An Evolutionary Algorithm for the Synthesis of Multilayer Coatings at Oblique Light Incidence

Jinn-Moon Yang and Cheng-Yan Kao

***Abstract*—A robust evolutionary approach is proposed for the synthesis of multilayer coatings at oblique light incidence. The proposed approach consists of global and local strategies by integrating decreasing mutations and self-adaptive mutations via family competition and adaptive rules. Numerical results calculated at normal and oblique angles of incidence indicate that the proposed approach performs very robustly and is very competitive with other approaches. Our approach, although somewhat slower, is very flexible and can easily be adapted to other application domains. This approach is able to generate binary-type solutions based on two materials and to generate inhomogeneous solutions with continuous refractive-index variations.**

***Index Terms*—Edge filter, evolutionary algorithms, family competition, nonpolarized coating, thin-film coatings.**

I. Introduction

THE design of optical coatings at oblique light incidence is an important topic. Many different approaches for thin-film designs are often required as the solutions of different types of problems. Specifically, the design of nonpolarizing coatings is of interest to many applications and is considered a difficult task for most coating design methods. Numerical methods are now the most widely used design techniques. These methods formulate the coating design problem as an optimization based on the use of merit functions distributed over coating design parameters such as indexes, layer thicknesses, number of layers, etc.

Refinement methods [1], [9] and synthesis methods [7], [15], [21] are two basic approaches to the design of numerical optical coatings. Refinement methods normally require a starting design that is not quite satisfactory. The quality of the solution of refinement methods is sensitive at the starting design. At the same time, to choose a good starting design is a difficult task for many coating applications, such as nonpolarizing coatings. Contrary to refinement methods, synthesis methods [15], [5] generate their own starting designs automatically. Synthesis methods are combined with refinement methods to improve the quality of the solutions. Therefore, development of an effective synthesis method is an important topic of research.

Recently, evolutionary algorithms [10], [16], [14], [23] have been applied successfully to some problems encountered in optical filters and coatings that are computationally complex. These references demonstrated that evolutionary algorithms were competitive with well-known synthesis methods. An evolutionary algorithm is based on ideas borrowed from genetics and natural selection. It is a generally adaptable approach for problem solving and is well suited for solving difficult optimization problems where traditional methods are less efficient.

There are three main independently developed but strongly related implementations of evolutionary algorithms: genetic algorithms [13], evolution strategies [3], and evolutionary programming [12]. For genetic algorithms, in both practice and theory [11], they entail disadvantages of applying binary-represented implementation to the design of optical coatings. To achieve better performance, real-coded genetic algorithms [17], [6] with a significantly large number of variables have been introduced. In contrast, evolution strategies [14], [4], [18] and evolutionary programming use mainly real-valued representation and focus on self-adaptive Gaussian mutations to the design of optical coatings. Recent work [25] has shown that self-adaptive Gaussian mutations may leave individual designs trapped near local optima for rugged functions, which is a potential problem for all synthesis methods.

In this paper, we apply a method called the family competition evolutionary algorithm (FCEA) to synthesize optical thin-film systems with various number of layers. This proposed approach has been successfully applied to global optimization [25] and flexible ligand docking [24]. FCEA combines decreasing-based Gaussian mutation, self-adaptive Gaussian mutation, and self-adaptive Cauchy mutation. The performance of these mutations depends heavily on the same factor, called *step size*. The self-adaptive mutations adapt the step size with a stochastic mechanism. Decreasing-based mutations decrease the step size with a fixed rate γ, where $\gamma < 1$. In order to balance exploration and exploitation in the design space, these operators are made to cooperate with one another by incorporating family competition and adaptive rules to construct a relationship among these mutations.

The rest of this paper is organized as follows. Section II describes the problem of optical thin-film coatings; Section III introduces the evolutionary nature of FCEA. Section IV applies FCEA to the synthesis of a filter with 0.0, 0.5, and 1.0 transmission regions at normal light incidence and of two edge filters at oblique light incidence in order to illustrate the performance of our proposed approach. Section V presents the results of applying FCEA to synthesize two nonpolarized edge filters. Section VI presents concluding comments.

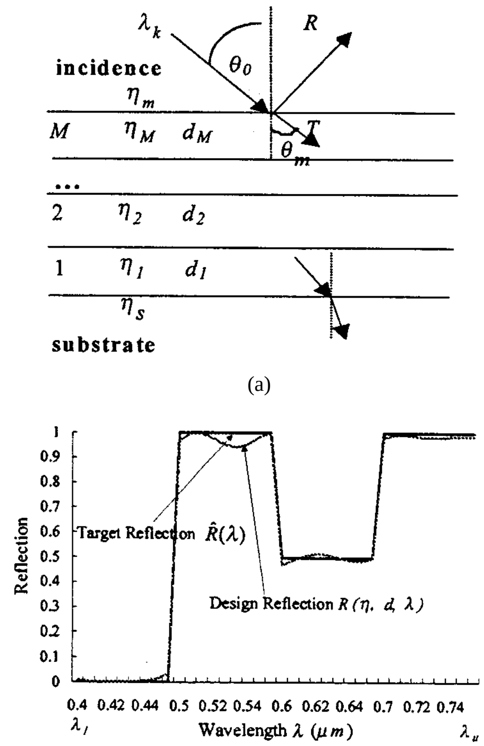
II. Problem Definition

The problem in numerical design of optical multilayer coatings is to find the construction parameters of a system that satisfy the desired optical specification. The construction parame-

Manuscript received June 1, 2000; revised December 27, 2000.

The authors are with the Department of Computer Science and Information Engineering, National Taiwan University, Taipei, Taiwan, R.O.C. (e-mail: moon@csie.ntu.edu.tw; cykao @csie.ntu.edu.tw).

Publisher Item Identifier S 0733-8724(01)02754-2.



(a)

(b)

Fig. 1. The construct parameters of a coating system and the profiles of target specification and a real coating system.

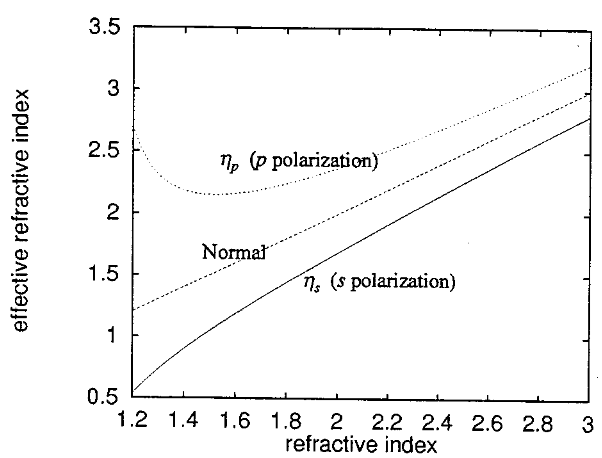
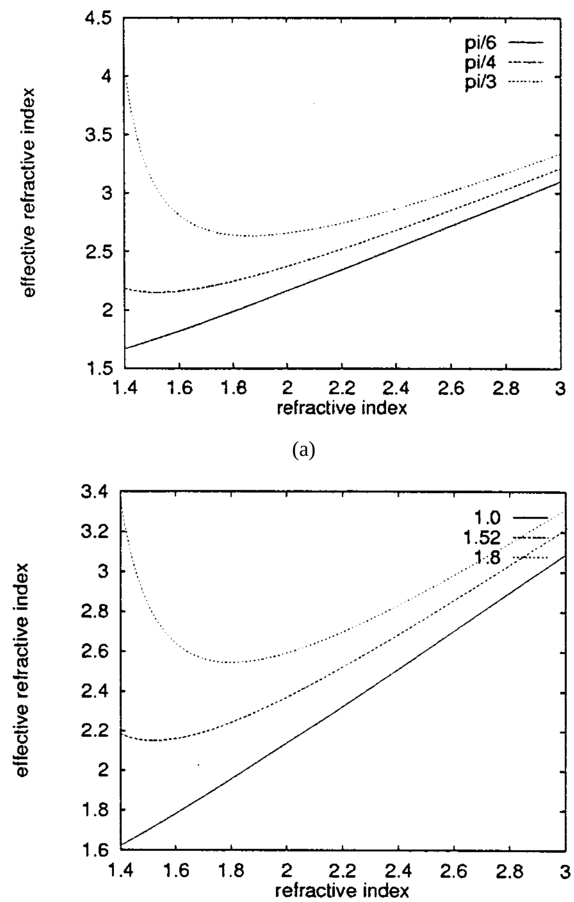


Fig. 2. Effective refractive indexes for s- and p-polarized light when ambient medium is glass and the incident angle is 45°.

ters includes the number of layers M, the refractive indexes η_j, and the thickness d_j of the jth layer, in order to match closely the specified performance where $1 \leq j \leq M$. Fig. 1(a) shows an M-layer coating system. It is necessary to define the desired optical specification when we design a multilayer coating system. Most often, this is defined by the target transmittance $\hat{T}$ or the target reflectance $\hat{R}$ at a number of wavelengths in the spectral region of interest. $\hat{T}$ equals $1-\hat{R}$ if the materials of a multilayer coating system are all nonabsorbing.

As defined in Fig. 1(a), a coating system is called a normal-incidence coating if θ_0 is zero; otherwise, it is called an oblique-incidence coating. On the other hand, a coating system can be divided into two categories according to the available values of refractive index $(\eta_1, \ldots, \eta_m)$. It may be called an inhomogeneous coating if the values of refractive index vector are continuously varying within each layer; otherwise, it is called a homogeneous coating based on materials. The inhomogeneous coating considered here presents a number of thin layers, whose refractive indexes are varied within certain limits.



(a)

(b)

Fig. 3. Effective refractive indexes of various incident angles and materials.

The merit function is one of the main elements of numerical coating design methods. Let the spectral reflectance of the M-layer system shown in Fig. 1(a), be denoted as $R(\eta, d, \lambda)$, where λ is the wavelength region of interest. Fig. 1(b) shows the example of a target reflection $(\hat{R}(\lambda))$ and of a respective design reflection $(R(\eta, d, \lambda))$. A suitable merit function is given by

$$f(\eta, d, \lambda_k) = \left\{ \frac{1}{W} \Sigma_{k=1}^{W} \frac{[R(\eta, d, \lambda_k) - \hat{R}(\lambda_k)]^2}{\delta R_k} \right\}^{1/2} \tag{1}$$

where

$R(\eta, d, \lambda_k)$, $\hat{R}(\lambda_k)$	respective desired and the target reflectance at wavelength λ_k;
η and d	refractive index and thickness vectors of a coating system, respectively;
δR_k	tolerance at the wavelength λ_k.

In general, δR_k is set to 0.01. Here W is the number of points where the merit function is evaluated. The merit function rep-

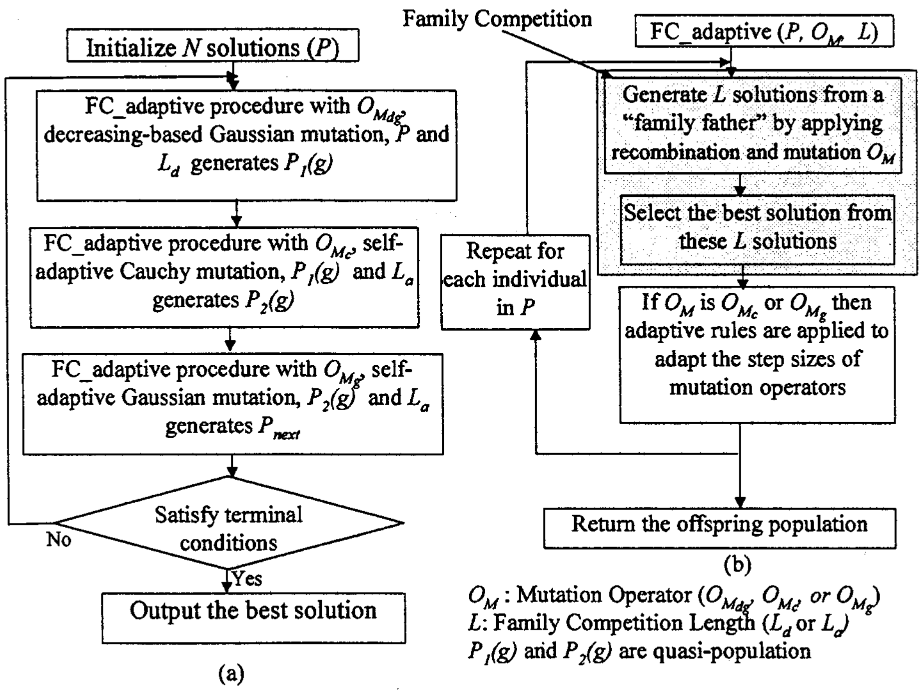


Fig. 4. Overview of our algorithm: (a) FCEA and (b) FC_adaptive procedure.

resents the root-mean-square error between the calculated reflectance and the target value. The most general method of calculating $R(\eta, d, \lambda_k)$ is based on a matrix formulation [8].

At oblique light incidence, the effective refractive index of a layer with a refractive index must be defined for each polarization as [8]

$$\eta^s = (\eta^2 - \eta_m^2 \sin^2 \theta_0)^{1/2} \quad (2)$$

$$\eta^p = \frac{\eta^2}{(\eta^2 - \eta_m^2 \sin^2 \theta_0)^{1/2}}. \quad (3)$$

Here η_m is the refractive index of the ambient medium, θ_0 is the incidence angle, and indexes η^s and η^p relate to the s and the p polarization, respectively. Fig. 2 shows the behavior of the effective indexes as functions of the film index for the case in which ambient medium is glass ($\eta_m = 1.52$) and the incident angle is 45°. It can be seen from Fig. 2 that the ratio of the effective refractive indexes of two arbitrary materials for s polarization is always greater than the ratio of the corresponding values at normal incidence. The ratio of the indexes for p polarization is always less than the ratio of the corresponding values at normal incidence. Fig. 3(a) and (b) shows that the difference between η^s and η^p becomes progressively larger, and this makes the designs more sensitive to the state of polarization when the incident angle (θ_0) or the incident index (η_m) increases. It implies that the difficulties are increasing in designing the nonpolarizing coating that optimizes both s- and p-polarization simultaneously as with increasing θ_0 or η_m.

Some observations should be considered when designing a coating system. According to the maximum principle [21], there is no advantage in using more than two materials with the lowest η_l and highest η_h refractive indexes at normal light incidence. That is, the best results can be achieved with the pair of materials having the lowest and highest refractive indexes. According to the concept of effective refractive indexes, problems for oblique light incidence can be reduced to the same form as in the case of normal incidence. However, it should be easier to achieve better synthesis results for the s case than for the p case. These postulates will be demonstrated in this paper. Second, the number of layers is limited because the difficulty in fabrication of coatings increases with the number of layers. Layer thicknesses cannot be negative for obvious reasons. Very thin layers will be eliminated if its thickness is lower than 0.001 μm, since this is difficult to control in fabrication.

III. Family Competition Evolutionary Algorithm

In this section, we present the detail of the FCEA for the optical thin-film designs. The basic structure of the FCEA is as follows (Fig. 4). N solutions are randomly generated as the initial population. Then FCEA enters the main evolutionary loop, in which each generation consists of three nearly identical procedures applied sequentially. Each procedure is realized by doing recombinations, mutations, family competition, and selection. These three procedures differ mainly in the mutations used: decreasing Gaussian mutation ($O_{M_{dg}}$), self-adaptive Cauchy mutation (O_{M_c}), and self-adaptive Gaussian mutation (O_{M_g}). Hence, we refer to such a combination of procedures as "FC_adaptive," which will be described later on in detail. The output is a new quasi-population with N solutions, which will be the input of the next FC_adaptive procedure.

The FC_adaptive procedure employs three parameters—the parent population (P, with N solutions), mutation operator (O_M), and family competition length (L)—to generate a new quasi-population [Fig. 4(b)]. The "FC_adaptive" procedure proceeds as follows to generate a quasi-population. Each

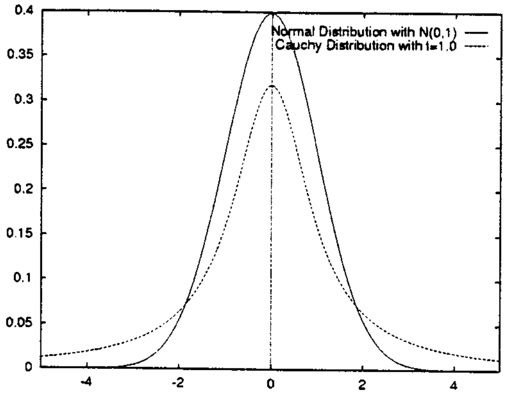


Fig. 5. Density functions of Gaussian and Cauchy distributions.

TABLE I
PARAMETERS OF FCEA AND NOTATION USED IN THIS PAPER

parameter name	the value and notation of parameter
recombination probability (p_c)	p_{cD} =0.8 (for $O_{M_{dg}}$), p_{cA} =0.2 (for O_{M_c} or O_{M_g}).
family competition length	L_d = 6 (for $O_{M_{dg}}$), L_a = 6 (for O_{M_c} or O_{M_g}).
step sizes	$v_i = \psi_i = 0.01$, $\sigma_i = 4v_i$.
population size (N)	50
other notation	M: number of layers, MF: value of merit function, $\sum \eta d$: total thickness of a solution.

individual in the population sequentially becomes the "family father." With a probability p_c, this family father and another solution randomly chosen from the rest of the parent population are used as parents to do a recombination operation. Then the new offspring or the family father (if the recombination is not conducted) is operated on by a mutation. For each family father, such a procedure is repeated L times. Finally, L children are produced, but only the one with the best objective value survives. Since we create L children from one "family father" and perform a selection, this is a "family competition" strategy. After the family competition, there are N parents and N children left. In each pair of father and child, the individual with a better objective value survives. This is called "family selection."

Regarding chromosome representation, we present a solution as$(M, I, x, \sigma, v, \psi)$ in FCEA when only one pair of materials with η_l and η_h is available. M is the number of layers of a coating system. We use an indicator I to represent the structure of the refractive indexes. The refractive index of first layer is equivalent to η_l when I is zero, and it is equivalent to η_h when I is one. The initial value I is randomly set to one or zero. The vector x is the thickness vector of a coating system to be optimized; σ, v, and ψ are the step-size vectors of decreasing-based mutation, self-adaptive Gaussian mutation, and self-adaptive Cauchy mutation, respectively. In other words, each solution x is associated with some parameters for step-size control. The number of elements of each vector x, σ, v, and ψ is M.

On the other hand, FCEA represents a solution as (M, x, σ, v, ψ) when the design problem is an inhomogeneous coating. The vector x includes both the thickness vector and the refractive indexes vector of a coating system to be optimized. The numbers of elements of the thickness vector and the refractive indexes vector are M. Therefore, the number of elements of each vector x, σ, v, and ψ is $2M$.

For both the two-material coatings and the inhomogeneous coatings, FCEA uses the same initialization procedure with the same initial values for parameters. The initial value M is randomly chosen from $[M_l, M_h]$, where M_l and M_h are the numbers of the lower bound and upper bound layers, respectively. The initial value of each entry x is randomly chosen over a feasible region, which depends on the properties of the specification of optical coatings. The initial values of each entry of σ, v, and ψ are set to 0.04, 0.01, and 0.01 according to experiments on problems studied in this paper with various values.

For the rest of this section, we use two-material coatings to explain each of the important components of the FC_adaptive procedure: recombination operators, mutation operations, and rules for adapting step sizes (σ, v, and ψ). For easy description of the operators, we use $a = (M^a, I^a, x^a, \sigma^a, v^a, \psi^a)$ to represent the "family father" and $b = (M^b, I^b, x^b, \sigma^b, v^b, \psi^b)$ as another parent (only for the recombination operator). The offspring, $c = (M^c, I^c, x^c, \sigma^c, v^c, \psi^c)$, is generated by a genetic operation. We also use the symbol x_j^d to denote the jth component of an individual d, $\forall j \in \{1, \ldots, M\}$.

A. Recombination Operators

The advantages or disadvantages of recombination for a particular objective function can hardly be accessed in advance [2]. Therefore, we implement two simple recombination operators to generate offspring: modified discrete recombination and intermediate recombination. With probabilities of 0.8 and 0.2, at each FC_adaptive procedure only one of the two operators is chosen. The values of probabilities were set in order to achieve the robust quality of the solution according to our previous results [25]. Here we would like to mention again that recombination operators are activated with only a probability p_c.

1) Modified Discrete Recombination: The original discrete recombination generates a child that inherits genes from two parents with equal probability. Here the two parents of the recombination operator are the "family father" and another solution randomly selected. Our experience indicates that FCEA can be more robust if the child inherits genes from the "family father" with a higher probability. Therefore, we modified the operator to be as follows:

$$x_j^c = \begin{cases} x_j^a \text{ with probability } 0.8 \\ x_j^a \text{ with probability } 0.2. \end{cases} \tag{4}$$

For a "family father," applying this operator in the family competition is viewed as a local search procedure because this operator is designed to preserve the relationship between a child and its "family father."

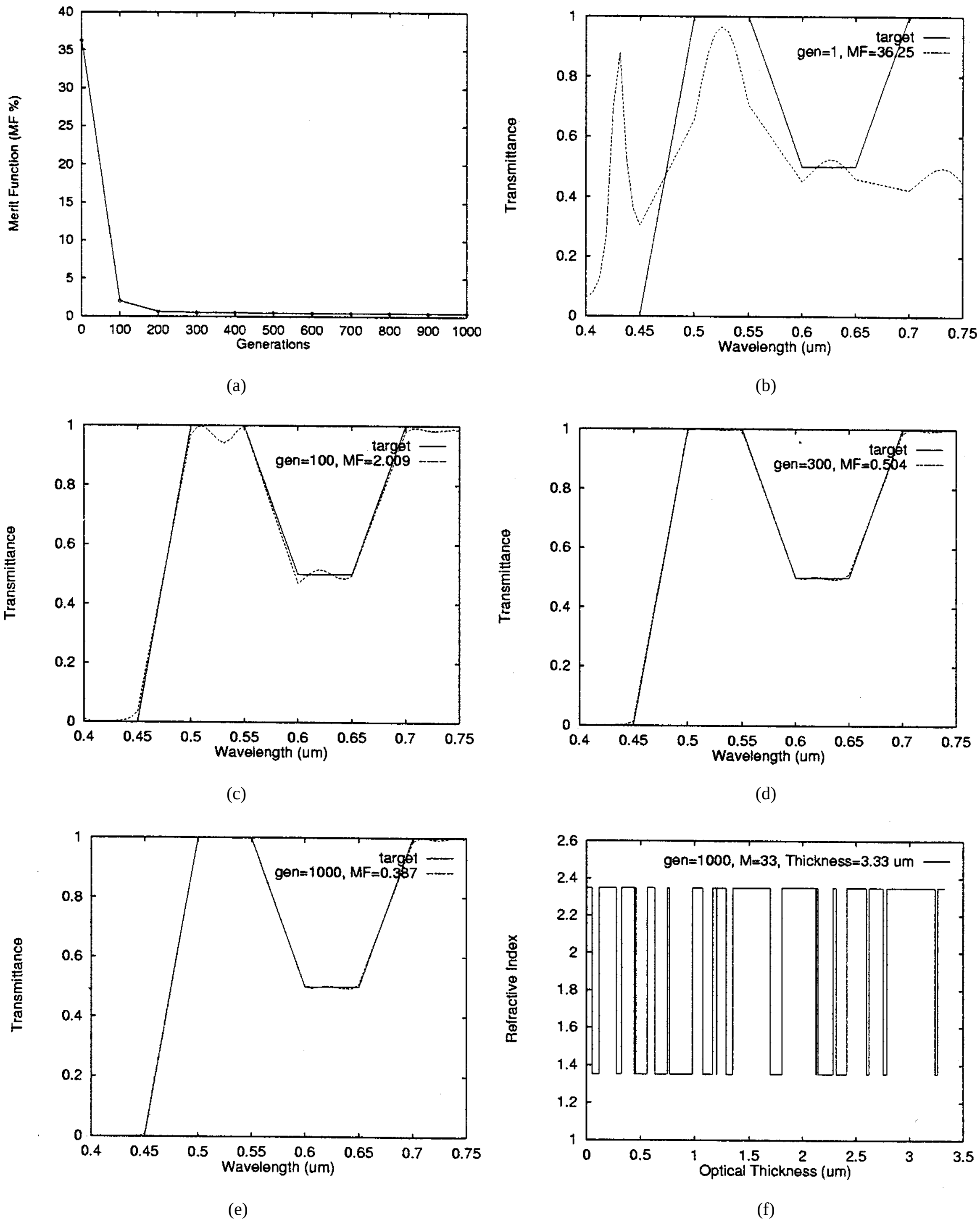


Fig. 6. Series of intermediate performance and the refractive-index profiles of our FCEA for the filter with 0.0, 0.5, and 1.0 transmission region $0.4 \leq \lambda \leq 0.75$ μm on an $\eta_s = 1.52$ substrate based on the refractive index pair 1.35 and 2.35.

2) Intermediate Recombination: We define intermediate recombination as

$$x_j^c = x_j^a + 0.5(x_j^b - x_j^a) \tag{5}$$

and

$$w_j^c = w_j^a + 0.5(w_j^b - w_j^a) \tag{6}$$

where w is v, σ, or ψ based on the mutation operator applied in the family competition. For example, if self-adaptive Gaussian mutation is used in this FC_adaptive procedure, x in (5) and (6) is v. We follow the work of the evolution strategies community [3] to employ only intermediate recombination on step-size vectors, that is, σ, v, and ψ.

B. Mutation Operators

Mutations are main operators of the FCEA. After the recombination, a mutation operator is applied to the "family father" or the new offspring generated by a recombination. In FCEA, the

TABLE II
COMPARISONS OF FCEA WITH SYNTHESIS METHODS AND REFINEMENT METHODS ON FILTER WITH 0.0, 0.5, AND 1.0 TRANSMISSION REGIONS

	FCEA			synthesis methods[15]				refinement methods[15]		
				Gradual-Evolution	Minus-Filter	Flip-Flop	Inverse-Fourier Transformation	Dumped-Least Square	Golden-Selection	Hooke Jeeves
M	32	33	32	9	37	34	43	35	35	35
$\sum \eta d(\mu m)$	1.96	3.33	2.805	2.350	4.550	1.99	2.37	1.975	2.671	3.434
MF(%)	1.72	0.387	0.587	5.10	2.13	2.56	2.20	1.86	3.81	4.970

mutation is performed independently on each vector element of the selected individual by adding a random value with expectation zero

$$x_i' = x_i + wD(\cdot) \tag{7}$$

where

- x_i thickness of the ith of x;
- x_i' ith variable of x' mutated from x;
- $D(\cdot)$ random variable;
- w step size.

In this paper, $D(\cdot)$ is evaluated as $N(0,1)$ or $C(1)$ if the mutations are, respectively, Gaussian mutation or Cauchy mutation.

1) Self-Adaptive Gaussian Mutation: We adapted Schwefel's [19] proposal to use self-adaptive Gaussian mutation. The mutation is accomplished by first mutating the step size v_j and then the thickness x_j

$$v_j^c = v_j^a \exp[\tau' N(0,1) + \tau N_j(0,1)] \tag{8}$$
$$x_j^c = x_j^a + v_j^c N_j(0,1) \tag{9}$$

where $N(0,1)$ is the standard normal distribution. $N_j(0,1)$ is a new value with distribution $N(0,1)$ that must be regenerated for each index j. For FCEA, we follow [3] in setting τ and τ' as $(\sqrt{2n})^{-1}$ and $(\sqrt{2\sqrt{n}})^{-1}$, respectively. The n equals M if the problem is the two-material coating; otherwise, n equals $2M$.

2) Self-Adaptive Cauchy Mutation: We follow previous works [26] to define self-adaptive Cauchy mutation as follows:

$$\psi_j^c = \psi_j^a \exp[\tau' N(0,1) + \tau N_j(0,1)] \tag{10}$$
$$x_j^c = x_j^a + \psi_j^c C_j(t). \tag{11}$$

In our experiments, t is one. Note that self-adaptive Cauchy mutation is similar to self-adaptive Gaussian mutation except that (9) is replaced by(11). That is, they implement the same step-size control but use different means of updating x.

3) Decreasing-Based Gaussian Mutations: Our decreasing-based Gaussian mutation uses the step-size vector σ with a fixed decreasing rate $\gamma = 0.97$ as follows:

$$\sigma^c = \gamma \sigma^a \tag{12}$$
$$x_j^c = x_j^a + \sigma^c N_j(0,1). \tag{13}$$

Previous results [25] demonstrated that self-adaptive mutations converge faster than decreasing-based mutations but, for rugged functions, self-adaptive mutations are more easily trapped into local optima than decreasing-based mutations.

Fig. 5 compares density functions of Gaussian distribution ($N(0,1)$) and Cauchy distributions ($C(1)$). Clearly, Cauchy mutation is able to make a larger perturbation than Gaussian mutation. This implies that Cauchy mutation has a higher probability of escaping from local optima than Gaussian mutation does. The main reason why we use these three types of mutations is that they can closely cooperate with one another to improve the performance of the overall search. The detailed reasons were discussed in [25]

C. Adaptive Rules

The performance of Gaussian and Cauchy mutations is largely influenced by the step sizes. FCEA adjusts the step sizes while mutations are being applied [e.g., (8), (10), and (12)]. However, such updates insufficiently consider the performance of the whole family. Therefore, after family competition, some additional rules are implemented.

1) *A-decrease rule:* Immediately after self-adaptive mutations, if objective values of all offspring are greater than or equal to that of the "family parent," we decrease the step-size vectors v (Gaussian) or ψ (Cauchy) of the parent

$$w_j^a = 0.95 w_j^a \tag{14}$$

where w^a is the step-size vector of the parent. In other words, when there is no improvement after self-adaptive mutations, we may propose a more conservative, that is, smaller step size, which tends to make an improvement in the next iteration.

2) *D-increase rule:* It is difficult, however, to decide the rate γ of decreasing-based mutations. Unlike self-adaptive mutations that adjust step sizes automatically, the step size of decreasing-based mutation goes to zero as the number of generations increases. It is essential to employ a rule that can enlarge the step size of decreasing-based mutations in some situations. The step size of the decreasing mutation should not be too small, when compared to step sizes of self-adaptive mutations. We propose to increase σ if one of the two self-adaptive mutations generates better offspring. To be more precise, after a self-adaptive mutation, if the best child with step size v is better than its "family father," the step size of the decreasing-based mutation is updated as follows:

$$\sigma^c = \max(\sigma^c, \beta v_{\text{mean}}^c) \tag{15}$$

where v_{mean}^c is the mean value of the vector v. β is 0.2 in our experiments. Note that this rule is applied in stages of self-adaptive mutations but not of decreasing mutations.

TABLE III

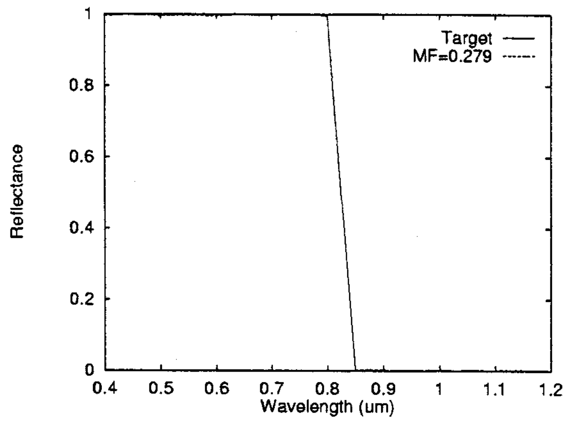
CONSTRUCTION PARAMETERS OF THE 0.0–0.5–1.0 TRANSMISSION FILTER AND THE s-POLARIZATION OF THE LONG-WAVE-PASS FILTER AND OF THE SHORT-WAVE-PASS FILTER. OBTAINED BY FCEA

	0.0-0.5-1.0 filter			long-wave-pass filter		short-wave-pass filter	
Layer	η	ηd	ηd	η	ηd	η	ηd
Subs(η_s)	1.52			1.52		1.52	
1	2.35	0.0912	0.0678	1.45	0.44943	2.35	0.01631
2	1.35	0.0633	0.0276	2.35	0.05743	1.45	0.01814
3	2.35	0.0168	0.4476	1.45	0.06604	2.35	0.11283
4	1.35	0.0050	0.0316	2.35	0.12593	1.45	0.20981
5	2.35	0.0411	0.1366	1.45	0.08192	2.35	0.11822
6	1.35	0.0031	0.0238	2.35	0.09485	1.45	0.23115
7	2.35	0.0066	0.1849	1.45	0.10638	2.35	0.01001
8	1.35	0.1315	0.0980	2.35	0.07607	1.45	0.02222
9	2.35	0.0048	0.0288	1.45	0.13735	2.35	0.01002
10	1.35	0.0546	0.1391	2.35	0.08491	1.45	0.22033
11	2.35	0.0898	0.0179	1.45	0.11492	2.35	0.11273
12	1.35	0.0466	0.0013	2.35	0.07759	1.45	0.20554
13	2.35	0.0260	0.3210	1.45	0.08634	2.35	0.13547
14	1.35	0.0256	0.1084	2.35	0.15958	1.45	0.02614
15	2.35	0.0610	0.3487	1.45	0.08878	2.35	0.13397
16	1.35	0.1523	0.0577	2.35	0.05881	1.45	0.20600
17	2.35	0.0876	0.0844	1.45	0.11170	2.35	0.11738
18	1.35	0.0783	0.0049	2.35	0.17752	1.45	0.23381
19	2.35	0.1230	0.0358	1.45	0.06772	2.35	0.03280
20	1.35	0.1045	0.0911	2.35	0.05008	1.45	0.04522
21	2.35	0.3301	0.0952	1.45	0.23398	2.35	0.03005
22	1.35	0.0939	0.2132	2.35	0.04599	1.45	0.24562
23	2.35	0.1212	0.0190	1.45	0.09472	2.35	0.01418
24	1.35	0.1382	0.1143	2.35	0.15562	1.45	0.01463
25	2.35	0.0397	0.0694	1.45	0.08259	2.35	0.11425
26	1.35	0.1223	0.1065	2.35	0.04518	1.45	0.21041
27	2.35	0.0051	0.0023	1.45	0.17860	2.35	0.11237
28	1.35	0.0019	0.0111	2.35	0.06825	1.45	0.20064
29	2.35	0.0862	0.1196	1.45	0.05959	2.35	0.10941
30	1.35	0.0013	0.0477	2.35	0.06566	1.45	0.19994
31	2.35	0.2120	0.1602	1.45	0.15079	2.35	0.10621
32	1.35	0.0169	0.0646	2.35	0.03476	1.45	0.09507
33	2.35		0.0503	1.45	0.09478		
34				2.35	0.13713		
35				1.45	0.15067		
36				2.35	0.01190		
medium (η_m)	1.0			1.0		1.0	
$\sum \eta d$		1.96	3.33		7.0		6.48
MF(%)		1.724	0.387		0.279		1.019

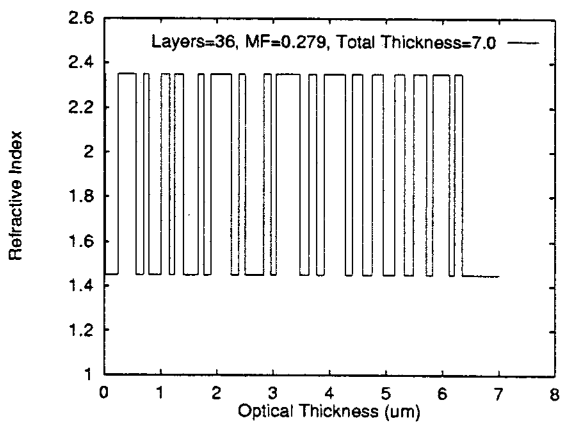
Instead of updating the value of γ, this rule increases directly the step size of decreasing-based mutations according to the step size of self-adaptive mutations when (15) is satisfied.

IV. EXPERIMENTAL RESULTS OF TWO-MATERIAL COATINGS

In this section, we present the numerical results of a filter with 0.0, 0.5, and 1.0 transmission at normal light incidence and of

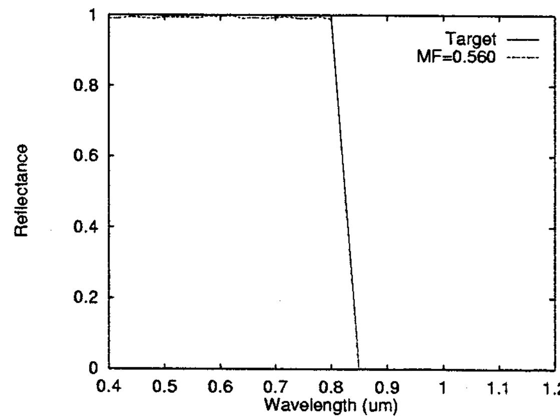


(a)

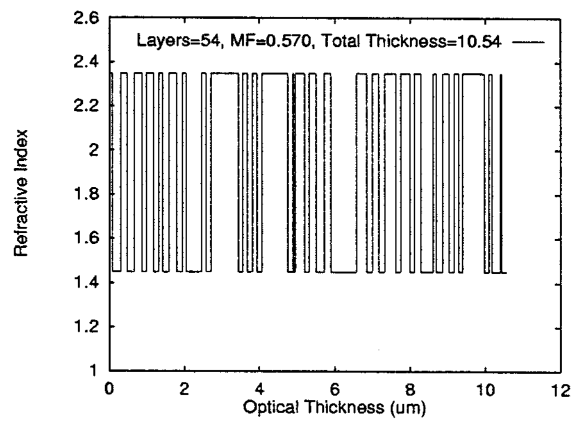


(b)

Fig. 7. Reflectance and refractive-index profile of a 36-layer long-wave-pass filter for s-polarized light obtained by FCEA at a 45° angle of incidence on interesting region $0.4 \leq \lambda \leq 1.2$ μm on an $\eta_s = 1.52$ substrate based on the refractive index pair 1.45 and 2.35.



(a)



(b)

Fig. 8. Reflectance and refractive-index profile of a 54-layer long-wave-pass filter obtained by FCEA for p-polarized light at a 45° angle of incidence on interesting region $0.4 \leq \lambda \leq 1.2$ μm on an $\eta_s = 1.52$ substrate based on the refractive index pair 1.45 and 2.35.

two edge filters at oblique light incidence to illustrate the proposed method. The first problem is used to compare the performance of FCEA with seven well-known coating approaches. The experimental results of edge filters indicated that FCEA is able to obtain good enough solutions for the problems at oblique light incidence.

Table I indicates the setting of FCEA parameters, such as initial step sizes, family competition lengths and recombination probabilities. These values were used for the synthesis problems defined in this paper. L_d, σ, and p_{cD} are the parameters for decreasing-based mutation; L_a, v, ψ, and p_{cA} are for self-adaptive mutations. The population size is 50. These parameters were selected after many attempts to design solutions for these three optical coatings with various initial values for the parameters. For each problem, FCEA was tested 30 times.

A. Filter with 0.0, 0.5, and 1.0 Transmission at Normal Light Incidence

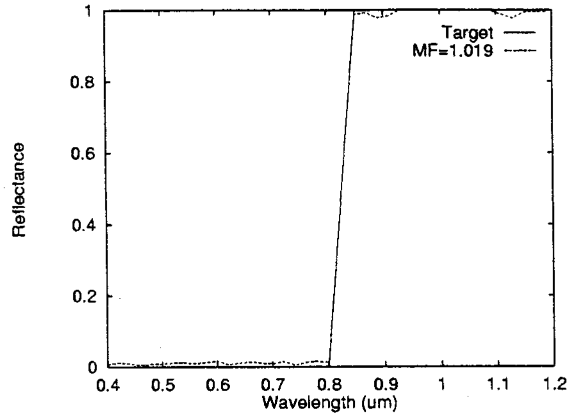
The first example concerns the synthesis of a filter in the region 0.4–0.75 μm at normal light incidence. This filter specification has been used previously for comparisons of synthesis methods [7], [15]. It was believed that it would be difficult to design such a coating system without the use of synthesis approaches because a suitable starting design for simple refinement is difficult to obtain. Such multiwavelength specifications might be required for laser and electrooptical systems. We implement FCEA on this problem to compare with seven well-known approaches. The desired performance is

$$0.4 \leq \lambda \leq 0.45 : T = 0.0 \text{ (reflector)}$$
$$0.5 \leq \lambda \leq 0.55 : T = 1.0 \text{ (antireflector)}$$
$$0.6 \leq \lambda \leq 0.65 : T = 0.5 \text{ (beam splitter)}$$
$$0.7 \leq \lambda \leq 0.75 : T = 1.0 \text{ (antireflector).}$$

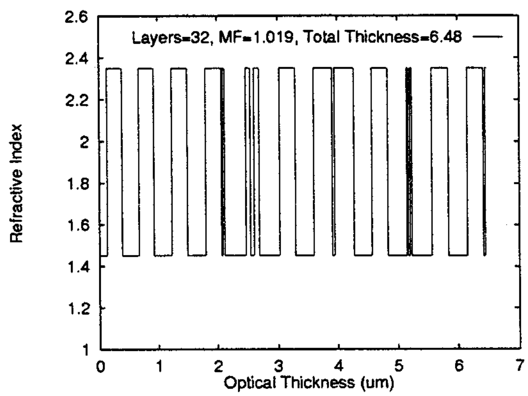
The spectral target is the solid curve shown in Fig. 6(b). Following previous works [7], [15], we had used only two coating materials with refractive indexes of $\eta_l = 1.35$ and $\eta_h = 2.35$. The substrate and medium indexes are $\eta_s = 1.52$ and $\eta_m = 1.0$, respectively. The merit function was defined at 36 points. That is, each region exists nine points.

The initial number of layers is randomly chosen from 25 to 35. The initial thickness of each layer was uniformly selected from the region from 0.01 to 0.2 μm. The maximum number of generations is 1000.

To illustrate the convergence, a series of intermediate solutions obtained by FCEA are shown in figures from Fig. 6(b) to Fig. 6(e). The dash curves represent designed results, and the

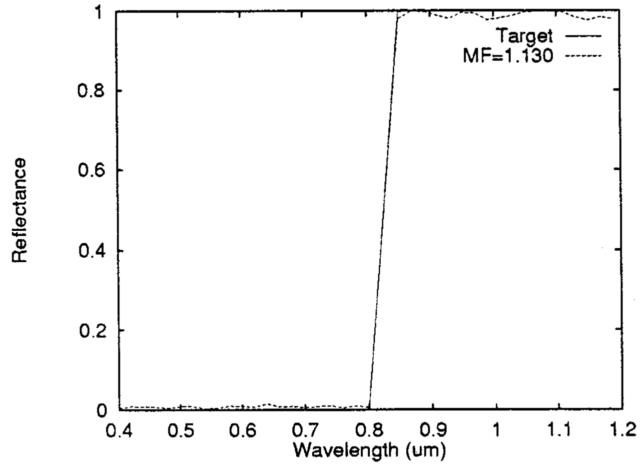


(a)

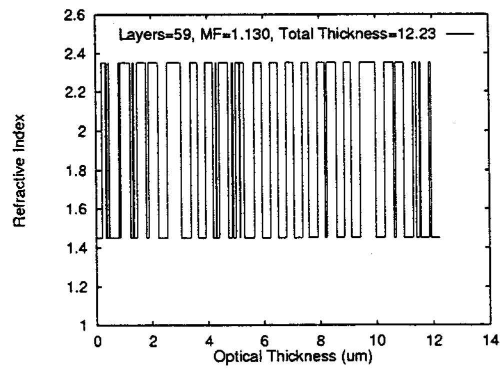


(b)

Fig. 9. Reflectance and refractive-index profile of a 32-layer short-wave-pass filter obtained by FCEA for s-polarized light at a 45° angle of incidence on interesting region $0.4 \leq \lambda \leq 1.2$ μm on an $\eta_s = 1.52$ substrate based on the refractive index pair 1.45 and 2.35.



(a)



(b)

Fig. 10. Reflectance and refractive-index profile of a 59-layer short-wave-pass filter obtained by FCEA for p-polarized light at a 45° angle of incidence on interesting region $0.4 \leq \lambda \leq 1.2$ μm on an $\eta_s = 1.52$ substrate based on the refractive index pair 1.45 and 2.35.

solid curve is the target design. Fig. 6(a) illustrates the relationship between the value of merit function and the number of generations. The value of merit function is 36.25% when the number of generations is one. The values become 2.009% and 0.504% when the numbers of generations are at the one-hundredth and three-hundredth, respectively. After FCEA has exhausted 1000 generations, the value of merit function is 0.387%, the number of layers is 33, and the thickness is 3.33 μm for the final solution. Its refractive-index profile is shown in Fig. 6(f).

Table II shows the comparisons of FCEA with well-known synthesis methods [15], such as gradual-evolution and inverse-Fourier transform methods; and refinement methods [15], such as dumped-least-square and Hooke and Jeeves methods, on this filter. FCEA is able to obtain more robust solutions than these comparative approaches.

The values of merit function are 1.72% and 0.387% for FCEA when the total thicknesses are 1.96 and 3.33 μm, respectively. The construction parameters of these two solutions of our FCEA are given in Table III.

B. Long-Wave-Pass Filter

The second design problem is to create a long-wave-pass filter for s- and p-polarized light at 45°. The solid curve in Fig. 7(a) indicates the target specification. The filter is a cold mirror that reflects as much as possible of the visible light incident upon them and transmits the remaining radiation. That is, the target reflectance of the s and p cases was 1.0 in the region from 0.4 to 0.8 μm and was zero from 0.85 to 1.2 μm. The incident medium is air ($\eta_m = 1$) and the substrate is glass with $\eta_s = 1.52$. The high- and the low-index refractive indexes were $\eta_h = 2.35$ and $\eta_l = 1.35$, respectively. The merit function was defined at 39 points.

The initial number of layers is randomly chosen from 30 to 60. The initial thickness of each layer was uniformly selected from 0.01 to 0.2 μm. The maximum number of generations is 1500.

Fig. 7 shows the reflectance and the refractive-index profile of an s-polarized case of a final coating system obtained by FCEA. Its number of layers is 36 and its value of merit function is 0.279%. The total thickness of this solution is 7.0 μm. On the other hand, the final solution of the p-polarized case is shown in Fig. 8. The total thickness is 10.54 μm, the number of layers is 54, and the value of merit function is 0.570% of this final solution. It can be seen that the total thickness and the number of layers of the final design in the p-polarized case are considerably more than in the s-polarized case. The construction parameters of an s-polarized solution generated by FCEA are given in Table III. According to the experimental results, FCEA is able

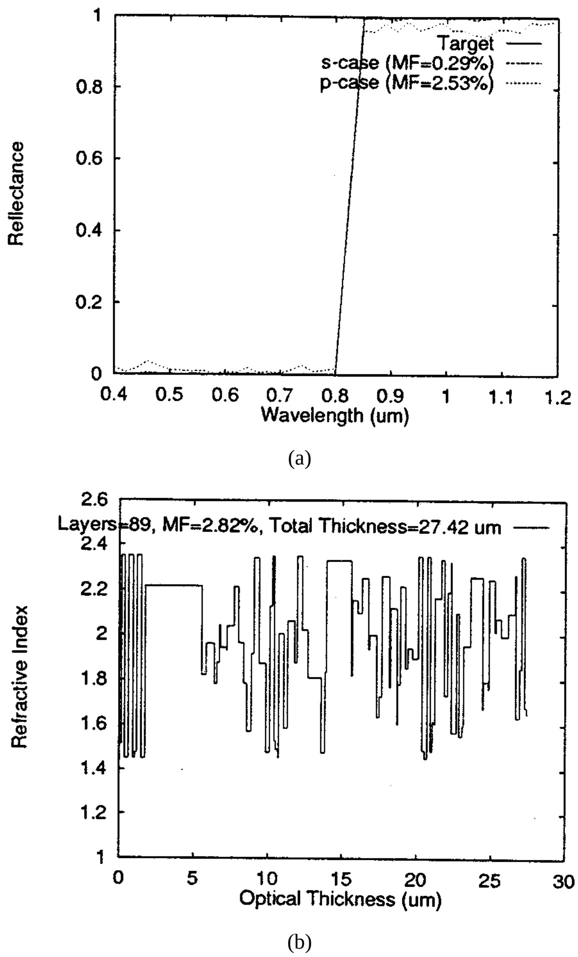


Fig. 11. The reflectance and refractive-index profile of an 89-layer short-wave-pass nonpolarized filter obtained by FCEA at a 45° angle of incidence on interesting region $0.4 \leq \lambda \leq 1.2$ μm. The available value of refractive index is continuous from 1.45 to 2.35.

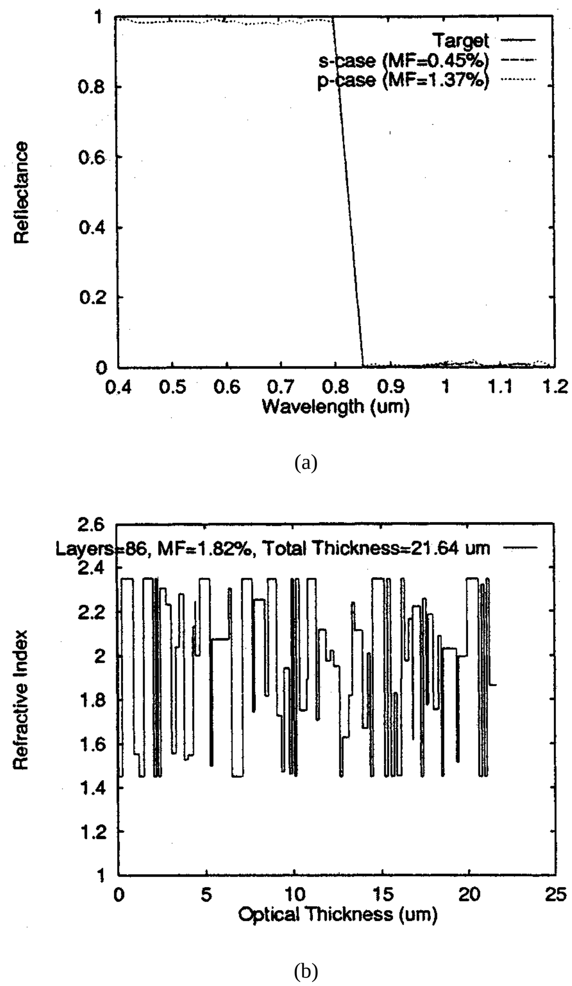


Fig. 12. Reflectance and refractive-index profile of an 86-layer long-wave-pass filter for nonpolarized light at a 45° angle of incidence on interesting region $0.4 \leq \lambda \leq 1.2$ μm. The refractive index is continuous from 1.45 to 2.35.

to obtain coating systems that are close to the target specification.

C. Short-Wave-Pass Filter

The synthesis of a short-wave-pass filter for s- and p-polarized light at 45° [22] is being discussed. The target reflectance is the solid curve shown in Fig. 9(a). The filter is a heat reflector (or called hot mirror), which is a special long-wavelength cutoff filter. That is, the filter transmits the visible radiation from 0.4 to 0.8 μm without disturbing the color balance. The width of the rejection region depends on the light source to be used. In this filter, the target reflectance was zero in the region from 0.4 to 0.8 μm and was 1.0 from 0.85 to 1.2 μm. The incident medium is air ($\eta_m = 1$), and the substrate is glass with $\eta_s = 1.52$. The high- and low-index refractive indexes were $\eta_h = 2.35$ and $\eta_l = 1.35$, respectively. The merit function was defined at 39 points. All initial values are set to the same values used by the long-wave-pass filter.

The final design obtained by FCEA is a 32-layer coating in the s-polarized case and a 59-layer coating in the p-polarized case for the short-wave-pass filter. Figs. 9 and 10 show the reflectance and refractive-index profiles of the s- and p-polarized case, respectively. This problem seems more difficult than the long-wave pass filter for FCEA. The total thickness of the design in the p-polarized case is also considerably more than in the s-polarized case. The construction parameters of an s-polarized solution generated by FCEA are given in the final column of Table III.

V. Inhomogeneous Coatings: Nonpolarized Edge Filters

At oblique light incidence, the most interesting design is that the coatings satisfy for both s and p polarizations simultaneously. In this section, FCEA is implemented to design nonpolarization edge filters including the long-wave-pass filter and the short-wave-pass filter. The target specifications of these two edge filters are the same ones defined in Section IV-B and Section IV-C. For example, the reflectance is zero in the region from 0.4 to 0.8 μm and 1.0 from 0.85 to 1.2 μm for the short-wave-pass filter. The incident angle is 45°, the incident medium is air, and the substrate is glass.

When s and p targets are specified simultaneously, the optimality of two-material designs may be no longer valid. Thus, if possible, it is worthwhile to use a set of materials with various indexes for nonpolarizing coatings. In this paper, we force the refractive indexes to satisfy the following constraints: $\eta_l \leq \eta_j \leq \eta_h$, where η_l and η_h correspond to the lowest and highest refractive indexes that are available for the problems, respectively. Therefore, FCEA is required to decide the refractive index η_j and the thickness d_j of the jth layer, in order to match closely the specified performance where $1 \leq j \leq M$.

Nonpolarizing coatings are much more difficult to design, and so a large number of layers may be required. Thus, the initial number (M) of layers was randomly chosen in the range of 70–90. The initial thickness (d_j) and refractive index (η_j) of each layer were uniformly selected from the region from 0.01 to 0.2 μm and from 1.45 to 2.35 μm, respectively. The maximum number of generations was set to 2000.

Figs. 11 and 12 show s and p reflectances and refractive-index profiles of short-wave-pass and long-wave-pass nonpolarizing edge filters, respectively. The final design of the short-wave-pass filter is an 89-layer coating with the value of merit function as 2.82%. In this final solution, the values of merit function of s- and p-polarized cases are 0.29% and 2.53%, respectively, as shown in Fig. 11(a) and (b).

On the other hand, the long-wave-pass design has 86 layers with its value of merit function as 1.82%, and the values of merit function of s- and p-polarized cases are 0.45% and 1.37%, respectively. Fig. 12(a) shows the s- and p-polarized reflectances, and Fig. 12(b) indicates the refractive-index profile of the solution. Again, these results show that designs for the s-polarization are easier to formulate than for p-polarization.

Conversion of an inhomogeneous system into a system with only two materials was described by Southwell [20]. In this method, the number of layers of the resulting system may increase. In practice, two-material systems are much easier to fabricate using a conventional deposition process. The design of a nonpolarized coating, discussed above, demonstrates that FCEA is able to find adequate solutions.

VI. Conclusion

This study demonstrates that FCEA is a stable synthesis approach for optical thin-film designs at both oblique and normal light incidence. From our experience, it is suggested that a global optimization method should consist of both global and local search strategies. In FCEA, decreasing-based mutations with a large initial step size is a global search strategy; self-adaptive mutations with family competition procedure and replacement selection are local search strategies. These strategies can closely cooperate with each other to improve the overall search performance.

The designs formulated as solutions for three difficult optical coating problems verify that the proposed approach is robust and is very competitive with other algorithms. FCEA is also able to obtain stable solutions for nonpolarizing coating problems. We believe that the flexibility and robustness of FCEA make it an effective synthesis method of optical thin-film designs.

We will continue to investigate a more flexible approach to adapt the number of layers of a coating system and will study a more diverse set of thin-film designs to determine the limits of FCEA.

Acknowledgment

The authors would like to Dr. C.-J. Lin and the anonymous referees for their constructive comments, which improved readability of the paper.

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Jinn-Moon Yang received the M.S. degree from National Central University, Chung-Li, Taiwan, in 1994, the M.B.A. degree from Tamkang University, Taipei, Taiwan, and the Ph.D. degree in computer science and information engineering from National Taiwan University, Taipei, Taiwan, in 2001.

In 1985-1998, he worked for PanChiao Telecommunications, Ministry of Transportation and Communications, Taiwan. Since 1998, he has been with Chunghwa Telecom Training Institute, Taipei, Taiwan. His current research interests include evolutionary computation, neural networks, global optimization, learning systems, and bioinformatics.

Cheng-Yan Kao, was born in Taipei, Taiwan, 1948. He received B.S. degree in mathematics from National Taiwan University, Taipei, Taiwan, in 1971, the M.S. degree in computer science in 1976, the M.S. degree in statistics in 1978, and the Ph.D. degree in computer science in 1981, all from the University of Wisconsin-Madison.

He worked for Ford Aerospace, the Unisys Corporation, and worked for General Electric from 1980 to 1989 at the Johnson Space Center, NASA, Houston, TX. He has been a Professor with the Department of Computer Science and Information Engineering, National Taiwan University since 1990. He has published more than 40 technical papers in various journals and conference records. His research interests include evolutionary computation, bioinformatics, optimization, and artificial intelligence.