FFH-BFSK Multiuser Detection in Uncoordinated Narrow-band FH Systems

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Abstract— **We propose a multiuser detection scheme for channelized fast frequency hopping with binary frequency shift keying in uncoordinated, narrow-band frequency hopping systems. "Multiuser" here means uncoordinated users and the desired user. This scheme consists of channel state detector and multiuser detector. Both detectors are Gaussian approximated energy combiners and are sub-optimal in maximum *a posteriori* probability sense. In a symbol interval, the channel state detector first detects the existence of narrow-band frequency hopping signals for each chip, and then the multiuser detector properly combines received chip energies to make the decision. Simulation results show the superiority of this scheme over other existing approaches.**

I. Introduction

Due to today's fast growing demand for data transmission, the information is transmitted by several collocated and uncoordinated sources. In frequency hopping (FH) multiple access systems, the multiuser interference due to interfering from narrow-band FH signals within other collocated and uncoordinated systems does exist and remains a threat to the desired system. The corresponding multiuser detector to counteract narrow-band, unequal power, uncoordinated FH signals, however, has not been investigated yet.

[1] proposed two-user multiuser detector, which can afford more simultaneous users. The constraint is that the two-user detector relies *a priori* design of hopping patterns of all active users and the detection is based on the known received signal amplitude in AWGN channel. However, these requirements are hard to achieve in the uncoordinated FH system and fading environment.

[2]–[4] proposed multilevel FSK multistage detectors based on co-channel interference (CCI) cancellation. The CCI cancellation also needs the knowledge of active users' hopping patterns, which are usually unavailable in uncoordinated FH interfering environment. Thus the detectors in this case are not suitable. Furthermore, their FHMA/MFSK structure requires received energies at all possible bands and has large expense in realization.

This paper proposes a scheme to deal with uncoordinated FH signals with narrower bandwidth and unequal power compared to the desired fast frequency hopping (FFH) signal. We adopt channelized [1] FH-BFSK system which has cheaper implementation. Note that *a priori* design of hopping patterns [1]–[4] for the uncoordinated interfering users is not possible in the consideration of this paper. Here we focus on two random hopping manner types of the uncoordinated FH signals: *pseudo-independent* type and *independent* type. We do not need to know the received envelopes of active users required in [1].

The detection of signal can be viewed as hypothesis testing problem. The channel state (i.e. the existence and the number of the narrow-band signals colliding with the desired signal) dominates the whole performance. However, due to the hopping manner, it has distribution and is not always constant under each hypothesis. The resultant composite hypothesis testing is often quite involved. It is clear that whatever test we design can never be better than a hypothetical test in which we are able to first acquire the channel state information, and then design the optimum likelihood ratio test [6]. Doing this also eliminates some parameters in the composite hypothesis so as to avoid complex calculation.

Specifically, the proposed scheme consists of channel state detector (CSD) and multiuser detector (MD). The CSD utilizes the fact that narrow-band FH signal dwells at a band longer than the desired FFH signal, to first detects the channel state for each FFH chip. After knowing this information, the MD then properly combines received chip energies to make the final decision. Both CSD and MD are sub-optimized by Gaussian approximation to reduce the complexity. We compare the proposed scheme with FH multilevel FSK detectors [2], two-user multiuser detectors [1], equal-gain and self-normalized diversity combining receivers. We also demonstrate the case that the CSD is skipped, to show the effectiveness of the CSD. Furthermore, the maximum-likelihood (ML) scheme is adopted in case of unknown number of the uncoordinated narrow-band signals. This ML scheme is invulnerable to the unknown number and remains superior to other existing approaches.

II. System Model

We adopt the channelized [1] non-coherent FFH-BFSK system, in which two FFH chips are hopped per symbol. That is, $T_b = 2T_c$, where T_b is symbol rate and T_c is chip rate.

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The entire spread spectrum band, W_{ss}, is partitioned into N_t tones, where $N_t \leq W_{ss}T_c$. These tones are equally spaced and further partitioned into non-overlapping FFH bands[1]. Let S be the power of the transmitted signal for the desired user. The chip energy $E_c = ST_c$. The channel is single path Rayleigh fading given by $h(t) = \beta\delta(t)\exp(j\theta)$, where β is the amplitude of the path and follows Rayleigh distribution. It is assumed that β and θ are statistically independent. The average power of the single path amplitude, $\overline{\beta_0^2}$, equals 1.

Timing recovery is assumed to be ideally done at the receiver. The desired signal received at the receiver of the desired user is

$$S(t) = \sum_{m\geq 0} \beta\sqrt{2S}p(t-mT_c)\cos(2\pi(f_c(m)+f_i(m))t+\theta), \quad (1)$$

where $p(t)$ is the normalized rectangular pulse function with interval T_c. $f_c(m)$ is the carrier frequency during the m-th chip interval and is controlled by the hopping pattern of the desired user. $f_0(m) = -1/(2T_c)$ corresponds to the symbol 0 and $f_1(m) = 1/(2T_c)$ corresponds to the symbol 1. Both symbols are equally probable. Let b_n be the desired user's n-th transmitted symbol, where $n = 1, 2, \cdots$. The first chip of b_n is hopped in the $(m = 2n-2)$-th chip interval and the second chip is hopped in the $(m = 2n-1)$-th chip interval. We assume $f_c(2n-2)$ and $f_c(2n-1)$ are different.

Consider there are J narrow-band FH users from another uncoordinated system sharing W_{ss} with the desired user. We assume each of the J FH users adopts non-coherent FH-BFSK modulation with signal power γS, and symbol rate $1/T$, where $T = 2T_c$ (i.e. twice the desired user's chip interval). In order to simplify the analysis, it is assumed that there is one hop per symbol for the uncoordinated FH signals, and one hopping interval of the uncoordinated FH signals is synchronized to one symbol interval of the desired user. The uncoordinated FH signal at the receiver of the desired user is $I(t) =$

$$\sum_{j=1}^{J}\sum_{m\geq 0} \beta\sqrt{2\gamma S}p(t-mT)\cos(2\pi(f_o^j(m)\pm\frac{1}{2T})t+\phi^j(m)+\theta). \quad (2)$$

Fig. 1 shows the relationship between the desired user's carriers and narrow-band user's carriers. Note that

$$f_o^j(m) \in \{\cdots, f_c(m)-\frac{3}{2T_c}, f_c(m)-\frac{1}{2T_c}, f_c(m)+\frac{1}{2T_c}, \cdots\}, \forall j, m.$$

We consider two hopping manners for these J users: (1) *pseudo-independent* type: In each hopping interval, the J FH signals randomly dwell at J distinct SFH[2] bands (i.e. there is at most one FH interfering signal dwelling at one SFH band). (2) *independent* type: In each hopping interval, the J FH signals hop independently (i.e. there can be two or more interfering FH signals dwelling at one SFH band). The received signal is $R(t) = S(t) + I(t) + n(t)$, where $n(t)$ is AWGN with two-sided power spectral density $N_0/2$. Let

[1]"FFH band" denotes a band used by the desired FFH user (e.g. C in Fig. 1).

[2]"SFH band" denotes a band used by uncoordinated users (e.g. C1 or C2 in Fig. 1).

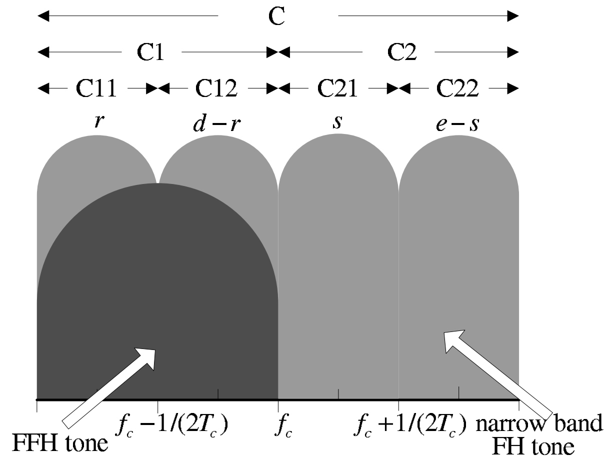


Fig. 1. FFH signal interfered by narrow-band FH signals

$\bar{y}_m = [y_{m,c0}\ y_{m,s0}\ y_{m,c1}\ y_{m,s1}]$ be the received vector for the m-th chip of the desired user. $y_{m,ci}$ and $y_{m,si}$ are respectively in-phase and quadrature components defined by

$$y_{m,ci} = \int_{mT_c}^{(m+1)T_c} R(t)\sqrt{2/T_c}\cos(2\pi(f_c(m)+f_i(m))t)dt, \quad (3)$$

$$y_{m,si} = \int_{mT_c}^{(m+1)T_c} R(t)\sqrt{2/T_c}\sin(2\pi(f_c(m)+f_i(m))t)dt. \quad (4)$$

The received vector for the first chip of b_n is $\bar{y}_{2n-2}$ and that for the second chip is $\bar{y}_{2n-1}$.

III. Channel State Detection

At each of $f_c(2n-2)$ and $f_c(2n-1)$, rather than receive the signal only in the corresponding chip interval, the desired user now keeps receiving at the chip frequency in the other chip interval, to get the channel state of that chip frequency. Doing this utilizes the fact that narrow-band FH signal's dwelling interval is longer than the chip interval of the desired user. The desired user can observe the interval in which only the interfering FH signals are present or not, to detect more reliably whether the chip frequency is dwelled by narrow-band FH users.

Specifically, let $\bar{x}_m = [x_{m,c0}\ x_{m,s0}\ x_{m,c1}\ x_{m,s1}]$ be the received channel state vector for the m-th chip of the desired user. For $m = 2n-2$ (i.e. the first chip of b_n), we have

$$x_{m,ci} = \int_{(m+1)T_c}^{(m+2)T_c} R(t)\sqrt{2/T_c}\cos(2\pi(f_c(m)+f_i(m))t)dt, \quad (5)$$

$$x_{m,si} = \int_{(m+1)T_c}^{(m+2)T_c} R(t)\sqrt{2/T_c}\sin(2\pi(f_c(m)+f_i(m))t)dt, \quad (6)$$

For $m = 2n-1$ (i.e. the second chip of b_n), we have

$$x_{m,ci} = \int_{(m-1)T_c}^{mT_c} R(t)\sqrt{2/T_c}\cos(2\pi(f_c(m)+f_i(m))t)dt, \quad (7)$$

$$x_{m,si} = \int_{(m-1)T_c}^{mT_c} R(t)\sqrt{2/T_c}\sin(2\pi(f_c(m)+f_i(m))t)dt, \quad (8)$$

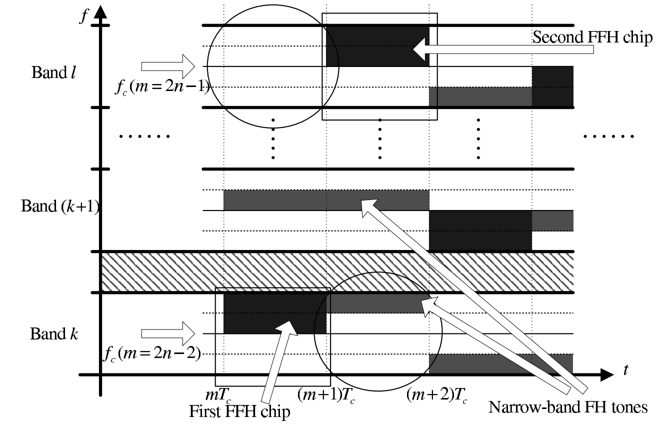


Fig. 2. Demodulation intervals and channel state observation intervals in the proposed scheme

Fig. 2 shows the differences between demodulation intervals (marked with rectangles) of $\bar{y}_m$, and channel state observation intervals of $\bar{x}_m$ (marked with circles). Note that during the same symbol period, $\bar{y}_m$ is received in a given chip interval while $\bar{x}_m$ is always received in the other chip interval.

Among J uncoordinated users, we assume r users transmit tones in C11, $d-r$ users transmit tones in C12, s users transmit tones in C21 and $e-s$ users transmit tones in C22, as indicated in Fig. 1. Pair $\{d,e\}$ is the channel state to detect. Given d and e, $E(r,s)$ denotes the event described in Fig. 1. It happens with probability $p(E(r,s)|\{d,e\}) = \binom{d}{r}\binom{e}{s}(1/2)^{d+e}$.

It can be shown that for the *pseudo-independent* FH type, $p(\{d,e\}) = \binom{2N_c-2}{J-d-e}/\binom{2N_c}{J}$ and $d,e \in \{0,1\}$.

For *independent* type narrow-band FH, $d,e \in \{0,1,\cdots,J\}$. We have $p(\{d,e\}) = \binom{J}{d}\binom{J-d}{e}(1/2N_c)^{(d+e)}(1-1/N_c)^{(J-d-e)}$.

Based on Bayes' formula, the *a posteriori* probability for $\{d,e\}$ can be written as:
$p(\{d,e\}|\bar{x}_m) = \sum_{r=0}^{d}\sum_{s=0}^{e} p(\{d,e\},E(r,s))|\bar{x}_m)$.

We can then derive the decision rule by maximizing *a posteriori* probability (MAP) for $\{d,e\}$ given $\bar{x}_m$:$\{\hat{d}(m),\hat{e}(m)\}$ $= \underset{\{d,e\}}{\arg\max}\ p(\{d,e\}|\bar{x}_m) = \underset{\{d,e\}}{\arg\max}\ W_1(\bar{x}_m|\{d,e\})$,
where $W_1(\bar{x}_m|\{d,e\}) =$

$$p(\{d,e\})\{\sum_{r=0}^{d}\sum_{s=0}^{e} p(\bar{x}_m|\{d,e\},E(r,s))p(E(r,s)|\{d,e\})\}. \quad (9)$$

The expression of $p(\bar{x}_m|\{d,e\},E(r,s))$ is in APPENDIX I. For the m-th chip, the CSD picks $\{\hat{d}(m),\hat{e}(m)\}$ such that $W_1(\bar{x}_m|\{\hat{d}(m),\hat{e}(m)\}) \geq W_1(\bar{x}_m|\{d,e\}), \forall d,e$.

The CSD is calculated by Gaussian approximation [5] and is sub-optimal in MAP sense. It performs multiple composite hypothesis test for $\{\hat{d}(m),\hat{e}(m)\}$ (hypothesis parameter is now reduced to $\{r,s\}$ pair only). Based on (9), Fig. 3 depicts the decision metric of $\{d,e\}$ in the CSD. The total number of components summed in (9) depends on $\{d,e\}$ pair. Given $\{d,e\}$, one can further prove this number is at most four for *pseudo-independent* type narrow-band FH signals, and at most $(\lfloor J/2\rfloor+1)(\lceil J/2\rceil+1)$ for *independent* type narrow-band FH, where $\lfloor x\rfloor$ denotes integer part of x, and $\lceil x\rceil = \lfloor x\rfloor + 1$.

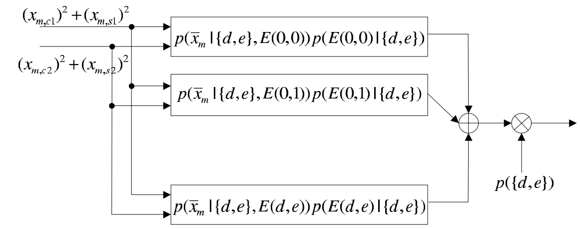


Fig. 3. Decision metric of $\{d,e\}$ in the CSD

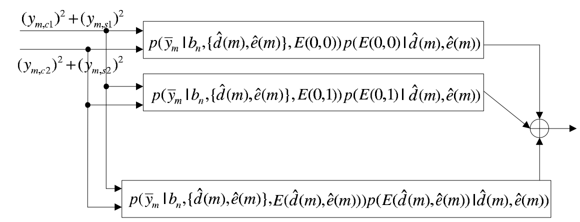


Fig. 4. Decision metric of b_n for the m-th chip in the MD

IV. Multiuser Detection

After getting $\{\hat{d}(m),\hat{e}(m)\}$, we proceed to detect b_n. It is assumed that the two received chips are uncorrelated and the MD picks $\hat{b}_n$ with the maximum $\prod_{m=2n-2}^{2n-1} W_2(\bar{y}_m|\hat{b}_n,\{\hat{d}(m),\hat{e}(m)\})$, where
$W_2(\bar{y}_m|\hat{b}_n,\{\hat{d}(m),\hat{e}(m)\}) = \sum_{r=0}^{\hat{d}(m)}\sum_{s=0}^{\hat{e}(m)}$

$$p(\bar{y}_m|b_n,\{\hat{d}(m),\hat{e}(m)\},E(r,s))p(E(r,s)|\{\hat{d}(m),\hat{e}(m)\}). \quad (10)$$

The expression of $p(\bar{y}_m|b_n,\{\hat{d}(m),\hat{e}(m)\},E(r,s))$ is in APPENDIX II. Like the CSD, the MD is a Gaussian approximated sub-optimal non-linear energy combiner. It performs binary composite hypothesis test for $\hat{b_n}$. Based on (10), Fig. 4 depicts the decision metric of b_n for the m-th chip in the MD. Given $\hat{d}(m)$ and $\hat{e}(m)$, the total number of components summed in (10) is $(\hat{d}(m)+1)(\hat{e}(m)+1)$.

Fig. 5 depicts the system block diagram of the proposed scheme. We can further see that at the receiver two energy detectors are required in order to simultaneously get $\bar{y}_m$ and $\bar{x}_{m+1}$ (with $m = 2n-2$), or $\bar{y}_m$ and $\bar{x}_{m-1}$ (with $m = 2n-1$). One is for $f_c(2n-2)$ and the other is for $f_c(2n-1)$.

V. Simulation Results

We adopt the data rate 32 kbps and the total bandwidth greater than 8.192 MHz as a representative example of the medium number of hopping bands for general use. We set N_c 128 in this case. Furthermore, we set $E_b/N_0 = 40$ *dB* and $\gamma = 3$. Repetition code (odd diversity $L = 9$) and hard decision with majority vote are adopted. Fig. 6 and Fig. 7 show the capacities in terms of coded bit error rate versus number of uncoordinated FH users. The coded bit error rate is calculated by $\sum_{l=(L+1)/2}^{L}\binom{L}{l}(P_b)^l(1-P_b)^{L-l}$, where P_b is the un-coded simulated bit error rate.

For multilevel FSK detectors [2], we adopt two hops per bit (number of hops is 14 for every 7 bits). M equals N_c, where M is the symbol alphabet size defined in [2]. Fig. 6(a) and Fig. 7(a) show that at bit error rate 10^{-4}, the proposed

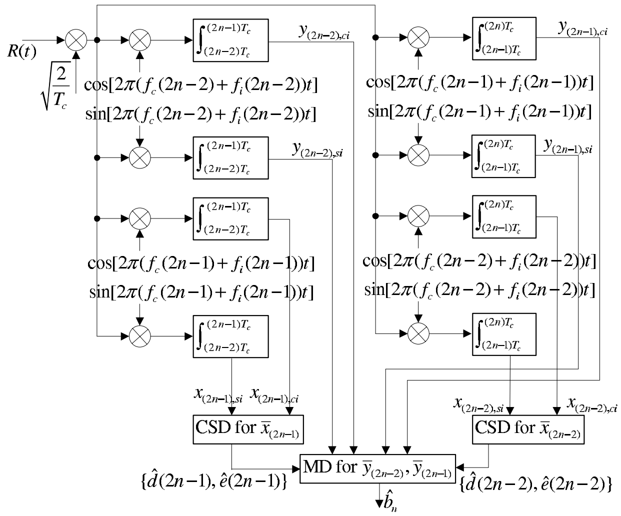


Fig. 5. Block diagram of the proposed scheme

TABLE I
TESTING FORMULATIONS IN 4 CASES

	J is known	J is unknown
MD only	**Case 1**: MD performs binary composite hypothesis test for $\hat{b}_n$ (hypothesis parameters: $\{d, e\}$ and $\{r, s\}$) and is sub-optimal in MAP sense	**Case 3**: MD performs binary composite hypothesis test for $\hat{b}_n$ (hypothesis parameters: $\{d, e\}$ and $\{r, s\}$) and is sub-optimal in ML sense
CSD and MD	**Case 2**: CSD performs multiple composite hypothesis test for $\{d, e\}$ (hypothesis parameters: $\{r, s\}$) and is sub-optimal in MAP sense. Given $\{\hat{d}, \hat{e}\}$, MD performs binary composite hypothesis test for $\hat{b}_n$ (hypothesis parameters: $\{r, s\}$) and is sub-optimal in MAP sense.	**Case 4**: CSD performs multiple composite hypothesis test for $\{d, e\}$ (hypothesis parameters: $\{r, s\}$) and is sub-optimal in ML sense. Given $\{\hat{d}, \hat{e}\}$, MD performs binary composite hypothesis test for $\hat{b}_n$ (hypothesis parameters: $\{r, s\}$) and is sub-optimal in MAP sense.

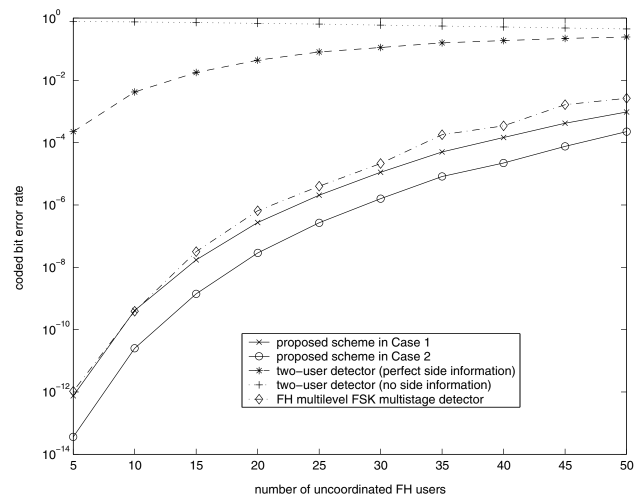
scheme can support around 15 users more than the multilevel FSK detector does.

For two-user detectors [1], we denote "side information" the knowledge of received envelope and whether the desired user uses the common band with uncoordinated users in each hop. In Fig. 6(a) and Fig. 7(a), we can see that the two-user detector is not suitable, because its assumption [1] that "any user is hit at most by one user" does not apply to this uncoordinated situation. In addition, the two-user detector requires the additional knowledge of received envelope and hopping patterns of interfering users.

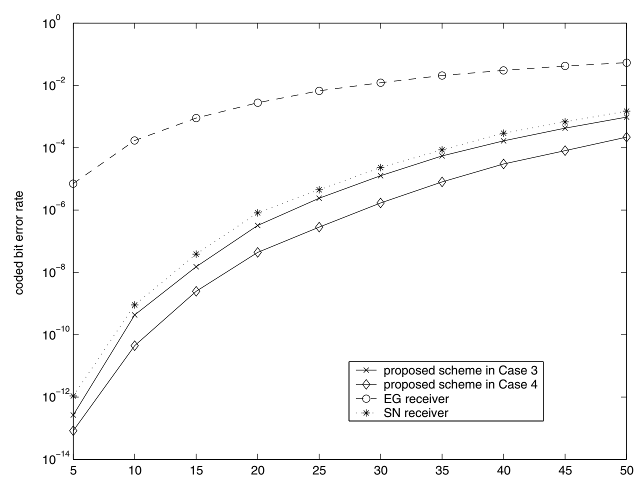
TABLE I summarizes the testing formulations of the proposed scheme in four cases. In case the total user number J is unknown, *a priori* probability of $\{d, e\}$ hypothesis is not available and it is assumed uniform for every possible $\{d, e\}$. We call the proposed scheme adopted in this case the maximum likelihood (ML) scheme. It is also assumed that there is at most one FH interfering user dwelling at one SFH band[3]. The ML scheme is compared to the EG receiver and SN receiver in Fig. 6(b) and Fig. 7(b). At bit error rate 10^{-4}, it supports around 10 users more than SN receiver, and 40 users more than the EG receiver in both types of narrow-band FH signals. By further comparing the MAP scheme and ML scheme in Fig. 6(a) and 6(b), or in Fig. 7(a) and 7(b), we can see that there is slight difference in performance curves. Therefore the proposed scheme is invulnerable to the unavailability of total user number J.

In addition, we demonstrate the effectiveness of CSD. In Fig 6(a) or 6(b), we compare the scheme with CSD and the scheme without CSD. Comparing these two curves we can see that there is performance degradation of the scheme without CSD, though it is still superior to other methods.

[3]Though this assumption does not satisfy the *independent* type, we use it since the number of hypotheses is unavailable in this case. In fact, this number is $(J+1)(J+2)/2$, which also results in huge complexity for large J. This assumption results in another sub-optimal solution while with much less complexity.



(a) Capacities of proposed schemes, two-user detectors, and FH multi-level FSK detector.



(b) Capacities of proposed schemes, SN receiver, and EG receiver.

Fig. 6. Capacity comparisons in *pseudo-independent* narrow-band FH

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VI. Conclusion

We propose a scheme composed of a channel state detector followed by a multiuser detector for channelized FFH-BFSK in uncoordinated narrow-band FH systems. The channel state detector observes the interval in which only the interfering FH signals are present or not (i.e. in the absence of desired FFH signal), to more reliably detect for each chip the existence and the number of narrow-band FH users dwelling in the same band as the desired FFH user does. Based on detected channel state information, multiuser detection is performed on both received chip vectors for each transmitted symbol. Simulation results show the superiority of the proposed scheme over the previous FH multilevel FSK detectors, two-user multiuser detectors, EG and SN receivers. In case of the unknown total number of narrow-band FH users, the ML scheme is adopted without apparent performance degradation.

In summary, combining the channel state detector and the multiuser detector results in more interfering resistance and is more applicable than the existing approaches [1]–[4] for the environment shared by several narrow-band uncoordinated FH signals.

Appendix I
Expression of $p(\bar{x}_m|\{d,e\},E(r,s))$

By Gaussian approximation [5], given $\{d,e\}$ and $E(r,s)$, $x_{m,ci}$ and $x_{m,si}$ are Gaussian random variables with zero mean and variance $\sigma_i^2(\{d,e\},E(r,s))$ in the following.
$\sigma_1^2(\{d,e\},E(r,s)) = E_c\bar{\beta}_0^2 Y_1/2 + N_0/2$,
$\sigma_2^2(\{d,e\},E(r,s)) = E_c\bar{\beta}_0^2 Y_2/2 + N_0/2$, where

$$Y_1 = (2\sqrt{2\gamma}/\pi)^2(d + s(1/3)^2 + (e-s)(1/5)^2), \qquad (11)$$

$$Y_2 = (2\sqrt{2\gamma}/\pi)^2(e + (d-r)(1/3)^2 + r(1/5)^2), \qquad (12)$$

It follows that $p(\bar{x}_m|\{d,e\},E(r,s)) =$

$$\frac{\exp(-((x_{m,c1})^2+(x_{m,s1})^2)/(2\sigma_1^2(\{d,e\},E(r,s))))}{2\pi\sigma_1^2(\{d,e\},E(r,s))} \times \frac{\exp(-((x_{m,c2})^2+(x_{m,s2})^2)/(2\sigma_2^2(\{d,e\},E(r,s))))}{2\pi\sigma_2^2(\{d,e\},E(r,s))}. \qquad (13)$$

Appendix II
Expression of $p(\bar{y}_m|b_n,\{\hat{d}(m),\hat{e}(m)\},E(r,s))$

Similarly, it can be shown that $y_{m,ci}$ and $y_{m,si}$ are Gaussian random variables with zero mean and variance in the following.

$$\sigma_1^2(b_n=1,\{\hat{d}(m),\hat{e}(m)\},E(r,s)) = E_c\bar{\beta}_0^2(1+Z_1)/2+N_0/2,$$

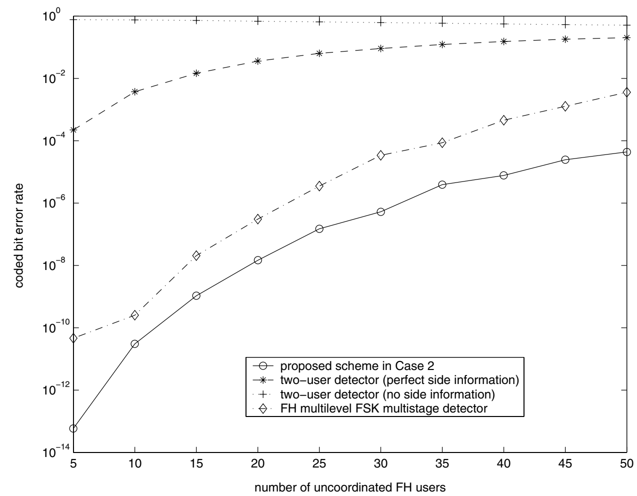
$$\sigma_2^2(b_n=1,\{\hat{d}(m),\hat{e}(m)\},E(r,s)) = E_c\bar{\beta}_0^2 Z_2/2+N_0/2,$$

$$\sigma_1^2(b_n=-1,\{\hat{d}(m),\hat{e}(m)\},E(r,s)) = E_c\bar{\beta}_0^2 Z_1/2+N_0/2,$$

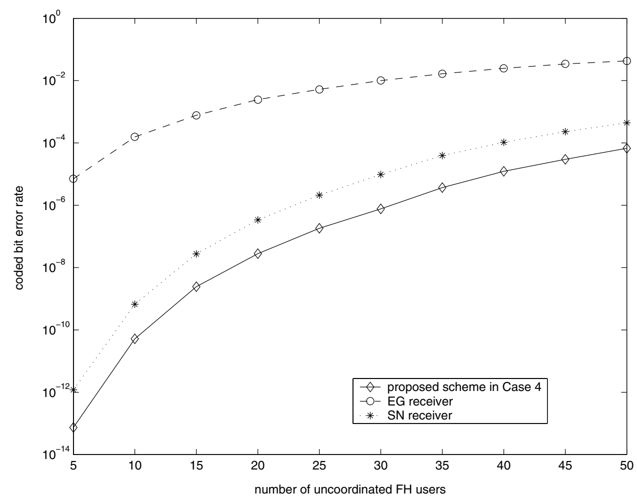
$$\sigma_2^2(b_n=-1,\{\hat{d}(m),\hat{e}(m)\},E(r,s)) = E_c\bar{\beta}_0^2(1+Z_2)/2+N_0/2,$$

where Z_i can be obtained by substituting $\hat{d}(m)$ for d, and $\hat{e}(m)$ for e in (11) and (12).

We have $p(\bar{y}_m|b_n,\{\hat{d}(m),\hat{e}(m)\},E(r,s))$ by substituting $\sigma_i^2(b_n,\{\hat{d}(m),\hat{e}(m)\},E(r,s))$ for $\sigma_i^2(\{d,e\},E(r,s))$ in (13).



(a) Capacities of proposed schemes, two-user detectors, and FH multi-level FSK detector.



(b) Capacities of proposed schemes, SN receiver, and EG receiver.

Fig. 7. Capacity comparisons in *independent* narrow-band FH

Acknowledgment

The authors would like to thank anonymous referees, for their comments and suggestions.

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