Rate, Sub-Carrier, and Power Allocations for Multi-Carrier CDMA with LMMSE Multiuser Detections

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Abstract— *Allocations of physical transmission rate, sub-carrier, and power in MC-CDMA systems are proposed to minimize total uplink power consumptions for users of different data rate and bit-error-rate (BER) requirements with the receiver using linear minimum-mean-square-error (LMMSE) MUD. Based on Multi-Code (MC) and Variable-Spreading-Length (VSL) strategies for multi-rate transmissions, we derive the discrete-rate transmission capacity of MC-CDMA and propose a simple admission control criterion for users of individual quality-of-service (QoS) demands. This criterion linearly relates the code length with the transmission rate and BER requrets of all users. An iterative power allocation algorithm is proposed after deciding all supportable transmission rates to allocate the sub-carrier power of each user and jointly solve the sub-carrier allocation problem. The results show that in either MC MC-CDMA or VSL MC-CDMA, the most power-efficient allocation is to distribute the power of each user over several sub-carriers of better quality and even concentrate on the best one, if it is exclusive.*

I. Introduction

Wireless multimedia communications requires not only effective transmission technique but also resource allocation to provide different quality-of-services (QoS) for users of various demands. Multi-Carrier CDMA (MC-CDMA) that combines Orthogonal-Frequency-Division-Multiplexing (OFDM) and CDMA was shown its advantages in transmitting broadband signals over frequency-selective fading channels and providing high capacity [1-2]. MC-CDMA data stream is spread by a given spreading code and each code chip modulates a different sub-carrier, which is so-called the frequency-domain spread spectrum [3] and all sub-carriers are mutually orthogonal. Power, sub-carrier, and spreading codes are available radio resources in MC-CDMA transmission.

Conventional MC-CDMA uniformly distributes power to all sub-carriers without optimization and obviously it is inefficient. Providing channel state information (CSI) in the transmitters, a method ever proposed to turn off deep-faded sub-carriers and uniformly distribute power to the remaining sub-carriers to improve the performance of maximum-ratio-combining (MRC) receiver [4]. However, it is not proven an optimal method in any sense. RRA in MC-CDMA is critical to multimedia traffic but it still lacks of systematic methodology to jointly utilize these radio resources. Most previous researches about resource allocation in CDMA systems only considered the conventional matched-filter-based single-user-detection (SUD) receivers, but it is well-known that MUD can significantly improve the performance and capacity of CDMA systems [5] and MUD is especially recommended in uplink MC-CDMA due to the serious distortion of code orthogonality in frequency-selective fading channels [6]. Hence, RRA with MUD in uplink channels is essential to practical MC-CDMA applications.

For multi-rate transmissions, Multi-Code (MC) and Variable-Spreading-Length (VSL) are two widely adopted multi-rate schemes in CDMA systems [7], and their combinations with MC-CDMA were previously studied in [8] with a LMMSE MUD effectively to mitigate the multiple access interference and orthogonality distortion in uplink receptions. In this work, we consider the MC- and VSL- based multi-rate MC-CDMA systems with the receiver applying the LMMSE MUD. All users in the systems have individual QoS demands on physical transmission rate and bit-error-rate (BER). To save critical power consumption for user terminals, the principle of allocations is to minimize the total uplink power consumption. Novel to conventional methodologies, the optimization presented in this paper is processed over the domain that users are decomposed as unit-rate virtual users such that either of the two multi-rate MC-CDMA systems exhibits structural regularity to make the whole problem easy to handle. We derive the user capacity of both multi-rate MC-CDMA schemes according to all users' QoS constraints and propose a simple and practical user admission criterion. To allocate power of each admitted user, an iterative power allocation algorithm to assign sub-carrier power and adjust signal phase is proposed, in which sub-carrier selection is jointly achieved.

II. System Description

MC MC-CDMA & VSL MC-CDMA

Suppose the data rates of all users are integer multiples of a pre-defined basic rate R_b, where $R_b = T_s^{-1}$ and define F as the basic spreading length, i.e. the code length of basic-rate users in the system. A "rate-m" user means a user whose data rate is mR_b. Basically in MC-CDMA, every symbol is duplicated to several copies and each is multiplied by a chip of the given spreading code, where these modulated copies are transmitted in parallel by orthogonal sub-carriers [1-2]. For a rate-m user, MC MC-CDMA multiplexes the source data stream into m basic-rate data streams and these data streams are transmitted respectively using different spreading codes by the MC-CDMA transmission procedure. Instead of source data multiplexing, VSL MC-CDMA adjusts the number of used sub-carriers in the normal MC-CDMA transmission procedure to fit various data rates, where F/m, an integer, sub-carriers are used for each symbol of a rate-m user.

A rate-m user in both MC MC-CDMA and VSL MC-CDMA can be decomposed as m basic-rate virtual users, where all virtual users have identical symbol durations and the lengths of spreading codes F [8]. The resulting spreading code for each virtual user is called a virtual spreading code, where all code bits are non-zero in MC scheme but some code bits are zeros

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for non-basic-rate users in VSL scheme. For example, if a double-rate user in MC scheme can be decomposed as two virtual users with spreading code [1 -1 1 -1] and [1 1 -1 -1], and then in VSL scheme the two virtual users are with spreading codes [1 -1 0 0] and [0 0 1 -1]. Based on virtual users, the received uplink signal in the receiver can be universally modeled for both access schemes as [8]:

$$r(t)=\sum_{k=1}^{K} A_{kf} b_k \sum_{f=1}^{F} \alpha_{kf} c_{kf} e^{j2\pi(f-1)\Delta f} + n(t),\ \ 0 \le t \le T_s, \quad (1)$$

where A_{kf} is the transmitted amplitude of the virtual user k at the sub-carrier f, b_k denotes the source symbol of the virtual user k within each OFDM symbol period, α_{kf} denotes the coefficient of the sub-channel passed by the fth chip of b_k, and $n(t)$ is the additive white Gaussian noise (AWGN) with power spectrum density $N_0/2$. We assume the number of total sub-carriers is large enough to yield flat fading in sub-channels, which is an essential operation requirement for MC-CDMA [1-3], and assume the fading is slow with respect to the elementary symbol period, FT_s.

LMMSE Multiuser Detection

After down-conversion to baseband, the received samples from all sub-carriers compose a $F \times 1$ vector $\mathbf{r}$, where

$$\mathbf{r}=\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_F \end{bmatrix} = \underbrace{\begin{bmatrix} A_{11}\alpha_{11}c_{11} & \cdots & A_{K1}\alpha_{K1}c_{K1} \\ A_{12}\alpha_{12}c_{12} & & A_{K2}\alpha_{K2}c_{K2} \\ \vdots & \cdots & \vdots \\ A_{1F}\alpha_{1F}c_{1F} & \cdots & A_{KF}\alpha_{KF}c_{KF} \end{bmatrix}}_{\mathbf{M}} \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_K \end{bmatrix}}_{\mathbf{b}} + \underbrace{\begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_F \end{bmatrix}}_{\mathbf{n}} \quad (2)$$

with $n_f = \int_0^{PT_s} n(t)\exp(-j2\pi T_s)dt$. Denote $\mathbf{W}$ as the linear filter operating over $\mathbf{r}$ that minimizes the mean-squared-error (MSE) $E[|\mathbf{Wr}-\mathbf{b}|^2]$, the LMMSE MUD is [8];

$$\begin{cases} \hat{\mathbf{b}} = \operatorname{sign}(\operatorname{Re}\{\mathbf{Wr}\}) \\ \mathbf{W} = \mathbf{M}^{H}\left(\mathbf{M}\mathbf{M}^{H} + {}^{N_0}\!/\!_2\,\mathbf{I}\right)^{-1} \end{cases}, \quad (3)$$

where $\hat{\mathbf{b}}$ is the decisions for $\mathbf{b}$ and the achieved MSE for virtual user k is:

$$\mathrm{MSE}_k = 1 - \mathbf{m}_k^{H}\left(\sum_{j=1}^{K}\mathbf{m}_j\mathbf{m}_j^{H} + {}^{N_0}\!/\!_2\,\mathbf{I}\right)^{-1}\mathbf{m}_k, \quad (4)$$

$\mathbf{m}_k$ is the kth column of the matrix $\mathbf{M}$.

The MUD in (3) is identically applicable to both MC MC-CDMA and VSL MC-CDMA. The error probability of any virtual user by Gaussian approximating [9] the interference from plenty of virtual users is

$$P_e = Q\left(\sqrt{\mathrm{SINR}}\right) = Q\left(\sqrt{\frac{(\mathbf{VM})_{kk}}{\sqrt{\sum_{i=1,i\neq k}^{K}\left((\mathbf{VM})_{ki}\right)^2 + {}^{N_0}\!/\!_2\left(\mathbf{VM}^{H}\right)_{kk}}}}\right), \quad (5)$$

and the SINR (signal-to-interference-with-noise power ratio) can be proved, by its similarity to the LMMSE MUD in DS-CDMA [10], that

$$\mathrm{SINR}_k = 1/\mathrm{MSE}_k - 1, \forall k. \quad (6)$$

Actually, it can be mathematically proved that the filter $\mathbf{W}$ operated on $\mathbf{r}$ is equivalent to the filter $\widetilde{\mathbf{W}}$ operated on $\mathbf{y}$, where

$$\widetilde{\mathbf{W}} = \left(\mathbf{R} + \tfrac{N_0}{2}\mathbf{I}\right)^{-1}, \quad (7)$$

$\mathbf{R} = \mathbf{M}^{H}\mathbf{M}$, and $\mathbf{y} = \mathbf{M}^{H}\mathbf{r}$. Such alternative avoids numerical singularity at high signal-to-noise ratio (SNR) environments when $F > K$ in practices. Hence, we adopt (7) in the receiver but its equivalence to (3) and (4) can be used to determine the user capacity and develop the admission criterion, shown in the next section. The achieved MSE for kth virtual user derived by (7) has the equivalent form:

$$\mathrm{MSE}_k = 1 - \mathbf{r}_k^{H}\left(\sum_{j=1}^{K}\mathbf{r}_j\mathbf{r}_j^{H} + {}^{N_0}\!/\!_2\,\mathbf{R}\right)^{-1}\mathbf{r}_k, \quad (8)$$

whose value is equal to (4) and $\mathbf{r}_k$ denotes the kth column of $\mathbf{R}$.

III. USER CAPACITY AND RATE ALLOCATION

Assume the requested quality of user i is $(\hat{m}_i, \gamma_i)$ to denote its rate demand $\hat{R}_i = \hat{m}_i R_b$ and SINR target γ_i. Note that $\hat{m}_i$ is an integer. By the unified signal model, the rate demand of each user can be translated to the minimum number of virtual users that have to be supported and all these virtual users have the same SINR target. Hence, if there are N users requesting $(\hat{m}_1, \gamma_1)$, …, and $(\hat{m}_N, \gamma_N)$ in either MC MC-CDMA or VSL MC-CDMA system, it results in an optimization problem in terms of virtual users as:

$$\mathit{Minimize}\ \sum_{k=1}^{K}\sum_{i=f}^{F} A_{kf}^2 \quad (9)$$

$$s.\,t.\ \mathrm{SINR}_k = \mathbf{r}_k^{H}\left(\sum_{j=1,j\neq k}^{K}\mathbf{r}_j\mathbf{r}_j^{H} + {}^{N_0}\!/\!_2\,\mathbf{R}\right)^{-1}\mathbf{r}_k \ge \gamma_k, \forall k. \quad (10)$$

$$m_i \ge \hat{m}_i, i = 1,2,\ldots,N. \quad (11)$$

$$K = \sum_{i=1}^{N} m_i \quad (12)$$

where K is the number of total requested virtual users seen in the receiver and $\gamma_1, \gamma_2, \cdots, \gamma_K$ are their SINR targets. At first, the existence of feasible solutions should be checked. For mathematical convenience, we replace the SINR constraints in (10) by the corresponding MSE constraints according to (6):

$$\mathrm{MSE}_k \le \frac{1}{\gamma_k + 1}, k = 1,2,\ldots,K. \quad (13)$$

and define φ_k as the equivalent MSE target of virtual user k by $\varphi_k \equiv 1 - (\gamma_k + 1)^{-1}$, which follows that:

$$\mathbf{m}_k^{H}\left(\sum_{j=1}^{K}\mathbf{m}_j\mathbf{m}_j^{H} + {}^{N_0}\!/\!_2\,\mathbf{I}\right)^{-1}\mathbf{m}_k \ge \varphi_k, k = 1,2,\ldots,K. \quad (14)$$

Proposition 1: $F \ge \sum_{k=1}^{K}\varphi_k$ is the necessary condition of (14).

***Pf*:** See Appendix. □

Proposition 2: If $F \ge \sum_{k=1}^{K}\varphi_k$, feasible solutions exist under the constraints of (14).

***Pf*:** (i) if $F \ge K$: There is always a feasible solution satisfying the constraints in (14). Because the number of available sub-carrier is more than the number of total virtual users, we can simply assign each virtual user an exclusive sub-carrier, regardless in MC MC-CDMA or VSL MC-CDMA, and the target SINR of each virtual user can be achieved given adequate power. Transmission by this way is actually a virtual-user-based OFDMA [11], which means OFDMA is one of the feasible solutions of MC-CDMA if applying resource allocation given $F \geq K$. (ii) if $F < K$: See Appendix.

Accordingly, "$F \geq \sum_{k=1}^{K} \varphi_k$" can be used as the admission criterion to decide if a new user j with request ($\hat{m}_j, \gamma_j$) can be transmitted or not and meanwhile keep guaranteeing the QoS of all users. For example, assume K the total number of minimum resulting virtual users from the currently transmitted users and identify the expected virtual users from the new user j as virtual user K+1, K+2, ..., with MSE targets φ_{K+1}, φ_{K+2},... respectively. We can decide the entrance of the user j by the following rule:

$$\begin{cases} F \geq \sum_{k=1}^{K+\hat{m}_j} \varphi_k, \text{ admit user } j \\ F < \sum_{k=1}^{K+\hat{m}_j} \varphi_k, \text{ reject user } j \end{cases}, \tag{15}$$

Note that $\varphi_{K+1} = \varphi_{K+2} = \ldots = \varphi_{K+\hat{m}_j}$. Translating this admission criterion back to the domain of original users, it defines the capacity region of MC MC-CDMA and VSL MC-CDMA systems. If there are N user with MSE targets $\varphi_1,...,\varphi_N$, this criterion is equivalent to:

$$\sum_{n=1}^{N} m_n \varphi_n = m_1\varphi_1 + m_2\varphi_2 + \cdots + m_N\varphi_N \leq F \tag{16}$$

We can find that the transmission capacity region is a hyper-lattice structure determined by the MSE targets of all requested users. Fig. 1 plots the capacity region of a two-user example. The lattice points represent the supportable transmission rates. Such discrete rate is the feature resulting from MC and VSL multi-rate access. This admission criterion also implies that the transmission capacity is determined by the number available sub-carriers and the BER demands of users.

IV. SUB-CARRIER AND POWER ALLOCATIONS

To minimize the total power consumptions, it costs minimum power if the granted users transmitting their signals according to their minimum requested rates, which results in totally $K = \sum_{i=1}^{N} \hat{m}_i$ virtual users for N admitted users whose minimum requested rates are $\hat{m}_1, \hat{m}_2, ..., \hat{m}_N$ respectively. The optimization problem is reduced to

$$\textit{Minimize} \sum_{k=1}^{K}\sum_{i=f}^{F} A_{kf}^2 \tag{17}$$

$$s.t.\text{: } \mathrm{MSE}_k \leq 1-\varphi_k, k = 1,2,..., K. \tag{18}$$

We have the *Lagrange* equation in terms of all virtual users:

$$\mathrm{L} = \sum_{k=1}^{K}\sum_{i=f}^{F} A_{kf}^2 - \sum_{k=1}^{K} \lambda_k \left(\mathbf{r}_k^{\mathrm{H}} \left(\sum_{j=1}^{K} \mathbf{r}_j \mathbf{r}_j^{\mathrm{H}} + {}^{N_0}\!/_{2} \mathbf{R} \right)^{-1} \mathbf{r}_k - \varphi_k \right) \tag{19}$$

where λ_k is the multiplier corresponding to the MSE constraint of virtual user k. Deriving a solution that minimizes such equation is generally difficult, and applying the steepest descent

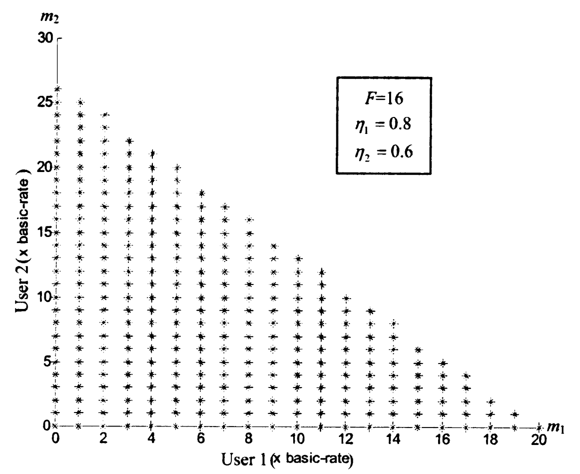

Fig. 1. The region of achievable transmission rates (m_1, m_2) of a two-user example in MC-CDMA with LMMSE MUD. The MSE targets of User 1 and User 2 are η_1=0.8 and η_2= 0.6 respectively.

method is still highly complicated to be used in practical applications. Alternatively, reformulate

$$\mathbf{r}_k^{\mathrm{H}} \left(\sum_{j=1}^{K} \mathbf{r}_j \mathbf{r}_j^{\mathrm{H}} + {}^{N_0}\!/_{2} \mathbf{R} \right)^{-1} \mathbf{r}_k = \tilde{\mathbf{w}}_k \mathbf{M}^{\mathrm{H}} \hat{\mathbf{C}}_k \mathbf{A}_k, \tag{20}$$

where $\tilde{\mathbf{w}}_k$ is the kth row of $\tilde{\mathbf{W}}$, i.e. the filter coefficients for the kth virtual user in the LMMSE MUD, $\hat{\mathbf{C}}_k = \begin{bmatrix} \alpha_{k1}c_{k1} & 0 & \cdots & 0 \\ 0 & \alpha_{k2}c_{k2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \alpha_{kF}c_{kF} \end{bmatrix}$, and $\mathbf{A}_k = \begin{bmatrix} A_{k1} \\ A_{k2} \\ \vdots \\ A_{kF} \end{bmatrix}$. This equation relates the sub-channel powers, the filter coefficients in the receiver, and the MSE target of each virtual user. Let

$$\mathbf{g}_k(n) \equiv \tilde{\mathbf{w}}_k(n)\mathbf{M}^{\mathrm{H}}(n)\hat{\mathbf{C}}_k(n), \tag{21}$$

we have an alternative *Lagrange* equation with iteration index n:

$$L = \sum_{k=1}^{K}\sum_{f=1}^{F} A_{kf}^2(n+1) - \sum_{k=1}^{K} \lambda_k \left(\mathbf{g}_k(n)\mathbf{A}_k(n+1) - \varphi_k \right). \tag{22}$$

Minimizing L with respect to A_{kf}, it derives the solution

$$\begin{cases} A_{kf}(n+1) = \varphi_k g_{kf}(n) \Big/ \sum_{f=1}^{F} g_{kf}^2, \forall k, f. \\ \mathbf{g}_k(n+1) = \tilde{\mathbf{w}}_k(n+1)\mathbf{M}^{\mathrm{H}}(n+1)\hat{\mathbf{C}}_k(n+1) \end{cases} \tag{23}$$

The derived A_{kf} is generally a complex number, which means that both signal amplitude and phase at each sub-carrier need to be adjusted in the transmitters to achieve the optimum condition. This power allocation algorithm is identically applicable to both MC MC-CDMA and VSL MC-CDMA, but a distinction should be noticed. In VSL MC-CDMA, the updated A_{kf} for unused sub-channel should be kept zero in iteration, since only partial sub-channels are used for a virtual user. By this algorithm, sub-carrier selection for each user is simultaneously solved, where "A_{kf}= 0" means the fth sub-carrier is not assigned to the virtual user k.

V. NUMERICAL SIMULATIONS

We simulate a system of two different data rates, where there are two basic-rate users (User 1, User 2) and two double-

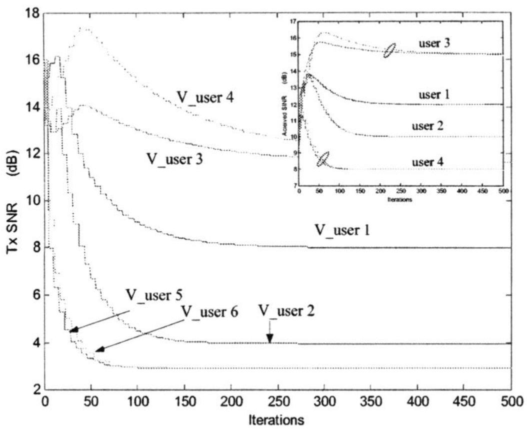

Fig. 2. The power update of user 1 (Virtual user 1), user 2 (Virtual user 2), user 3 (Virtual user 3 & 4), and user 4 (Virtual user 5 & 6) to achieve received SINR targets of 12 dB, 10 dB, 14 dB, and 8 dB respectively in MC MC-CDMA.

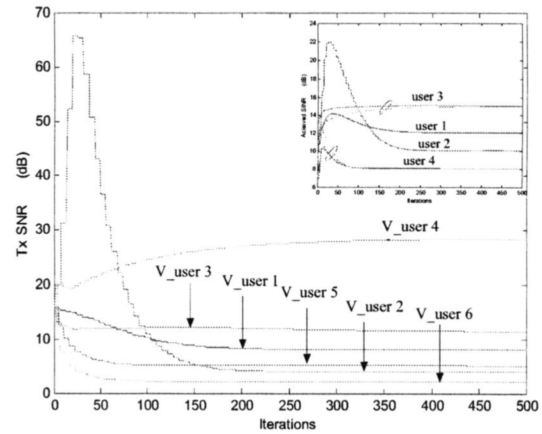

Fig. 3. The power update of user 1 (Virtual user 1), user 2 (Virtual user 2), user 3 (Virtual user 3 & 4), and user 4 (Virtual user 5 & 6) to achieve received SINR targets of 12 dB, 10 dB, 14 dB, and 8 dB respectively in VSL MC-CDMA.

rate users (User 3, User 4), i.e. 6 virtual users, with the SINR targets (12 dB, 10 dB, 15 dB, 8 dB) respectively. Without loss of generality for other codes, random spreading codes of length 16 and 8 are respectively assigned to basic-rate and double-rate users in simulations, i.e. the basic spreading factor $F = 16$ in this system. For simplicity, we assume all propagation path losses to be unity without loss of generality and the sub-channels experienced by each user are uncorrelated Rayleigh fading with unit variance. The initial power allocation of each user is uniformly distributed to all sub-carriers as convention under E_b/N_0 = 10dB, where E_b denotes the energy per data bit.

Fig. 2 shows the power update of each virtual user in MC MC-CDMA with the achieved SINR for each user (each curve represents one virtual users) in the receiver. Fig. 3 shows the results in VSL MC-CDMA. Note that there should be 16 power update curves for each virtual user, where each curve corresponds to a sub-carrier, but we plot their power sum for not complicating the figures. The SINR of each virtual user is shown approaching to its target value and we can find that the power curves of the virtual users from the same user (we call them "homogenous virtual users") have similar behaviors in MC MC-CDMA but behave different in VSL MC-CDMA. This is because homogenous virtual users experience identical sub-channels in MC MC-CDMA but not in VSL MC-CDMA. Furthermore, it has the phenomenon of power concentration at several sub-carriers for each virtual user. For example, Fig.4 specifically shows the 16 sub-carrier power-update curves of the two virtual users from User 4 in MC and VSL schemes respectively. The powers of the two virtual users concentrate on identical two sub-carriers in MC access while each concentrates on one individual sub-carrier in VSL access.

To demonstrate another characteristic, Fig. 5 shows the power update curves in a MC MC-CDMA simulation run where a random channel perturbation occurs, a user quits, and a user joins. We can see that in addition to converging to the steady solution, it fast adjusts the solution to adapt the channel and user variations. However, it is interesting that the power curves of the remaining virtual users do not change when the User 4 quits, and the steady solutions of these virtual users are identical to the previous after the participating of User 5. This

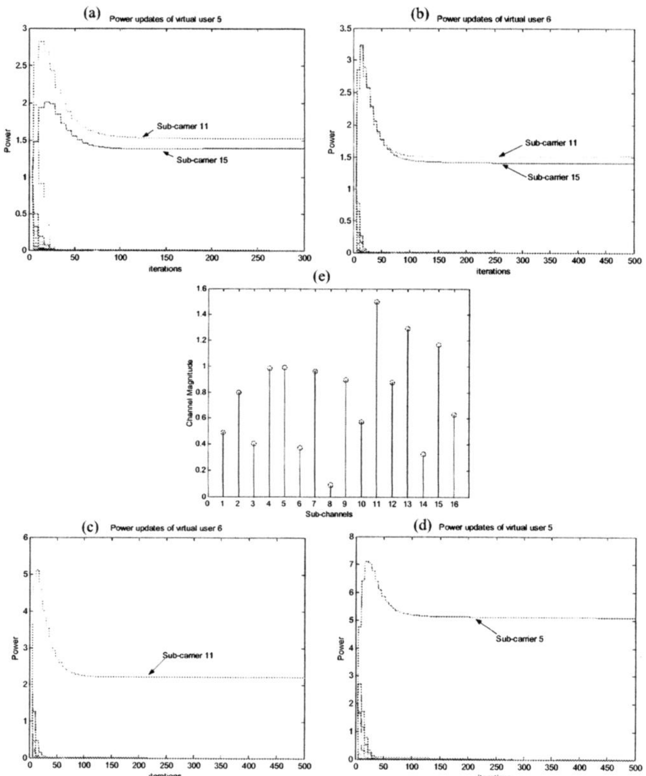

Fig. 4. Sub-carrier power allocated to V_User 5 & V_User 6 in MC MC-CDMA (a)(b) and VSL MC- CDMA(c)(d) in a simulation run, where sub-channel magnitudes of User 4 is plotted in (e). Note: V_user 5 can only use sub-carrier 1 to 8 and V_user 6 can only use sub-carrier 9 to 16 in VSL scheme.

characteristic is actually the consequence of the power concentration, where the power of each virtual user in either MC MC-CDMA or VSL MC-CDMA concentrates on best sub-channels. The 'best' is in terms of magnitude response.

VI. Conclusions

In this paper, we proposed the resource allocation of MC-CDMA with LMMSE MUD for multi-rate quality-guaranteed uplink transmissions. The user capacity of MC-CDMA with

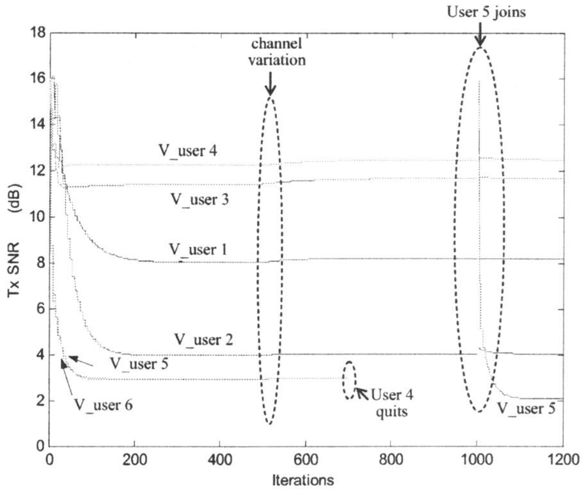

Fig. 5. Transmit power adjustments of MC MC-CDMA. V_user 3 and V_user 4 are the virtual users from user 3; V_user 4 and V_user 5 are the virtual users from user 4. The virtual user from the new user 5 is numbered as '5' after the quit of user 4.

MC and VSL multi-rate schemes was determined and we derived a useful admission criterion to control the physical-layer signal transmissions under quality constraints. This criterion indicates a simple linear decision rule that relates the admissible rate and BER demands with the number of sub-carriers used in the systems. It in practice can be used to not only determine the supportable transmission rates of the system but also decide the entrance admission of new data streams other than new users. It helps to exploit the system capacity for multimedia traffics. To minimize the total power consumption and meanwhile maintain the QoS demands, the best-quality sub-channels with respect to each user shall be selected and power concentrated. This principle is valid if the best sub-channels are all exclusive. Otherwise, these best sub-channels are shared by codes. The proposed low-complexity iterative algorithm jointly selects the sub-carriers and allocates the power for all users. In terms of resource utilization, MC-CDMA is shown advantageous in contrast to other pure OFDM-based multiple access schemes. For example, each user in OFDMA can only exclusively select its own sub-carriers without the flexibility to share with others, which is generally a sub-optimal solution.

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Appendix

Proof of Proposition 1: If (14) is satisfied, it follows that

$$\sum_{k=1}^{K} \mathbf{m}_k^{\mathrm{H}} \left(\sum_{j=1}^{K} \mathbf{m}_j \mathbf{m}_j^{\mathrm{H}} + {}^{N_0}\!/_2 \mathbf{I} \right)^{-1} \mathbf{m}_k \geq \sum_{k=1}^{K} \varphi_k , \tag{A1}$$

$$\mathrm{trace}\left\{ \mathbf{M}\mathbf{M}^{\mathrm{H}} \left(\mathbf{M}\mathbf{M}^{\mathrm{H}} + {}^{N_0}\!/_2 \mathbf{I} \right)^{-1} \right\} \geq \sum_{k=1}^{K} \varphi_k . \tag{A2}$$

By eigenvalue decomposition, $\mathbf{MM}^{\mathrm{H}}$ can be decomposed as

$$\mathbf{MM}^{\mathrm{H}} = \mathbf{V\Lambda V}^{-1} \tag{A3}$$

where $\mathbf{\Lambda}$ is a diagonal matrix whose diagonal elements $\{\lambda_1, \lambda_2, \cdots, \lambda_F\}$ are the eigenvalues of $\mathbf{M}^{\mathrm{H}}\mathbf{M}$. Taking (A3) into (A2), we have

$$\sum_{f=1}^{F} \frac{\lambda_f}{\lambda_f + {}^{N_0}\!/_2} \geq \sum_{k=1}^{K} \varphi_k . \tag{A4}$$

Since $N_0 \geq 0$, it implies $F \geq \sum_{k=1}^{K} \varphi_k$.□

Proof of Proposition 2: Making use of the mathematical proof in [12] that dealt with an equation of identical structure, it follows that if $F \geq \sum_{k=1}^{K} \varphi_k$, we can always construct a $F \times K$ matrix $\mathbf{L}$ whose columns are orthonormal and

$$\mathrm{diag}(\mathbf{LL}^{\mathrm{H}}) = [e_1, e_2, \ldots e_K], \tag{A5}$$

where $\varphi_k < e_k \leq 1, \forall k$, and $\sum_{k=1}^{K} e_k = F$. Let $\mathbf{M} = g\mathbf{L}$, where g is an arbitrary constant, then

$$\mathrm{diag}\left[\mathbf{M}^{\mathrm{H}} \left(\mathbf{MM}^{\mathrm{H}} + \tfrac{N_0}{2}\mathbf{I} \right)^{-1} \mathbf{M} \right] = \frac{g^2}{g^2 + \frac{N_0}{2}} [e_1, e_2, \cdots, e_K]. \tag{A6}$$

Selecting g large enough, $[\mathbf{M}^{\mathrm{H}}(\mathbf{MM}^{\mathrm{H}} + \frac{N_0}{2}\mathbf{I})^{-1}\mathbf{M}]_{kk}$ can be arbitrarily close to e_k and larger than φ_k for all k, and hence the constraints in (14) are satisfied. □