The Mixed Fuzzy Controller for Anti-braking Systems with Quasi-Nonholonomic Constraints

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Abstract

The quasi-constrained dynamics is composed of motions not only onto the constraint space but also onto the unconstraint or the "freedom" space, which may occur when the system escapes or slips away from its constraint manifold during high-speed motion. Traditionally, slippage phenomenon, which has been an issue in the automobile industry, is usually ignored because of its high frequency and strong nonlinear features. Conventional Frobenius theorem is focused on the holonomic dynamics, which are integrable on the freedom space. On the other hand, a complementary Frobenius theorem (CFT) is proposed to release conventional constraints from "hard" to "soft' models. In this paper, we derive a geometric formulation instead of its algebraic counterpart for acatastatically nonholonomic systems in the viewpoint of topology. Besides, we propose a mixed fuzzy controller (MFC) for the nonholonomic system with escaping motions, which includes a traditional controller for the hard subsystem and a non-traditional controller with fuzzy rules for another soft subsystem on the constraint manifold. The closed-loop stability of the nonholonomic system with an MFC scheme will be proved under admissible conditions. Finally, th proposed algorithm is applied to a wheeled vehicle with an anti-lock braking system (ABS) under the assumption of Coulomb's viscous friction. Computer simulation is used to justify the results.

Keywords: Topology, Manifold, Creep, Nonholonomy and ABS.

I. Introduction

Nonholonomic systems most commonly arise from mechanical systems where some of the constraints are not integrable. Physical models of versatile nonholonomic constraints have been significant topics because few of them can be classified under the existing theories in literature. Since 1890's, Ferrers, Korteweg, Bloch, MaCclamroch et al. [1,2,7,10,11,12,18] have devoted to the formulation of nonholonomic systems by means of Lagrange equation and Gaussian theorem in terms of algebraic viewpoint. They constructed equations of motion by reducing degrees of freedom from constraints and simulated the dynamics by prescribing additional given constraints. Traditionally, such an algebraic approach may successfully analyze nonholonomic systems for discrete points in state space. However, the coordinate transformation is so time-consuming that the real-time computation is hard to be realized if high-speed driving input is desired. The infinitesimal displacements of constrained systems on the tangent space of the geometric manifold are allowed to move under the satisfaction of modified Frobenius theorem (MFT) [5,8,9]. Those hard constraints are soften in the real world, such as compliance or deformations from the viscous friction on ABS (anti-braking system) and TCS (tracking control system). Some literatures handled the system with ABS in terms of sensor-based models [16,17]. The violation of flexible displacement on the constraint is in the pseudo sense, which includes the normal space and the tangent space of the manifold. In 1996, Huang and Wang [4,5,8] proposed the scheme of complementary distribution. Under the assumption of MFT, the nonsingular neighborhood instead of discrete points is formulated as the driving flow of constraint submanifolds.

This paper is organized as follows. In the next section, we will discuss the integrability of the Pfaffian forms on the geometric space. Besides, the admissible conditions of the reachability and the escaping dynamics on the annihilated space for dissipative mechanical systems with Pfaffian-one form constraints will be illustrated by different Frobenius theorems. In section three, Nonholonomically constrained systems will be decoupled into the constraint space and the unconstraint space. Meanwhile, the Lagrange equation with nonholonomic constraints will also be mapped from "normal form" to the annihilated space or the normal space of the constraint manifold. In section four, the well-known ABS is introduced by incorporating with Coulomb friction. In section five, we propose a mixed fuzzy controller (MFC), which includes a fuzzy controller for the escaping motion and a linearized controller for the "hard" constraint motion. In section six, the computer simulation of the creeping motion and linearized tracking will be illustrated. Finally, conclusion is followed.

II. Nonholonomic Dynamics on the Geometric Space

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Consider the Lagrange equation with nonholonomic constraints using two control inputs $f(\dot{q},q,\tau)=[B(q)\tau \quad f_c(\dot{q},q,\tau)]^T$. The system dynamics are given as

$$M(q)\ddot{q}(t)+c(\dot{q},q)=N^T(q))\lambda(t)+f(q,\tau) \\ =N^T\lambda(t)+[B(q)\tau \quad f_c(\dot{q},q,t)]^T \tag{1}$$

where $M(q)\in R^{n\times n}$ is the inertia matrix; $c(\dot{q},q)\in R^{n\times n}$ is the nonlinear damping terms; $N_{ij}\in C^{\infty}$ is the component of an m-dimensional constraint matrix or the normal matrix in $q(t)\in R^n$ space; $\lambda(t)\in R^m$ is the Lagrange multiplier or the constraint force. The term $f_c(\dot{q},q,t)$ is the control input of the soft part of the system, which comes from the contact force of four pneumatic tires. While $\tau(t)\in R^p$ is the control input form the hard part of the system and $B(q)\in R^{n\times p}$ is the hard gain matrix. The $m<n$ nonholonomically independent constraints in Pfaffian-one form $\varpi_i(q),\ i=1,\ldots,m$ is described as

$$\varpi_i(q)=\sum_{j=1}^{m}N_{ij}(q)dq_j+N_{0i}(q) \tag{2}$$

where $N_{0i}\in R^m$ means "creep" or acatastatic terms [10,11,12]. Define

$$dh_i(q)=\varpi_i(q)=L_g h_i(q)=\nabla h_i(q)\cdot g(q) \\ =\sum_{j=1}^{n}N_{ij}(q)dq_j=N_{i1}(q)dq_1+\cdots+N_{in}(q)dq_n=0 \tag{3}$$

Equation (1) is called holonomic or integrable if it can be reduced by differentiation of function $h_i(q)$, and $h_i(q)=c_i=const$ is called exact. Otherwise, equation (1) is called nonholonomic or nonintegrable. Let $\Omega_m(q)=[\varpi_1\cdots\varpi_m]$ be in the normal space of the constraint manifold $M\subset M_T$, where $M_T\in R^n$ is an n-dimensional geometric space. There exists a distribution $\Delta_{n-m}(q)\in R^{n-m}$ and $\Delta_{n-m}(q)=\Omega_m^{\perp}(q)$. The vector $V_i(q)\in\Delta_{n-m}(q),\ i=m+1,\ldots,n$ in the tangent space of the constraint manifold is defined in the direction of a vector function $g_i(q)\in\Delta_{n-m}(q)$ as

$$V_i(q)=\sum_{j=1}^{n}T_{ij}(q)\partial/\partial q_j \tag{4}$$

where $T_{ij}(q)\in R^{(n-m)\times n}$ is the tangent component of the constraint manifold. Using the Kronecker symbol $\delta_{ik}=0,\ i\neq k$, we have

$$\varpi_i(q)\cdot V_k(q)=\sum_{j=1}^{n}N_{ij}(q)T_{jk}(q)=\delta_{ik}=0 \tag{5}$$

The so-called "hard" constraints can be softened when the equality in Eq.(5) no longer holds.

Theorem 1: Modified Frobenius Theorem (MFT) [4,5]

Given a distribution $\Delta_1(q)=span[g_1(q),\cdots,g_{m-k}(q)]$ in an m-dimensional topological space, where $Rank(\Delta_1)=k<m\leq n$. There exists the involutive closure with the Lie bracket $[g_i,g_j]$ in $\Delta_1(q)$ and $g_i(q)\in R^m$ in the topological space $\Delta\in R^{n\times m}$. Then, $\Delta=\Delta_1\oplus\Delta_1^{\perp}=span[g_1,\cdots,g_m]$ is nonsingular and involutive if, and only if $[g_i,g_j]=(\nabla g_j)g_i-(\nabla g_i)g_j\in\Delta$.

□

MFT gives the sufficient and necessary conditions to the nonholonomic system such that the diffeomorphism of different submanifolds may hold on the tangent space.

Theorem 2: Complementary Frobenius theorem (CFT) [4,5,8]

Suppose that $\varpi_i^c\in M^c,\ i=m+1,\ldots,n$ represents an m-dimensional complementary vector of $\varpi_i(q),\ i=1,\ldots,m$, where $M^c\subset M_T$, and $\Delta_m^c(q)$ is the complementary distribution of the syetem. The terms $g_i^c(q),g_j^c(q)\in\Delta_m^c(q)$ are the complementary vectros of $g_i(q),\ g_j(q)$. Then, the Lie bracket $[g_i^c(q),g_j^c(q)]\in\Delta_m^c(q)$ exists if, and only if the complementary distribution $\Delta_m^c(q)$ is in the involutive closure.

□

The difference between the constraint manifold $M\in R^m$ and the complementary manifold M^C is the natural field and artificial field of the environment.

III. Acatastatic or "Creep" Nonholonomic Problems

Neglecting the higher order infinitesimal terms of the Pfaffian one-form equation, the acatastatically nonholonomic constraints with escaping motion can be described as

$$\varpi_k^c(q')=\sum_{j=1}^{n}N_{kj}^c(q)d\delta q_j+\delta N_{kj}^c(q)dq_j=0 \tag{6}$$

where "δ" is the notation of the exterior derivative. $\sum_{j=1}^{n}\delta N_{kj}^c(q)dq_j\neq 0$ is called the virtual constraint for the escaping motion. Assume that $N(q)\equiv[I_m \quad \bar{N}(q)]\in R^{m\times n}$ and $q(t)\equiv[q^c \quad q^u]^T$ and the full rank matrix $T_1(t)\in R^{n\times(n-m)}$ is defined as the normal form mapping, the original system is decomposed into the constrained subsystem $q^c(t)\in R^m$ and the unconstrained subsystem $q^u(t)\in R^{n-m}$ as

$$\dot{q}(t)=\begin{bmatrix}\dot{q}^c(t)\\ \dot{q}^u(t)\end{bmatrix}=\xi(q,t)+T_1(t)\dot{q}^u(t) \tag{7}$$

where $\xi(q,t)=[-\bar{N} \quad 0]^T,\ T_1(t)=[-\bar{N}^T(q) \quad I_{n-m}]^T$. Then, we have

$$\ddot{q}(t)=\frac{\partial\xi}{\partial q}\dot{q}(t)+\frac{\partial\xi}{\partial t}+T_1(t)\ddot{q}^u(t)+\dot{T}_1(t)\dot{q}^u(t) \tag{8}$$

Let $\bar{N}(q)\in R^{m\times n-m}$ be a full rank matrix, then, the normal form mapping $T_1(t)\in R^{n\times(n-m)}$ is described as

$$T_1(t)=[-\bar{N}^T(q) \quad I_{n-m}]^T=span\ [\phi_{t_1}^{g_1}\ \cdots\ \phi_{t_{n-m}}^{g_{n-m}}] \tag{9}$$

Suppose that $N(q)\in R^{m\times n}$ is spanned on the normal distributions, then,

$$N(q)\equiv[I_m \quad \bar{N}(q)]=span[\phi_{t_1}^{g_1}\ \cdots\ \phi_{t_m}^{g_m}]^{\perp} \tag{10}$$

where $\phi_{t_{i+1}}^{g_{i+1}},i=1,\ldots,n-m$ is the drift flow in the tangent direction $g_{i+1}(q)$ of the constrained manifold within time step t_{i+1}. Then,

$$T_1^T(t)\{M(q)T_1(t)[\frac{\partial \xi}{\partial q}\dot{q}(t)+\frac{\partial \xi}{\partial t}+T_1(t)\ddot{q}^u(t)+\dot{T}_1(t)\dot{q}^u(t)] \quad (11)$$
$$+c(\dot{q},q)-N^T\lambda(t)\}=T_1^T(t)[B(q)\tau(t) \quad f_c(\dot{q},q,t)]^T$$

Let $\tilde{u}(\tau)=[T_1^T M(q)T_1]^{-1}B(q)$ and $\tilde{f}_c(\tau)=[T_1^T M(q)T_1]^{-1}[f_c(\dot{q},q,t)-C(\dot{q},q,t)]$ be ABS control inputs. Then, we have

$$\dot{\tilde{q}}^c(t)=F(q^c,q^u,t)+G(q^c,q^u)\dot{q}^u(t) \quad (12)$$
$$\ddot{q}^u(t)\equiv \tilde{u}(t)$$

where $\tilde{q}^c(t)\in R^{2m}$, $F(q^c,q^u,t)\equiv[N_0(q,t_0)0000]^T$ and $G(q^c,q^u)\equiv[\tilde{N}(q,t_0)0000]^T$. While $\hat{u}(\tau)\in R^p$ is the inertial force in the unconstrained space. The constraint forces can also be described as the Lagrange multipliers $\lambda(t)\in R^m$, which can be found by substituting the generalized forces into the system

$$\lambda(t)=[N(q)N^T(q)]^{-1}N(q)\{M(q)\ddot{q}(t)+c(\dot{q},q) \quad (13)$$
$$-[B(q)\tau(t) \quad f_c(\dot{q},q,t)]^T\}$$

By defining a nonsingular matrix $T_2(t)\in R^{2m\times 2m}$, the normal form is transformed into a quasi-linear equation, i.e., SSEL equation

$$z(t)=T_2(t)\tilde{q}^c(t) \quad (14)$$

Substituting the Lagrange equation into the normal form mapping, the generalized coordinates are similarly transformed into SSEL space. Note that the variable $z(t)$ is decoupled into the "hard" subset and the "soft" subset as

$$\dot{z}(t)=\begin{cases}\dot{z}_H(t)=A_H z_H(t)+B_H v(\dot{q}^c,q^c,t)\\ \dot{z}_S(t)=L_1(\dot{z},z,t)+L_2(t)\tilde{f}_c(\dot{z},z,t)\end{cases} \quad (15)$$

where A_H is the state matrix and B_H is the control gain of the 2m-dimensional hard-constrained subsystem $z_H(t)\in R^h$. While $z_S(t)\in R^{2m-h}$ is the escaping coordinate with soft constrained subsystem. Then, the internal dynamics $v(\dot{q}^c,q^c,t)$ is formulated as.

$$v(\dot{q}^c,q^c,t)=L_f^r h(q^c)+L_g L_f^{n-1}h(q^c)+B_H\tau(\dot{q}^c,q^c,t) \quad (16)$$

The reachability and the stability of the controller can not be assured if the inverse mapping $T_1^{-1}(t)$ does not exist. The nonlinearity of the internal dynamics will be assumed to be "zero dynamics", i.e., $L_f^r h(q^c)+L_g L_f^{n-1}h(q^c)\tau(t)=0$. Namely, it can be settled down in the steady state.

$$\lim_{t\to\infty} v(\dot{q}^c,q^c,t)=L_f^r h(q^c)+L_g L_f^{n-1}h(q^c)+B_H\tau(t)=B_H\tau(t) \quad (17)$$

In the next section, an ABS algorithm is introduced under the assumption of "soft" constraints, which allows escaping motion from the Coulomb's viscous friction.

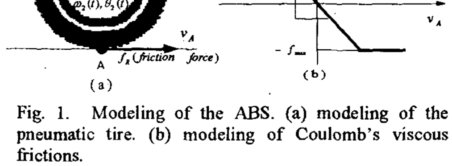

Fig. 1. Modeling of the ABS. (a) modeling of the pneumatic tire. (b) modeling of Coulomb's viscous frictions.

IV. Anti-braking Systems with Coulomb's Friction

The tires are the only part of a vehicle in contact with the road. When a car is in the process of braking or accelerating, longitudinal forces occur between the tire and the road. During a cornering maneuver, those forces are lateral counterparts. Both the longitudinal and the lateral forces lead to tire deformation and escaping motions over its entire circumstance [16,17], which is illustrated in Fig.1. Suppose that the Coulomb's viscous braking forces in the lateral and the transverse directions are

$$f_1(t)=-b_R v_A(t)=-b_R[v_1(t)-r\omega_2(t)] \quad (18)$$

where $f_1(t)$ and b_R represent the friction force and the friction coefficient in the radial directions, respectively. The term $v_A(t)$, $v_1(t)$ are the ground velocity and the axle velocity of the wheel, respectively. Similarly, the term r and $\omega_2(t)$ are radius and the angular speed of the wheel, respectively. Meanwhile,

$$f_2(t)=-b_T v_2(t) \quad (19)$$

where $f_2(t)$ and b_T are the friction force and the friction coefficient in the transverse direction, respectively. On the other hand, ABS control inputs $\tilde{f}_c(\dot{q},q,t)$ of the controller given by the numerical results of the pedal command from the car driver are in form of Fourier's series. The model of ABS commanded by ways of Fourier series is an approximate but effective method in comparison to the sensor-based model. It is illustrated as

$$\tilde{f}_c(\dot{q},q,t)\approx f_c(nT_n)=\sum_{n=1}^{\infty}a_n\cos(\omega_n T_n)+b_n\sin(\omega_n T_n) \quad (20)$$

where a_n, b_n are magnitudes or coefficients of the Fourier series, ω_n is the ABS braking frequency and T_n is the real-time braking period of the ABS, as given in Fig.1.

The Nonlinear term of the escaping dynamics is reformulated by incorporating of the parameters of versatile creeping coefficient α_I, β_R and γ_R. In order to simplify formulations derived above, we rearrange creep equations of motion in forms of dimensionless algorithm by appending a subscript notation "$_0$". Namely,

$$z''_{10}+(1-\gamma_R)z'_{10}+z_{10}=\omega_{10}z'_{20}+\gamma_R\omega_{20} \quad (21)$$

where $z_{i0}(t)$ is the dimensional state of the i-th degree of freedom in the longitudinal or the lateral direction on wheels. On the other hand, the rotational creeping dynamics $\theta_{i0}(t)$ is described as the dimensional form by imposing on coefficients α_I, β_R and γ_R as

$$\alpha_J\theta''_{20}(t)+(\beta_R+\gamma_R)\theta'_{20}(t)+\theta_{20}(t)=\gamma_R z'_{10}(t) \quad (22)$$

where α_I, β_R are tire-escaping coefficients on the ground and γ_R is the proportional coefficient of the tire escaping motion. Assume that both the speed v_{A0} and v_{10} at the contact point are specified as

a) $v_{A0}=0$ and $v_{10}=0$, or
b) constant value $v_{A0}=const$ and $v_{10}=const$, or
c) proportional to ω_{10}, or
d) $v_{A0}=const$ and $v_{10}=const$ plus linear ω_{10} terms.

The related parameters for the optimal procedure can be obtained by a series of multistage decision-making criterion.

The Algorithm of the Mixed Fuzzy Controller (MFC)

MFC is basically a composite nonholonomic controller, which is composed of a state space exact linearization (SSEL) controller and a non-traditional fuzzy controller. The former is designed along an admissible trajectory on the "hard" constraint. The latter is designed to regulate the Coulomb friction force as "soft" constraints from the tires of versatile vehicle. The decisions of the fuzzy logic are based on inputs in the form of linguistic variables derived from membership functions. To perform the computational rules of various inferences, the response of each rule is weighted according to the confidence or degree of membership of its inputs. The algorithm consists of knowledge base, inference engine, data base, fuzzification and defuzzification.

A) The knowledge base of the "soft" controller

A set of the database and a set of the rule base constitute the knowledge base of the "soft" controller. The database of the knowledge base consists of three subsets, which are small (S), medium (M) and large (L). According to the empirical data from the Ford motor company and the international journal of automobile [16,17], the membership function can be classified as different levels, as shown in Fig.2

Level	#1:			Detect forces	<	-50N
Level	#2:	-50N	<	Detect forces	<	-40N
Level	#3:	-40N	<	Detect forces	<	-30N
Level	#4:	-30N	<	Detect forces	<	0N
Level	#5:	0N	<	Detect forces	<	30N
Level	#6:	30N	<	Detect forces	<	40N
Level	#7:	40N	<	Detect forces	<	50N
Level	#8 :			Detect forces	<	50N

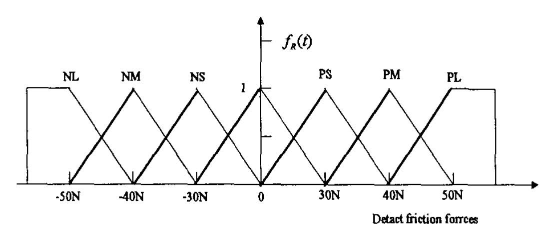

Fig. 2 Membership function of the friction force

B) Inference Engine

The inference engine of the "soft" controller is a forward data-driven inference. Suppose that there exists a multi-input and multi-output (MIMO) system with the following linguistic formulas

$$R = \{R^1_{MIMO},\quad R^2_{MIMO},\quad R^3_{MIMO}\} \tag{23}$$

where the i-th fuzzy logic rule is

$$R_{MIMO} = \quad IF\ (x_1\ is\ A_1 and\ \cdots\ and\ x_p\ is\ A_p) \tag{24}$$

The general rule can be decomposed into a series of multi-input and single input (MISO) rules. For instance,

"IF the trajectory is *no bias* and the anti-lock braking system is *forward slip*, then brake ABS button starts."

C) State Space Exact Linearization for the "hard" controller

The SSEL equations of motion are on the "hard" constraint space

$$z_H(t) = A_H z_H(t) + B_H v(t) \tag{25}$$

While the control law $k(t)$ is defined as

$$v(t) = k(t) * z_H(t) \tag{26}$$

where "*" represents the convolution of two analytical functions. In designing the SSEL controller, we propose a well-known proportional-integral-derivative (PID) algorithm to tune different kinds of dominant poles and zeros [14]. Defining $e_H(t) = y^d_H(t) - y_H(t)$ as the error or the difference of the hard-constraint space between desired outputs and the system outputs

$$v(t) = k_P e_H(t) + k_I \int e_H(\tau) d\tau + k_D \cdot [de_H(t)/dt] \tag{27}$$

where k_P, k_I and k_D are the proportional constant, the integral constant and the derivative constant, respectively. They will be properly design according to various circumstances of the escaping motion.

Theorem 3: The stability of the quasi-constrained system

Suppose that there exists an m-dimensional state equation in the complementary space $\varpi^c_i(q) \in M^c$, $i = m+1, \ldots, n$, where $\dot{z}(t)$ is SSEL state, $M^c \subset M_T$ and $\Delta^c_m(q) = T_q M^c \subset M_T$. Namely,

$$\dot{z}(t) = \begin{bmatrix} z_H(t) \\ z_S(t) \end{bmatrix} = \begin{cases} A \cdot z_H(t) + Bv(t) \\ L_1(\dot{z}, z, t) + L_2(t) f_c(\dot{z}, z, t) \end{cases} \tag{28}$$

By prescribing an admissible norm $\|z_s(t) - z_0\| < L_0$, the outputs of the soft subsystem are bounded for bounded inputs $\|f_c(\dot{z}, z, t)\| < L_c$. The global system is closed-loop Lyapunov stable if, and only if the PID controller is closed-loop stable in the hard states and all soft states of the fuzzy controller are bounded within the time interval $t_0 \le t \le t_f$.

Proof:

A) Necessary condition

Consider the state equation of the hard space of the nonholonomic system as

$$\dot{z}_H(t) = A_H z_H(t) + B_H(t) v(t) = A_H z_H(t) + B_H(t) v(t)$$
$$y_H(t) = C_H(t) z_H(t) \tag{29}$$

where $C_H(t)$ is the output gain of the hard constrained system. Under the assumption of the modified Frobenius theorem, the control system of the SSEL plant is nonsingular. Besides, the drift flows are involutive closures and the system is reachable within the admissible region and the time interval $t_0 \le t \le t_f$. Thus, the PID controller can be defined as

$$v(t) = k_P e_H(t) + k_I \int e_H(\tau) d\tau + k_D \cdot [de_H(t)/dt] \tag{30}$$

By appropriately tuning the PID constants k_P, k_I and k_D, the closed-loop poles can be placed in the left hand side of the s-plane, i.e.,

$$\dot{z}_H(t) = A_H z_H(t) + Bv(t) = (\sigma_i I - A_H) z_H(t) \tag{31}$$

The output of the time-varying plant becomes

$$y_H(t) = C_H(t)\int_{t_0}^{t_f} \Phi(t,\tau) B_H(\tau) v(\tau) d\tau \tag{32}$$
$$= \int_{t_0}^{t_f} [C_H(t)\Phi(t,\tau)B_H(\tau)] v(\tau) d\tau = \int_{t_0}^{t_f} G_H(t,\tau) v(\tau) d\tau$$

where $\Phi(t,\tau)$ is the state transition matrix and $G_H(t,\tau)$ is defined as the impulse/ response matrix of the system, i.e.,

$$G(t,\tau) = \begin{cases} C_H(t)\Phi(t,\tau)B_H(\tau) & t \ge \tau \\ 0 & t < \tau \end{cases} \tag{33}$$

Besides, all states of the fuzzy controller and the soft plant are bounded, which are assumed to be the inputs of the hard plant. Then, we conclude that the system is causal and dissipative if all eigen modes of the system is negative or $G_H(t,\tau) < 1$.

B) Sufficient condition

The normal form of the hard space is mapped into a quasi-linear equation with $\phi_{t_{i+1}}^{g_{i+1}}, i = 1,\dots,n-m$. Defining a nonsingular matrix $T_2(t)$ in the soft space, we have

$$T_1(t) = [-N^T(q) \;\; I_{n-m}]^T = span\,[\phi_{t_1}^{g_1}(q) \;\cdots\; \phi_{t_{n-m}}^{g_{n-m}}(q)] \tag{34}$$
$$T_2(t) = [-N^{\beta}(q) \;\; I_m]^T = span\,[\eta_1^{g_1^c}(q) \;\cdots\; \eta_m^{g_m^c}(q)]$$

Thus, the closed-loop stability of the global escaping system is invariant under the assumption of diffeomorphic mapping. Namely,

$$T(t) = \begin{bmatrix} T_1(t) & 0 \\ 0 & T_2(t) \end{bmatrix}, \quad q(t) = T(t)^{-1}[z_H(t) \;\; z_c(t)] \tag{35}$$

Reviewing the driving flow of the escaping motion, which will be bounded in the composition of $\phi_1^{g_1^c}(q)$ to $\phi_m^{g_m^c}(q)$ with bounded inputs and bounded outputs. According to the CFT, we know that $\partial\Theta(z,q_0)/\partial z_i$, $i = 1,\cdots,m$ are independent and lie in the involutive distribution Δ_m^c of the virtually complementary manifold M^C. Namely,

$$z(t) = \Theta(z, q_0) = \phi_1^{g_1^c}(q) \circ \cdots \circ \phi_m^{g_m^c}(q) \tag{36}$$

where $\|z(t) - z_0\| < L_0$ and L_0 is the admissible norm of the system. Thus, we conclude that the local hard system is closed-loop stable and the local soft system is bounded if, and only if the global system is BIBO stable within the time interval $t_0 \le t \le t_f$.

□

V. Examples of Wheeled Vehicles

A wheeled vehicle with pneumatic tires on the ground is depicted with generalized coordinates $q(t) = [q_1 \;\; q_2 \;\; q_3 \;\; q_4]^T$ and the virtual displacement $z(t) = [z_1 \;\; z_2 \;\; z_3 \;\; z_4]^T$, as shown in Fig. 3.

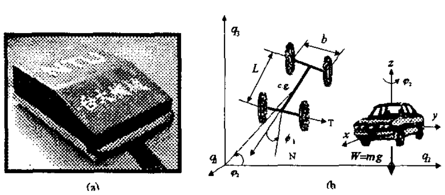

Fig.3. Modeling of wheeled vehicle (a) an AGV on a prescribed trajectory, (b) modeling of the lumped system for a four-wheeled vehicle

part 1: The construction of the nonholonomic constraints

The system can be obtained as

$$\dot{x}\sin(\phi_1 + \phi_2) - \dot{y}\cos(\phi_1 + \phi_2) = 0 \tag{37}$$
$$\dot{x}\cos(\phi_1 + \phi_2) + \dot{y}\sin(\phi_1 + \phi_2) - (l/\sin\phi_1)\dot{\phi}_1 = 0$$

where l is the length of the wheel axles and ϕ_0 is the initial value. The constraints are not integrable if $[g_3, g_4](q) \notin \Delta$. Thus, we map the system into normal form using the Lagrange equation

$$T_1^T(t) = \begin{bmatrix} -\sin(\phi_1 + \phi_2) & -\cos(\phi_1 + \phi_2) & 1 & 0 \\ \cos(\phi_1 + \phi_2) & -\sin(\phi_1 + \phi_2) & 0 & 1 \end{bmatrix} \tag{38}$$

The normal form can be derived from the unconstrained dynamics $\dot{q}^u(t) \equiv u(t) \in R^{n-m}$ and $\dot{q}^c(t) = \alpha_0 \dot{q}(t) + \beta_0$, where

$$\alpha_0 \approx \begin{bmatrix} 0.89 s_{12}/c_{12} - 1.31/s_{12} & -2.81c_{12} & -0.02 & 0 \\ 0.25c_{12} & -2.54c_{12}/s_{12} - 0.25/c_{12} & 0 & 1.02 \end{bmatrix} \tag{39}$$
$$\beta_0 \approx [1.05c_{12}/t_{12} - 0.04/s_{12} \;\; -0.75c_{12}]^T$$

Part 2: Fuzzy controller of the creeping motion

Define the predictive fuzzy controller as

$$z(k+1) = f[z(k),\cdots,z(k-m), u(k),\cdots,u(k-m)] \tag{40}$$

where $z_e(i)$, $\bar{z}(i)$ are the estimated and the prescribed trajectory, respectively.

Table 1. Multistage parameters of rule base for MFC

Fuzzy Logical Parameters	Factors (A)	DOF (f)	Variation (S)	Covariant (V)	Contributions (100%)
Friction Forces (N)	B	2	8.45	2.1	33.33
Creeping (m/s)	C	3	2.35	0.5	33.33
ABS (N)	D	2	6.14	1.51	33.33

Part 3: PID design of the "hard" subsystem

Let the control law be defined as $v(t) = k(t) * z_H(t)$. Let $e_H(t) = y_H^d(t) - y_H(t)$ be the error or the difference of the hard-constraint space between desired outputs and the system outputs. Then

$$v(t) = k_P e_H(t) + k_I \int e_H(\tau) d\tau + k_D \cdot [de_H(t)/dt] \tag{41}$$

Using the ultimate period method [14], the integral time

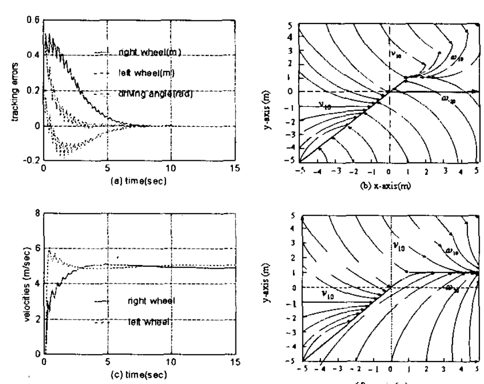

Fig. 4. Computer simulations (a) tracking errors the vehicle, (b) velocities of the wheels, (c) the creeping motion for u_{R0} =0.3, (d) the creeping motion u_{R0} = 1.0.

constant is assumed as $T_I = \infty$, the derivative time constant is assumed as $T_D = 0$ and the proportional parameter is assumed as $k_P = 0.005$. Tuning k_P slightly until the "hard" system becomes marginally stable, where $k_P = (0.55 \sim 0.62) \cdot k_U$, $T_I = 0.9\pi / \omega_D \sim 1.05\pi / \omega_D$, $T_D = 0.242\pi / \omega_D \sim 0.306\pi / \omega_D$. The computer simulation for the PID design is given in Fig.4.

VI. Conclusion

Observing the simulations in Fig.4, the slippage phenomenon of the geometric scheme is damped out by using admissible parameters. Comparing to the traditionally algebraic approach, the advantages of MFC include less computational time, more effective controllers, significant physical meaning for the driving flow, higher speed motions, and geometric formulations for the tracking or regulating problems. Besides, a PC-based instead of a sensor-based analysis may be a better choice in the ABS research. It may reduce experimental cost in the laboratory. However, the tracking approximation of the MFC algorithm does not behave excellent if we ignore unmodeled effects by eliminating nonlinearity of the escaping motion. In addition, the mechanical structure is modeled as lumped subsystems, which ignores the complexity of the suspension system and the lateral friction of their four tires. The trend to develop better mobile systems may be focused on multi-stage controllers, which include effects from both the reaction forces of tracking control system (TCS) and the escaping motion of an ABS. By applying the structural dynamics, acatastatically escaping problems can be considered as a flexible model instead of a lumped model, which is a challenging job to research in the future.

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