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Target Tracking in an Environment of Nearly Stationary and Biased Clutter

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Abstract

The Probabilistic Data Association (PDA) filter deals with tracking a single target in an environment of randomly distributed clutter. Significant performance degradation occurs when the measurements originate from a biased and nearly stationary clutter rather than a non-stationary nor non-biased one. We propose a modified PDA filter to achieve successful tracking in such an environment. Simulation results demonstrate the feasibility of the proposed approach.

Keywords: Probabilistic data association, Kalman Filter, Visual tracking.

1 Introduction

In many tracking problems, a vision sensor provides the coordinates of each detected target. Measurements are affected by noise which is modeled as Gaussian. As the sensor repeats the process of detection, the sequence of images can be processed in a proper filter to smooth the measurement noise, thus providing target tracks. In the practical case, the problem is complicated by the false alarms owing to system noise and clutter. By clutter we refer to the undesirable detections of the nearby objects, electromagnetic interference, etc. This may lead to several measurements in the "validation region" [1] of a single target. The sensor information of the target of interest is called the "target-originated measurement" in the Probabilistic Data Association (PDA) literature. The PDA Filter [1][2] is a technique which handles the difficulty of false data, where the measurements are processed with a probabilistic weighting within the state estimation procedure.

The original PDA filter assumed a single *a priori* probability density function for the false measurements [1]. Recently, the work [4] establishes a new false measurement model of multiple *a priori* probability density functions. Although these models might work for radar/sonar tracking missions, they are not valid in visible-light object tracking, which is of great concern in the robotics and automation society. Data distortion factors, such as partial occlusion and presence of clutter, can lead to false alarms whose distribution are unknown *a priori* [2], but which usually occur at almost the same position in consecutive frames with small variance. In other words, false measurements should be modeled with nearly stationary, biased, but unknown probability density function.

In this paper, we propose a new filter to remedy the weakness of the PDA filter when

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dealing with nearly stationary and biased clutter. This method does not require *a priori* knowledge about the distribution of the clutter.

In section 2, we present a stochastic model based on which all the underlying objects are represented. Section 3 briefly describes the elements of the standard PDA filter. Section 4 describes the proposed modified PDA filter featured in a new construction of the data association probabilities. Section 5 details the comparison of the standard approach and the proposed one through a number of simulations. Finally, some concluding remarks are given in section 6.

2 The Stochastic Model

The dynamics of the target of interest are modeled by the equation

$$\mathbf{x}(k+1) = \mathbf{F}\,\mathbf{x}(k) + \mathbf{v}(k)$$

where the process noise vector $\mathbf{w}(k)$ is assumed to be white Gaussian with mean zero, and

$$E\{\mathbf{w}(j)\mathbf{w}(k)^{T}\} = \delta_{jk}\mathbf{Q}\ .$$

The measurement system is modelled as follows. If the measurement originates from the target (instead of from clutter), then

$$\mathbf{z}(k) = \mathbf{H}\mathbf{x}(k) + \mathbf{w}(k)\ .$$

and the measurement noise is taken to be white Gaussian with mean zero, and

$$E\{\mathbf{w}(j)\mathbf{w}(k)^{T}\} = \delta_{jk}\mathbf{R}\ .$$

Note that both the matrices **Q** and **R** are assumed known.

3 The PDA Filter

The Probabilistic Data Association (PDA) filter [1][2] is used to handle the problem of measurement origin uncertainty. It computes the posterior association probabilities for all current candidate measurements in a validation gate and uses them to form a weighted sum of innovations for updating the target's state in a suitably modified version of the Kalman Filter. Assume that the PDA filter has been initialized using one of several available techniques [1], and false measurements enter the system since time step k.

3.1 The Validation Region

The measurements are reduced to a set of m_k validated measurements by defining the following validation region

$$\widetilde{V}_k(\gamma) = \{\mathbf{z}_i : \mathbf{v}_i^{T}(k)\,\mathbf{S}^{-1}(k)\,\mathbf{v}_i(k) \le \gamma\}$$

where $\mathbf{v}_i(k) = \mathbf{z}_i(k) - \hat{\mathbf{z}}(k\,|\,k-1),\ i = 1,\ldots,m_k,$ is the innovation. Each measurement $\mathbf{z}_i$ that lies within this region is considered validated. The threshold γ is obtained from tables of the chi-square distribution [1], since the weighted norm of the innovation that defines the validation region is chi-square distributed with the number of degrees of freedom equal to the dimension of the measurement.

3.2 State Estimation

Suppose that at time k there are a number of m_k validated measurements. The set of validated measurements at time k is denoted by

$$Z(k) = \{\,\mathbf{z}_i(k)\}_{i=1}^{m_k}$$

and the cumulative set of measurements up to time k is

$$Z^{k} = \{Z(j)\}_{j=1}^{k}$$

Define the events $\theta_i(k) = \{\mathbf{z}_i(k)$ is the target originated measurement$\}$, $i = 1,\ldots,m_k$, and $\theta_0(k) = \{$none of the measurements at time k is target-originated$\}$ with probability

$$\beta_i(k) = P\left\{\theta_i(k)|Z^k\right\}, \quad i=0,1,\ldots,m_k.$$

The procedure that yields these probabilities is called Probabilistic Data Association, and will be given in the next section. At the moment, we assume they are known.

By the total probability theorem [1], the conditional mean of the state can be written as

$$\begin{aligned}\hat{\mathbf{x}}(k\,|\,k) &= E\left\{\mathbf{x}(k)\,|\,Z^k\right\} \\ &= \sum_{i=0}^{m_k} E\left\{\mathbf{x}(k)\,|\,\theta_i(k), Z^k\right\} P\left\{\theta_i(k)\,|\,Z^k\right\} \\ &= \sum_{i=0}^{m_k} \hat{\mathbf{x}}_i(k\,|\,k)\beta_i(k)\end{aligned}$$

where $\hat{\mathbf{x}}_i(k\,|\,k)$ is the updated state estimate conditioned on the event that the i^{th} validated measurement is correct. This is given by the standard Kalman Filter as

$$\hat{\mathbf{x}}_i(k\,|\,k) = \hat{\mathbf{x}}(k\,|\,k-1) + \mathbf{W}(k)\,\mathbf{v}_i(k)$$

where $\mathbf{v}_i(k) = \mathbf{z}_i(k) - \hat{\mathbf{z}}(k\,|\,k-1)$ is the corresponding innovation, and $\mathbf{W}(k)$ is the standard Kalman gain. The error covariance associated with the updated state estimate is defined as

$$\mathbf{P}(k\,|\,k) = \left\{\left[\mathbf{x}(k) - \hat{\mathbf{x}}(k\,|\,k)\right]\left[\mathbf{x}(k) - \hat{\mathbf{x}}(k\,|\,k)\right]^{\mathrm{T}}\middle|Z^k\right\}$$

and can be evaluated [1] by

$$\mathbf{P}(k\,|\,k) = \beta_0(k)\mathbf{P}(k\,|\,k-1) + [1-\beta_0(k)]\mathbf{P}^c(k\,|\,k) + \tilde{\mathbf{P}}(k)$$

where

$$\tilde{\mathbf{P}}(k) = \mathbf{W}(k)\left(\sum_{i=1}^{m_k} \beta_i(k)\,\mathbf{v}_i(k)\,\mathbf{v}_i^{\mathrm{T}}(k) - \mathbf{v}(k)\,\mathbf{v}^{\mathrm{T}}(k)\right)\mathbf{W}^{\mathrm{T}}(k)$$

and

$$\mathbf{P}^c(k\,|\,k) = \left(\mathbf{I} - \mathbf{W}(k)\mathbf{H}\right)\mathbf{P}(k\,|\,k-1).$$

Now that we have obtained $\hat{\mathbf{x}}_i(k\,|\,k)$, it remains to find out the values of $\beta_i(k)$.

3.3 The PDA Weights

The *association probabilities* $\beta_i(k)$ are developed in detail in [1] and require knowledge of the probability mass function of the number of false measurements. Assuming a nonparametric diffuse prior [1], the association probabilities are given by

$$\beta_i(k) = \frac{e_i}{\sum_{i=0}^{m_k} e_i}\,, \quad i=0,1,\ldots,m_k \tag{1}$$

where

$$e_i = \exp\left(-\frac{1}{2}\mathbf{v}_i^{\mathrm{T}}(k)\mathbf{S}^{-1}(k)\mathbf{v}_i(k)\right), \quad i=1,\ldots,m_k$$

$$e_0 = (2\pi/\gamma)^{n_z/2} m_k c_{n_z}\left(1 - P_D P_G\right)/P_D \tag{2}$$

with P_G being the probability that the target-originated measurement falls within the validation gate, P_D being the probability that the correct measurement is detected, and c_{n_z} being the volume of the n_z dimensional unit hyper-sphere$\left(c_1 = 2, c_2 = \pi, c_3 = 4\pi/3, etc.\right)$[1].

4 The Modified PDA Filter

In this section, we state the main contribution of the paper. The derivation of the association probabilities (1) required knowledge of the probability mass function of the number of false measurements [1]. Both parametric and non-parametric models [1][3][4] were used for this PMF, but as argued in section 1, these models are not realistic due to the nearly stationary and biased characteristics of clutter in certain kinds of tracking missions, especially in visible-light target tracking,. We explicitly take these characteristics into account in modifying the derivation of the association probability.

Define

$$m_i(k) = \min_j \left\|z_i(k) - z_j(k-1)\right\|$$

$$i=1,\ldots,m_k\,,\ j=1,\ldots,m_{k-1}$$

The points $m_i(k)$, $i=1,\ldots,m_k$, can be separated into two clusters in one-dimensional space: the cluster with smaller values is originated from current-previous measurement pairs,

($z_i(k), z_j(k-1)$), with both measurements being nearly stationary; the cluster with larger values comes from either "nonstationary-nonstationary" pairs, or "nonstationary-stationary" pairs. The target-originated measurement will belong to the latter cluster due to the motion of the target.

We could use hypothesis testing or statistical pattern recognition techniques to find the decision boundary for the two clusters, and the discriminant function would be helpful in deriving the modified association probabilities. The idea is that the larger the probability a measurement belongs to the non-stationary class is, the larger the probability that this measurement is the target-originated one will be.

By converting the distribution of $m_i(k)$ into a histogram, the hypothesis testing problem reduces to an one-dimensional thresholding problem. We suggest the iterative threshold selection scheme [5] for finding the threshold value:

Step 1. Assuming no knowledge about if a measurement is stationary (Class A) or non-stationary (Class B), arbitrarily select one measurement from $i=1,\dots,m_k$ and assign it to Class A, the others to Class B. Assign $T^0=0$.

Step 2. At step t, compute

$$\mu_{st}^t = \frac{\sum_{i\in Class\ A}|m_i(k)|}{\#\text{of Class A measurements}}$$

$$\mu_{nst}^t = \frac{\sum_{i\in Class\ B}|m_i(k)|}{\#\text{of Class B measurements}}$$

$$T^{t+1} = \frac{\mu_{st}^t + \mu_{nst}^t}{2}$$

Step 3. Use T^{t+1} as the threshold for classification:

$$\begin{array}{ll} \text{if } |m_i(k)| < T^{t+1}, & i \in \text{Class A} \\ \text{else} & i \in \text{Class B} \end{array}$$

Step 4. If $T^{t+1} \neq T^t$, return to step 2; else, the optimal threshold is found, and all measurements are classified into either Class A or B.

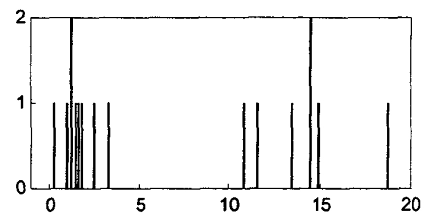

Fig. 1: A typical histogram of $m_i(k)$. We use the iterative threshold selection scheme to find the optimal threshold.

After obtaining the optimal threshold, we could define the discrimination function as

$$\zeta_i(k) = \begin{cases} 1, & \text{if } i \in \text{non-stationary measurement} \\ 0, & \text{if } i \in \text{stationary measurement} \end{cases}$$

$$i=1,\dots,m_k,$$

and

$$\zeta_0(k)=1.$$

Finally, we define the *modified association probabilities* as

$$\beta_i(k) = \frac{1}{C} \times \zeta_i(k) \times \frac{e_i}{\sum_{i=0}^{m_k} e_i}, \quad i=0,1,\dots,m_k$$

where

$$C = \sum_{i=0}^{m_k} \left(\zeta_i(k) \times \frac{e_i}{\sum_{i=0}^{m_k} e_i} \right)$$

with e_i, $i=0,1,\dots,m_k$ being defined as in (2).

Question arises if such a decision boundary or threshold does not exist or is ill-defined. In fact, this implies that the oscillation of the nearly stationary false measurements is too large to be discriminated from the non-stationary ones. In this case, all measurements belong to the same class (non-stationary), and the discriminant function will assign equal probability to all measurements. The proposed modified PDA filter then reduces back to the standard PDA filter. This argument suggests that the proposed modified PDA filter would work at least as good as the original standard PDA filter in a pure non-stationary clutter environment, while superior in a hybrid (non-stationary and stationary) clutter environment.

5 Simulation Results

Simulations were done for a target modeled as moving with constant velocity in a plane with process noise accounting for slight changes in velocity. The plant equation for this model, discretized with time interval T, is

$$\mathbf{x}(k+1) = \mathbf{F}\mathbf{x}(k)$$

where the state is

$$\mathbf{x} = [x \quad \dot{x} \quad y \quad \dot{y}]^{T}$$

and

$$\mathbf{F} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The measurements originated from the target in track are given by

$$\mathbf{z}(k) = \mathbf{H}\mathbf{x}(k) + \mathbf{w}(k)$$

where

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

and

$$E\{\mathbf{w}(k)\} = 0;\ E\{\mathbf{w}(j)\mathbf{w}(k)^{T}\} = \mathbf{R}\delta_{jk}$$

The simulation used the following numerical values: sampling interval $T = 1\,\mathrm{s}$, $R_{11} = R_{22} = 200\,\mathrm{m}^2$, $R_{12} = R_{21} = 0$, $P_D = 1$, $P_G = 0.95$, $\mathbf{x}(0) = [200\,\mathrm{m} \quad 0 \quad 10^4\,\mathrm{m} \quad -15\,\mathrm{m/s}]^{T}$,.

Initialization was done in a clean environment (with no false measurements) with two-point differencing [1] and false measurements are introduced into the system at time t = 10s. The non-stationary false measurements are generated at each time step independently and uniformly distributed within the whole two-dimensional space. The stationary false measurements are generated uniformly distributed and biased in a confined region and keep a small oscillation at every sampling time. The confined region is a rectangle with corners (170m, 9850m), (190m, 9850m), (170m, 6850m), and (190m, 6850m). The small oscillation is simulated by Gaussian noise with zero mean and identity covariance. The total number of samples is 200.

Fig. 2(a)(b) shows the snapshot taken at the final sample. The proposed modified PDA filter successfully tracks the target while the standard PDA filter fails.

A number of 50 Monte Carlo simulations were run over a wide range of clutter density as shown in fig. 3 and fig. 4. The modified PDA filter outperforms the standard PDA filter in mean square error (fig. 3) and percentage of lost tracks (fig. 4).

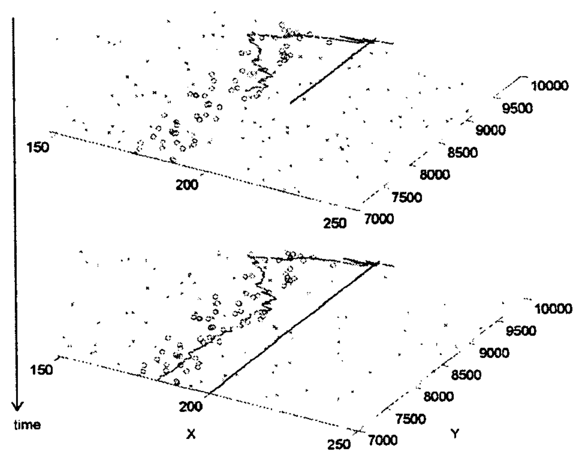

Fig. 2(a): Standard PDA Filter

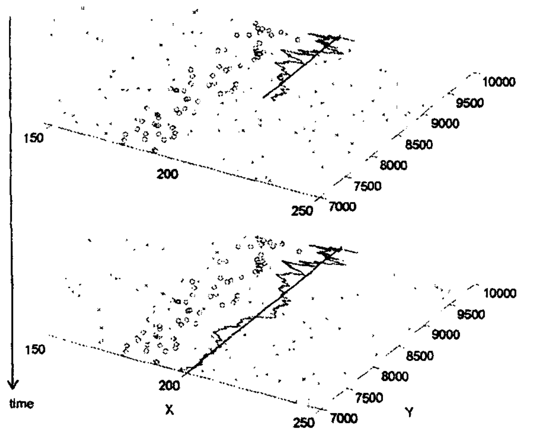

Fig. 2(b): Proposed modified PDA Filter

Fig. 2(a)(b): The target moves along X=200 from top to bottom. The *x-marks* denote the non-stationary false measurements which are re-distributed uniformly in the whole space at every time step. The *circles* are the stationary and biased false measurements which vary smally by Gaussian with zero mean and identity covariance. The *plus* signs show the filtered trajectory.

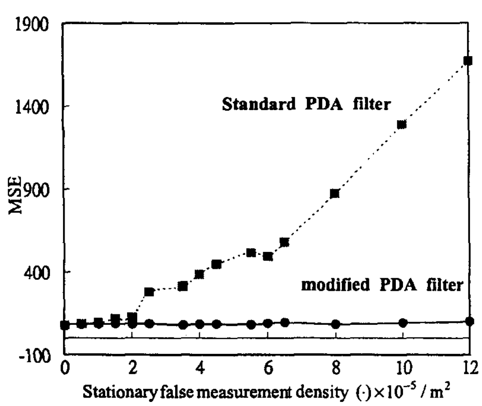

Fig. 3: MSE performance comparison. The non-stationary false measurement density is fixed at $10\times10^{-5}/m^2$. The stationary and biased false measurement density varies from 0 to $12\times10^{-5}/m^2$.

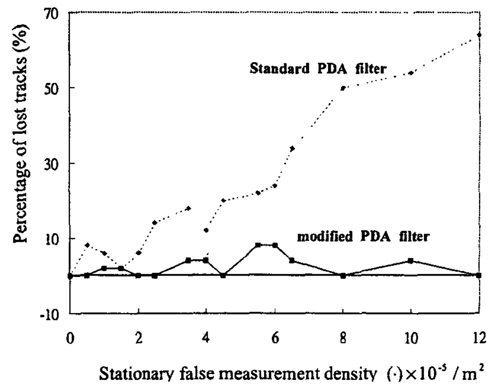

Fig. 4: Percentage of lost tracks comparison. The density settings are the same as in fig. 2.

6 Conclusion

The new filter is shown to be as robust as the standard PDA filter in a non-stationary clutter environment, while superior in a nearly stationary and biased clutter environment. Future work should include its use for multiple targets, the incorporation of target maneuver, and its development for multiple sensors data fusion.

7 References

[1] Y. Bar-Shalom and T. E. Fortmann, *Tracking and Data Association*, Academic Press, 1988, Chapter 5 and 6.

[2] M. Boshra, and B. Bhanu, "Predicting Performance of Object Recognition," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 22, no. 9, pp. 956-969, Sep 2000.

[3] J. Chen, H. Leung, T. Lo, J. Litva, M. Blanchette, "A Modified Probabilistic Data Association in a Real Clutter Environment," *IEEE Trans. Aerospace and Electronic Systems*, vol. 32, no. 1, pp. 300-313, Jan 1996.

[4] S. B. Colegrove and S. J. Davey, "The Probabilistic Data Association Filter with Multiple Nonuniform Clutter Regions," *IEEE Int. Radar Conf.*, pp. 65-70, 2000.

[5] T. W. Ridler, and S. Calvard, "Picture Thresholding Using an Iterative Selection Method," *IEEE Trans. Systems, Man, and Cybernetics*, vol. 8, no. 8, pp. 630-632, Aug 1978.