z_2 = \frac{1}{\Delta}\left\{-j\sum_{n=0}^{K-1}\cos(n\theta)\sin(n\theta) \cdot \sum_{n=0}^{K-1}[\cos^2(n\theta)+\sin^2(n\theta)] + j\sum_{n=0}^{K-1}\cos(n\theta)\sin(n\theta)\cdot K\right\}. \quad (A.16)$$

Since $\cos^2(n\theta)+\sin^2(n\theta)=1$, we can prove that

$$Z_2 = \frac{1}{\Delta}\cdot 0 = 0. \quad (A.17)$$

APPENDIX C

The parameters of the simulated system are shown as follows:

Equivalent source impedance

$$Z_{S1} = 0.238 + 15\ (\Omega) \quad Z_{S0} = 2.738 + 26.6\ (\Omega)$$
$$Z_{R1} = 0.238 + 16\ (\Omega) \quad Z_{R0} = 0.833 + 13.6\ (\Omega).$$

Transmission-line parameters

$$R_1 = 0.0196\ (\Omega/\mathrm{km}) \quad R_0 = 0.2079\ (\Omega/\mathrm{km})$$
$$L_1 = 1.011\ (\mathrm{mH/km}) \quad L_0 = 2.552\ (\mathrm{mH/km})$$
$$C_1 = 14.167\ (\mathrm{nF/km}) \quad C_0 = 7.668\ (\mathrm{nF/km}).$$

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