SINR for DS-CDMA with Random Spreading

Ping-Hung Chiang
Graduate Institute of Communication
Engineering
National Taiwan University
Taipei, Taiwan, R.O.C.
d1942011@ee.ntu.edu.tw

Ding-Bing Lin
Department of Electronic Engineering
National Taipei University of Technology
Taipei, Taiwan, R.O.C.
dblin@ntut.edu.tw

Hsueh-Jyh Li
Department of Electric Engineering
National Taiwan University
Taipei, Taiwan, R.O.C.
hjli@ew.ee.ntu.edu.tw

Abstract—For wireless multipath fading channels, the performance of a direct-sequence code-division multiple-access (DS-CDMA) system is degraded by the interferences, including the multipath interference (MPI) and the multiuser interference (MUI). In this paper, for the downlink DS-CDMA with random spreading, we derive the variances of the interferences and obtain the signal-to-interference-plus-noise ratio (SINR). The simple and elegant expressions for the variances of the interferences provide useful insights, while the corresponding SINR representation is helpful in evaluating the bit-error-rate (BER) performance that takes the impact of the interferences into consideration. In order to demonstrate our approach, an example regarding the performance analysis of the two-input-single-output (2ISO) space-time block coded DS-CDMA with RAKE reception is also provided.

Keywords-Antenna diversity, DS-CDMA, MPI, MUI, RAKE reception, random spreading, SINR.

I. INTRODUCTION

For wireless multipath fading channels, the performance of a direct-sequence code-division multiple-access (DS-CDMA) system is degraded by the interferences, which are classified into two categories [1]: 1) the multipath interference (MPI), consisting of the intersymbol interference (ISI) and the interpath interference (IPI), due to multipath propagation; 2) the multiuser interference (MUI) from other active users.

Recently, Bjerke *et al.* analyzed the bit-error-rate (BER) performances of the downlink DS-CDMA with various antenna diversity schemes [2], including both transmit diversity and receive diversity. However, they assumed perfect spreading and hence the impacts of the MPI and the MUI on the system performance are not taken into account.

In this paper, for the downlink DS-CDMA with random spreading, we derive the variances of the interferences and obtain the signal-to-interference-plus-noise ratio (SINR). The simple and elegant expressions for the variances of the interferences can provide useful insights. Moreover, the impacts of the interferences can be included in the derived BER expressions by applying the concept of *effective signal-to-noise ratio* (SNR) to the results in [2]. To demonstrate our strategy, an example regarding the performance analysis of the two-input-single-output (2ISO) space-time block coded DS-CDMA (STBC-DS-CDMA) with RAKE reception is also provided.

The rest of this paper is organized as follows. In Section II, the system model and the statistical properties are described.

An example for demonstrating our strategy is given in Section III. Then some numerical results are shown in Section IV, whereas conclusions are drawn in Section V.

II. PRELIMINARIES

In this section, the system model of the DS-CDMA is described and some statistical properties including the variances of the interferences are given. To simplify the derivation, we consider the *SISO synchronous downlink*.

A. System Model

Let $X_{k,i}$ denote the ith information symbol of the kth user, and let T_S represent the symbol duration. After short code spreading, the transmitted sequence for all K users in the ith symbol interval is expressed as

$$\tilde{s}_{i,l} = \sum_{k=0}^{K-1} X_{k,i} b_{k,l}, \text{ for } 0 \le l \le L-1, \tag{1}$$

where $\{b_{k,l}, 0 \le l \le L-1\}$ is the random spreading sequence, taking on the values of $\pm 1/\sqrt{L}$ with equal probability. Thus, $T_S = LT_C$, where L and T_C represent the spreading factor and the chip duration (i.e. the reciprocal of the system bandwidth), respectively. Assuming that $\tilde{s}_{i,l}$ is zero for $l < 0$ and $l \ge L$, the total transmitted baseband sequence is

$$s_l = \sum_{i=-\infty}^{\infty} \tilde{s}_{i,l-iL}. \tag{2}$$

In this paper, we consider the WSSUS Rayleigh fading channel which can be modeled as a tapped delay line model [3] with fixed tap spacing T_C. Assuming perfect synchronization and power control, the received baseband sequence of the desired user, denoted by the dth user, can be expressed as

$$r_{d,l} = \sum_{m=0}^{M-1} h_{d,m,l} s_{l-m} + n_{d,l} = \sum_{i=-\infty}^{\infty} \sum_{m=0}^{M-1} h_{d,m,l} \tilde{s}_{i,l-m-iL} + n_{d,l}, \tag{3}$$

where M is the number of channel taps, $h_{d,m,l}$ is the tap coefficient with mean zero and variance $\sigma_{d,m}^2$, and $n_{d,l}$ is the AWGN with mean zero and variance N_0. Assume the channels corresponding to different users are identical. Subsequently, the subscript d of $h_{d,m,l}$, $\sigma_{d,m}^2$, and $n_{d,l}$ is omitted for simplicity. As shown in Fig. 1, given $M < L$, rearranging $r_{d,l}$ symbol by symbol as $r_{d,i,l} = r_{d,iL+l}$ yields

$$r_{d,i,l} = \begin{cases} \sum_{m=0}^{l} h_{m,i,l}\tilde{s}_{i,l-m} + \sum_{m=l+1}^{M-1} h_{m,i,l}\tilde{s}_{i-1,l-m+L} + n_{i,l}, \\ \qquad\qquad\qquad\qquad \text{if } 0 \leq l \leq M-2; \\ \sum_{m=0}^{M-1} h_{m,i,l}\tilde{s}_{i,l-m} + n_{i,l}, \text{ if } M-1 \leq l \leq L-1, \end{cases} \quad (4)$$

with $h_{m,i,l} = h_{m,iL+l}$ and $n_{i,l} = n_{iL+l}$. Inserting (1) into (4) gives

$$r_{d,i,l} = \begin{cases} \sum_{k=0}^{K-1} X_{k,i} \sum_{m=0}^{l} h_{m,i,l}b_{k,l-m} + \sum_{k=0}^{K-1} X_{k,i-1} \sum_{m=l+1}^{M-1} h_{m,i,l}b_{k,l-m+L} \\ +n_{i,l}, \qquad\qquad\qquad\qquad \text{if } 0 \leq l \leq M-2; \\ \sum_{k=0}^{K-1} X_{k,i} \sum_{m=0}^{M-1} h_{m,i,l}b_{k,l-m} + n_{i,l}, \text{ if } M-1 \leq l \leq L-1. \end{cases} \quad (5)$$

Assume the RAKE receiver has F ($F \leq M$) fingers and, for the finger f, $0 \leq f \leq F-1$, define (see Fig. 1)

$$\tilde{r}_{d,f,i,l} = \begin{cases} r_{d,i,l+f}, & \text{if } 0 \leq l \leq L-f-1; \\ r_{d,i+1,l+f-L}, & \text{if } L-f \leq l \leq L-1. \end{cases} \quad (6)$$

Then substituting (5) into (6) produces

$$\tilde{r}_{d,f,i,l} = \begin{cases} \sum_{k=0}^{K-1} X_{k,i} \sum_{m=0}^{l+f} h_{m,i,l+f}b_{k,l+f-m} \\ +\sum_{k=0}^{K-1} X_{k,i-1} \sum_{m=l+f+1}^{M-1} h_{m,i,l+f}b_{k,l+f-m+L} + n_{i,l+f}, \\ \qquad\qquad\qquad\qquad \text{if } 0 \leq l \leq M-f-2; \\ \sum_{k=0}^{K-1} X_{k,i} \sum_{m=0}^{M-1} h_{m,i,l+f}b_{k,l+f-m} + n_{i,l+f}, \\ \qquad\qquad\qquad\qquad \text{if } M-f-1 \leq l \leq L-f-1; \\ \sum_{k=0}^{K-1} X_{k,i+1} \sum_{m=0}^{l+f-L} h_{m,i,l+f-L}b_{k,l+f-m-L} \\ +\sum_{k=0}^{K-1} X_{k,i} \sum_{m=l+f-L+1}^{M-1} h_{m,i,l+f-L}b_{k,l+f-m} + n_{i,l+f-L}, \\ \qquad\qquad\qquad\qquad \text{if } L-f \leq l \leq L-1. \end{cases} \quad (7)$$

Accordingly, at the receiver of the desired user, the code-matched filter corresponding to the fth path performs despreading on $\{\tilde{r}_{d,f,i,l}, 0 \leq l \leq L-1\}$ and outputs signal for the ith symbol as

$$R_{d,f,i} = \sum_{l=0}^{L-1} \tilde{r}_{d,f,i,l}b_{d,l} \qquad (8)$$
$$= H_{f,i}X_{d,i} + P_{d,f,i} + S_{d,f,i} + U_{d,f,i} + N_{f,i},$$

where the multiplicative distortion (MD) $H_{f,i}$, the IPI $P_{d,f,i}$, the ISI $S_{d,f,i}$, the MUI $U_{d,f,i}$, and the noise $N_{f,i}$ are given by

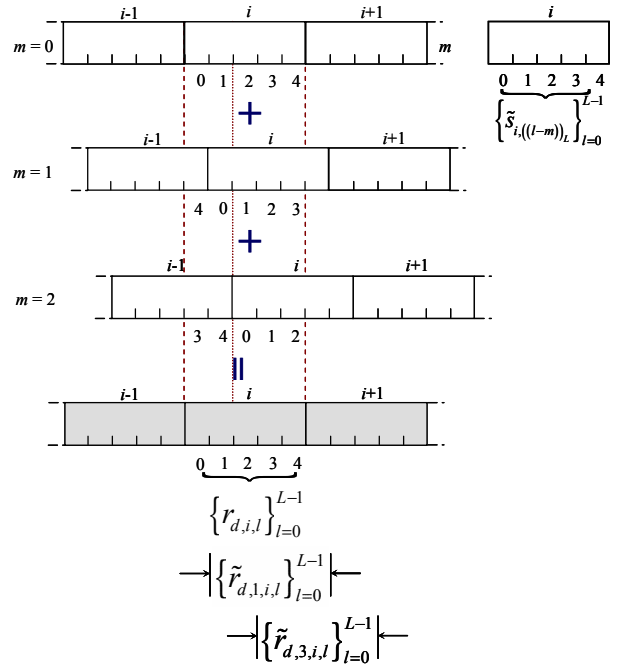$$H_{f,i} = \frac{1}{L} \sum_{l=0}^{L-1} h_{f,i,l}, \qquad (9)$$

Fig. 1 Graphical illustration of the received signals for M=3, L=5.

$$P_{d,f,i} = X_{d,i} \left(\sum_{\substack{m=0 \\ m\neq f}}^{M-1} \sum_{l=m}^{L-1} h_{m,i,l}b_{d,l-m}b_{d,l-f} \right.$$
$$+\sum_{\substack{m=1 \\ m\neq f}}^{M-1} \sum_{l=0}^{m-1} h_{m,i,l}b_{d,l-m+L}b_{d,l-f+L} - \sum_{m=0}^{f-1} \sum_{l=m}^{f-1} h_{m,i,l}b_{d,l-m}b_{d,l-f+L} \qquad (10)$$
$$\left. -\sum_{m=f+1}^{M-1} \sum_{l=f}^{m-1} h_{m,i,l}b_{d,l-m+L}b_{d,l-f} \right),$$

$$S_{d,f,i} = X_{d,i-1} \sum_{m=f+1}^{M-1} \sum_{l=f}^{m-1} h_{m,i,l}b_{d,l-m+L}b_{d,l-f}$$
$$+X_{d,i+1} \sum_{m=0}^{f-1} \sum_{l=m}^{f-1} h_{m,i,l}b_{d,l-m}b_{d,l-f+L}, \qquad (11)$$

$$U_{d,f,i} = \sum_{\substack{k=0 \\ k\neq d}}^{K-1} \left[X_{k,i} \left(\sum_{m=0}^{M-1} \sum_{l=m}^{L-1} h_{m,i,l}b_{k,l-m}b_{d,l-f} \right. \right.$$
$$+\sum_{m=1}^{M-1} \sum_{l=0}^{m-1} h_{m,i,l}b_{k,l-m+L}b_{d,l-f+L} - \sum_{m=0}^{f-1} \sum_{l=m}^{f-1} h_{m,i,l}b_{k,l-m}b_{d,l-f+L}$$
$$\left. -\sum_{m=f+1}^{M-1} \sum_{l=f}^{m-1} h_{m,i,l}b_{k,l-m+L}b_{d,l-f} \right) \qquad (12)$$
$$+X_{k,i-1} \sum_{m=f+1}^{M-1} \sum_{l=f}^{m-1} h_{m,i,l}b_{k,l-m+L}b_{d,l-f}$$
$$\left. +X_{k,i+1} \sum_{m=0}^{f-1} \sum_{l=m}^{f-1} h_{m,i,l}b_{k,l-m}b_{d,l-f+L} \right],$$

$$N_{f,i} = \sum_{l=0}^{L-1} n_{i,l} b_{d,((l-f))_L} . \tag{13}$$

Notice that (9)-(13) are derived by changing variables, switching the order of the summations, and rearranging the resultant terms. Here $((\cdot))_L$ denotes the modulo-L operation, and the subscript d of $H_{d,f,i}$ and $N_{d,f,i}$ is omitted.

From (13), for random spreading, it is readily to recognize that $N_{f,i}$ is still the AWGN with mean zero and variance N_0. It is assumed that the symbols $X_{k,i}$ are independently and identically distributed (i.i.d.) with mean zero and variance E_S (symbol energy) and the tap coefficients $\{h_{m,i,l}, 0 \le m \le M-1\}$ are mutually uncorrelated. Thereupon, even though the distributions of the interferences, including $P_{d,f,i}$, $S_{d,f,i}$, and $U_{d,f,i}$ are not easy to identify, it is readily to verify: 1) they are mutually uncorrelated; 2) they are all uncorrelated to the MD $H_{f,i}$ and the noise $N_{f,i}$, as well; 3) they are all of mean zero. Furthermore, since independent Gaussian noise results in the smallest capacity, it is reasonable to model these interferences as Gaussian random variables and hence to achieve the performance bound [4]. Indeed, (8) can be rewritten as

$$R_{d,f,i} = H_{f,i} X_{d,i} + W_{d,f,i} , \tag{14}$$

where $W_{f,i} = P_{d,f,i} + S_{d,f,i} + U_{d,f,i} + N_{f,i}$ is the equivalent AWGN with mean zero and variance $\sigma_{W_{d,f}}^2 = \sigma_{P_{d,f}}^2 + \sigma_{S_{d,f}}^2 + \sigma_{U_{d,f}}^2 + N_0$.

B. Statistical Properties

Here the statistical properties regarding the channel tap coefficient, the MD, and the interferences are clarified.

For WSSUS Rayleigh fading channels with classical Doppler spectrum, the correlation of the tap coefficients is [3]

$$E\left[h_{m,i,l} h_{m',i',l'}^* \right] = \sigma_m^2 J_0 \left\{ 2\pi f_D T_C \left[(i-i')L + (l-l') \right] \right\} \delta_{mm'}, \tag{15}$$

where σ_m^2 is the fading power of the mth tap, $J_0(\cdot)$ is the zero-order Bessel function of the first kind, f_D is the maximum Doppler frequency, and δ_{ij} is the Kronecker delta. For the exponential power delay profile (PDP) [3] with the constraint $\sum_{m=0}^{M-1} \sigma_m^2 = 1$, we have $\sigma_m^2 = \lambda^m (1-\lambda)/(1-\lambda^M)$ where $\lambda = e^{-1/\mathcal{D}}$. The delay control $\mathcal{D}$ dominates the normalized root-mean-square (RMS) delay spread τ_{rms}/T_C.

1) Variance of the Multiplicative Distortion

In light of (9), it is clear that the MD $H_{f,i}$ is of mean zero and hence variance

$$\sigma_{H_f}^2 = E\left[\left| H_{f,i} \right|^2 \right]$$
$$= \frac{\sigma_f^2}{L^2} \sum_{l=-L+1}^{L-1} (L - |l|) J_0(2\pi f_D T_C l). \tag{16}$$

2) Variances of the Interferences

From (10) and (11), utilizing the fact that the information symbols, the channel tap gains, and the random spreading sequences are mutually independent, the variances of the IPI $P_{d,f,i}$ and the ISI $S_{d,f,i}$ are calculated as follows.

$$\sigma_{P_{d,f}}^2 = E\left[\left| P_{d,f,i} \right|^2 \right] = \frac{E_S}{L^2} \left\{ \sum_{\substack{m=0 \\ m \ne f}}^{M-1} \left[(L-m) + m \right] \sigma_m^2 \right.$$
$$\left. - \sum_{m=f+1}^{M-1} (m-f) \sigma_m^2 - \sum_{m=0}^{f-1} (f-m) \sigma_m^2 \right\} \tag{17}$$
$$= \frac{E_S}{L^2} \sum_{\substack{m=0 \\ m \ne f}}^{M-1} (L - |m-f|) \sigma_m^2 ,$$

$$\sigma_{S_{d,f}}^2 = E\left[\left| S_{d,f,i} \right|^2 \right] = \frac{E_S}{L^2} \sum_{\substack{m=0 \\ m \ne f}}^{M-1} |m-f| \sigma_m^2. \tag{18}$$

Thereupon, the variance of the MPI is

$$\sigma_{P_{d,f}}^2 + \sigma_{S_{d,f}}^2 = \frac{E_S}{L} \sum_{\substack{m=0 \\ m \ne f}}^{M-1} \sigma_m^2 = \frac{E_S}{L} \left(1 - \sigma_f^2 \right) \le \frac{E_S}{L} . \tag{19}$$

Similarly, by (12), the variance of the MUI $U_{d,f,i}$ is derived as

$$\sigma_{U_{d,f}}^2 = E\left[\left| U_{d,f,i} \right|^2 \right] = \frac{E_S}{L^2} \sum_{\substack{k=0 \\ k \ne d}}^{K-1} \sum_{m=0}^{M-1} (L - |m-f| + |m-f|) \sigma_m^2 \tag{20}$$
$$= \frac{E_S}{L} (K-1).$$

Consequently, the power of the total interference can be expressed by a simple formula as

$$\sigma_{P_{d,f}}^2 + \sigma_{S_{d,f}}^2 + \sigma_{U_{d,f}}^2 = \frac{E_S}{L} \left[(1 - \sigma_f^2) + (K-1) \right]. \tag{21}$$

Noteworthily, an upper bound of (21) can be obtained by considering the most dispersive condition of the exponential PDP. In this extreme case, $\mathcal{D} \to \infty$, $\lambda \to 1$, and $\sigma_f^2 \to 1/M$, i.e., the exponential PDP reduces to the uniform PDP. Hence, for this extreme channel condition, the power of the total interference is given by

$$\sigma_{P_{d,f}}^2 + \sigma_{S_{d,f}}^2 + \sigma_{U_{d,f}}^2 = \frac{E_S}{L} \left[\left(1 - \frac{1}{M} \right) + (K-1) \right]$$
$$\le \frac{E_S}{L} \left[1 + (K-1) \right] = \frac{E_S}{L} K, \tag{22}$$

which reveals that the MPI can be treated as the MUI from an additional user. In other words, the total interference can be approximated as the MUI contributed by K users. This confirms the assumption in [1]. Moreover, it is evident that the number of users K dominates the quantity of the total interference whereas the spreading factor L stands for the inherent interference rejection ability of a DS-CDMA system.

III. 2ISO DS-CDMA

To demonstrate our strategy, an example for evaluating the performance of the 2ISO space-time block coded DS-CDMA system in the quasi-static (QS) channel is provided. Subsequently, the subscript d is omitted for simplicity.

Denote the spatially identical and independent MD, for the gth transmit antenna ($g = 0,1$), as $H_{f,i}^{(g)}$ and assume the channel is QS over a space-time codeword duration, then the piecewise-constant MD for the ith space-time codeword is defined as $\mathcal{H}_{f,i}^{(g)} \triangleq H_{f,2i+0}^{(g)} = H_{f,2i+1}^{(g)}$ with mean zero and variance σ_f^2. Accordingly, from (14), the output for the fth finger is [2] [5]

$$
\mathbf{r}_{f,i} = \mathbf{H}_{f,i}\mathbf{x}_i + \mathbf{w}_{f,i}
$$

$$
\begin{bmatrix} R_{f,2i+0} \\ R_{f,2i+1}^* \end{bmatrix} = \begin{bmatrix} \mathcal{H}_{f,i}^{(0)} & \mathcal{H}_{f,i}^{(1)} \\ \mathcal{H}_{f,i}^{(1)*} & -\mathcal{H}_{f,i}^{(0)*} \end{bmatrix} \begin{bmatrix} X_{2i+0} \\ X_{2i+1} \end{bmatrix} + \begin{bmatrix} W_{f,2i+0} \\ W_{f,2i+1}^* \end{bmatrix},
\tag{23}
$$

where $\mathbf{w}_{f,i} \sim \mathcal{CN}\left(\mathbf{0}, \sigma_{W_f}^2 \mathbf{I}_2\right)$ and $\sigma_{W_f}^2 = 2\left(\sigma_{P_f}^2 + \sigma_{S_f}^2 + \sigma_{U_f}^2 + N_0\right)$.

Performing the space-time matched filtering gives

$$
\begin{aligned}
\hat{\mathbf{r}}_{f,i} &= \left(\boldsymbol{\Lambda}_{f,i}\mathbf{H}_{f,i}\right)^H \mathbf{r}_{f,i} \\
&= \left(\boldsymbol{\Lambda}_{f,i}\mathbf{H}_{f,i}^H\mathbf{H}_{f,i}\right)\mathbf{x}_i + \left(\boldsymbol{\Lambda}_{f,i}\mathbf{H}_{f,i}^H\right)\mathbf{w}_{f,i} \\
&= \hat{\mathbf{H}}_{f,i}\mathbf{x}_i + \hat{\mathbf{w}}_{f,i},
\end{aligned}
\tag{24}
$$

where $\boldsymbol{\Lambda}_{f,i} = \alpha_{f,i}^{-1}\mathbf{I}_2$, $\hat{\mathbf{H}}_{f,i} = \alpha_{f,i}\mathbf{I}_2$, $\alpha_{f,i} = \sqrt{\sum_{g=0}^{1}\left|\mathcal{H}_{f,i}^{(g)}\right|^2}$, $\hat{\mathbf{r}}_{f,i} = \begin{bmatrix} \hat{R}_{f,2i+0} & \hat{R}_{f,2i+1} \end{bmatrix}^T$, $\hat{\mathbf{w}}_{f,i} = \begin{bmatrix} \hat{W}_{f,2i+0} & \hat{W}_{f,2i+1} \end{bmatrix}^T$ and $\hat{\mathbf{w}}_{f,i} \sim \mathcal{CN}\left(\mathbf{0}, \sigma_{W_f}^2 \mathbf{I}_2\right)$.

Thereupon, (24) can be separated into two equations as

$$
\hat{R}_{f,2i+t} = \alpha_{f,i}X_{2i+t} + \hat{W}_{f,2i+t}, \quad \text{for } t = 0, 1.
\tag{25}
$$

A. Maximum-Likelihood Detection

From (25), stacking up $\left\{\hat{R}_{f,2i+t}, 0 \le f \le F-1\right\}$ gives

$$
\hat{\mathbf{r}}_{2i+t} = \boldsymbol{\alpha}_i X_{2i+t} + \hat{\mathbf{w}}_{2i+t}, \quad \text{for } t = 0, 1,
\tag{26}
$$

where $\hat{\mathbf{r}}_{2i+t} = \begin{bmatrix} \hat{R}_{0,2i+t} & \hat{R}_{1,2i+t} & \cdots & \hat{R}_{F-1,2i+t} \end{bmatrix}^T$, $\boldsymbol{\alpha}_i = \begin{bmatrix} \alpha_{0,i} & \alpha_{1,i} & \cdots & \alpha_{F-1,i} \end{bmatrix}^T$ and $\hat{\mathbf{w}}_{2i+t} = \begin{bmatrix} \hat{W}_{0,2i+t} & \hat{W}_{1,2i+t} & \cdots & \hat{W}_{F-1,2i+t} \end{bmatrix}^T$. Since $\hat{\mathbf{w}}_{2i+t} \sim \mathcal{CN}\left(\mathbf{0}, \boldsymbol{\Sigma}\boldsymbol{\Sigma}\right)$ and $\boldsymbol{\Sigma} = \text{diag}\left\{\sigma_{W_0}, \sigma_{W_1}, \cdots, \sigma_{W_{F-1}}\right\}$, then the maximum-likelihood (ML) detection is given by

$$
\begin{aligned}
\hat{X}_{2i+t} &= \arg\min_X \left(\hat{\mathbf{r}}_{2i+t} - \boldsymbol{\alpha}_i X\right)^H \left(\boldsymbol{\Sigma}\boldsymbol{\Sigma}\right)^{-1} \left(\hat{\mathbf{r}}_{2i+t} - \boldsymbol{\alpha}_i X\right) \\
&= \arg\min_X \left\|\boldsymbol{\Sigma}^{-1}\left(\hat{\mathbf{r}}_{2i+t} - \boldsymbol{\alpha}_i X\right)\right\|^2, \text{ for } t = 0,1,
\end{aligned}
\tag{27}
$$

where $\boldsymbol{\Sigma}^{-1}$ represents the *noise balancing*. Let

$$
\hat{\mathbf{r}}_{2i+t} = \boldsymbol{\Sigma}^{-1}\hat{\mathbf{r}}_{2i+t} = \boldsymbol{\alpha}_i X_{2i+t} + \hat{\mathbf{w}}_{2i+t}, \quad \text{for } t = 0, 1,
\tag{28}
$$

with $\boldsymbol{\alpha}_i = \boldsymbol{\Sigma}^{-1}\boldsymbol{\alpha}_i$, $\hat{\mathbf{w}}_{2i+t} = \boldsymbol{\Sigma}^{-1}\hat{\mathbf{w}}_{2i+t}$. Then, (27) is rewritten as

$$
\hat{X}_{2i+t} = \arg\min_X \left\|\hat{\mathbf{r}}_{2i+t} - \boldsymbol{\alpha}_i X\right\|^2 = \arg\min_X \left|Y_{2i+t} - X\right|^2,
\tag{29}
$$

where

$$
Y_{2i+t} = \frac{\boldsymbol{\alpha}_i^T \hat{\mathbf{r}}_{2i+t}}{\left\|\boldsymbol{\alpha}_i\right\|^2} = X_{2i+t} + Z_{2i+t},
\tag{30}
$$

and $Z_{2i+t} = \boldsymbol{\alpha}_i^T \hat{\mathbf{w}}_{2i+t} \big/ \left\|\boldsymbol{\alpha}_i\right\|^2$. From (29), it is clear that the $2F^{th}$ order diversity channel is transformed into the equivalent AWGN channel via the maximum-ratio combining (MRC).

B. SINR and BER

From (30), the total instantaneous SINR is derived as

$$
\gamma = \frac{E\left[\left|X_{2i+t}\right|^2\right]}{E\left[\left|Z_{2i+t}\right|^2\right]} = \left\|\boldsymbol{\alpha}_i\right\|^2 E_s = \left\|\boldsymbol{\Sigma}^{-1}\boldsymbol{\alpha}_i\right\|^2 E_s
$$

$$
= \sum_{f=0}^{F-1}\frac{\alpha_{f,i}^2 E_s}{\sigma_{W_f}^2} = \sum_{g=0}^{1}\sum_{f=0}^{F-1}\frac{\left|\mathcal{H}_{f,i}^{(g)}\right|^2 E_s}{\sigma_{W_f}^2} = \sum_{g=0}^{1}\sum_{f=0}^{F-1} r_f^{(g)},
\tag{31}
$$

where $r_f^{(g)}$ is the instantaneous SINR of the $(g,f)^{th}$ branch. Also, γ is of probability density function (pdf) as [2, eq.(18)]

$$
p_r(\xi) = \sum_{f=0}^{F-1}\left(\frac{B_{f1}}{\Gamma_f} + \frac{B_{f2}}{\Gamma_f^2}\xi\right)e^{-\frac{\xi}{\Gamma_f}}, \quad \xi \ge 0,
\tag{32}
$$

where $B_{f1} = 2A_f^2 \sum_{\substack{m=0 \\ m\ne f}}^{F-1}\Gamma_m\big/\left(\Gamma_m - \Gamma_f\right)$, $B_{f2} = A_f^2$, $A_f = \prod_{\substack{m=0 \\ m\ne f}}^{F-1}\Gamma_f\big/\left(\Gamma_f - \Gamma_m\right)$ and $\Gamma_f = E\left[\gamma_f^{(g)}\right]$.

From (29), for BPSK modulation, $\hat{X}_{2i+t}$ is of conditional BER $Q\left(\sqrt{2\gamma}\right)$ and hence its average BER is as [2, eq.(19)]

$$
\begin{aligned}
P_b &= \int_0^{\infty} p_r(\xi) Q\left(\sqrt{2\xi}\right)d\xi \\
&= \sum_{f=0}^{F-1}\left[\frac{B_{f1}}{2}\left(1-\mu_f\right) + \frac{B_{f2}}{4}\left(2 - 3\mu_f + \mu_f^3\right)\right],
\end{aligned}
\tag{33}
$$

where $\mu_f = \sqrt{\Gamma_f \big/ \left(1 + \Gamma_f\right)}$.

IV. NUMERICAL RESULTS

For all numerical results, the parameters are detailed as follows. Firstly, the system bandwidth is 800 KHz, thus the chip duration is $T_C = 1.25\mu$ seconds; secondly, for exponential PDP, the number of uncorrelated paths and the delay control are $M = 6$ and $\mathcal{D} = 2$, respectively, which correspond to the RMS delay spread $\tau_{rms} = 1.74\mu$ seconds; thirdly, the channel is QS; fourthly, the modulation is BPSK; finally, the number

of fingers is $F = 1, 2,..., 6$. In addition to the BERs of the 2ISO DS-CDMA system, the BERs of the SISO system are also provided as the benchmarks.

Fig. 2 is the BERs of various DS-CDMA systems with $K = 10$ and $L = 127$. It is evident that the interferences, comprising the MPI and the MUI, have great impacts on the system performances such that the irreducible error floors occur. In addition, it is clear that the 2ISO systems benefit from the larger diversity gain and hence perform better than the SISO systems. Thereupon, combining the path diversity and the antenna diversity does enhance the signal component and provide the better interference reduction.

Fig. 3 is the error floors corresponding to different spreading factors (i.e. $L = 15, 31, 63, 127, 255, 511, 1023,$ and 2047), provided that $K = 10$. It confirms that enlarging the spreading factor results in a better interference rejection ability of the DS-CDMA system and hence a lower BER value.

Fig. 4 is the error floors with respect to different number of active users, given that $L = 127$. For $F = 6$, if the permissible BER is 10^{-3}, then the maximum numbers of active users for the SISO and the 2ISO systems are about 12 and 18, respectively. Consequently, by adding one transmit antenna at the base station, the system capacity is increased about 6 users.

V. CONCLUSION

In this paper, for the downlink DS-CDMA with random spreading, we derive the variances of the interferences and obtain the SINR. The simple and elegant expressions for the variances of the interferences provide useful insights, while the resulting SINR representation is helpful in evaluating the BER performance. An example regarding the performance analysis of the 2ISO space-time block coded DS-CDMA with RAKE reception is also provided. Based on our approach, by applying the concept of effective SNR to the results in [2], one can theoretically investigate the impacts of the MPI and the MUI on the performance of the DS-CDMA systems.

REFERENCES

[1] T. Eng and L. Milstein, "Coherent DS-CDMA performance in Nakagami multipath fading," *IEEE Trans. Commun.*, vol. 43, pp. 1134–1143, Feb./Mar./Apr. 1995.

[2] B.A. Bjerke, Z. Zvonar, and J. G. Proakis, "Antenna diversity combining schemes for WCDMA systems in fading multipath channels," *IEEE Trans. Wireless Commun.*, vol. 3, pp. 97-106, Jan. 2004.

[3] G. L. Stüber, *Principles of Mobile Communication*, 2nd ed. London: Kluwer Academic Publishers, 2001.

[4] C. E. Shannon, "Communication in the presence of noise," *Proc. IRE*, Vol. 37, pp. 10–21, 1949.

[5] S. M. Alamouti, "A simple transmit diversity scheme for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1451-1458, Oct. 1998.
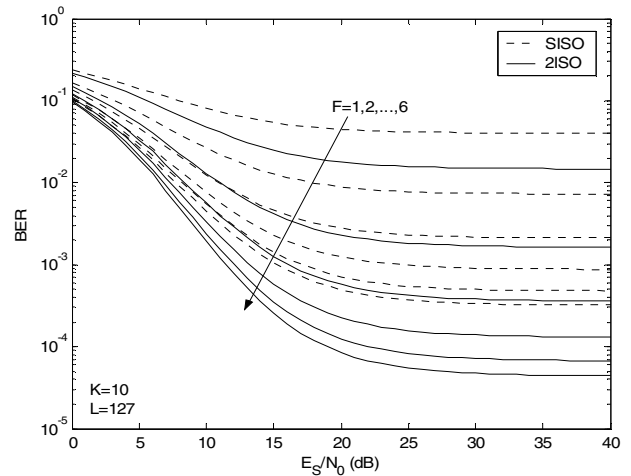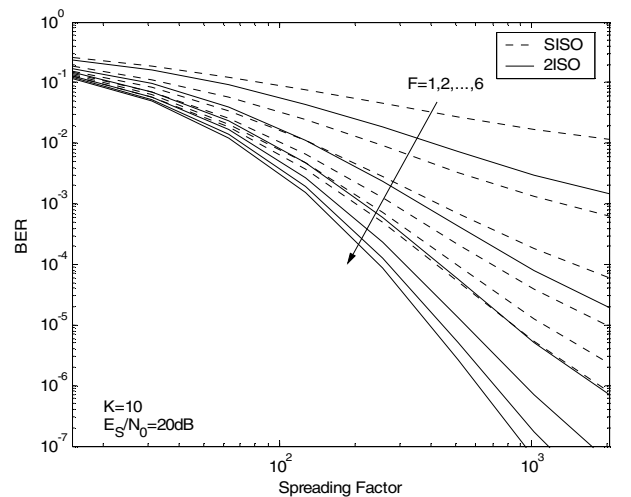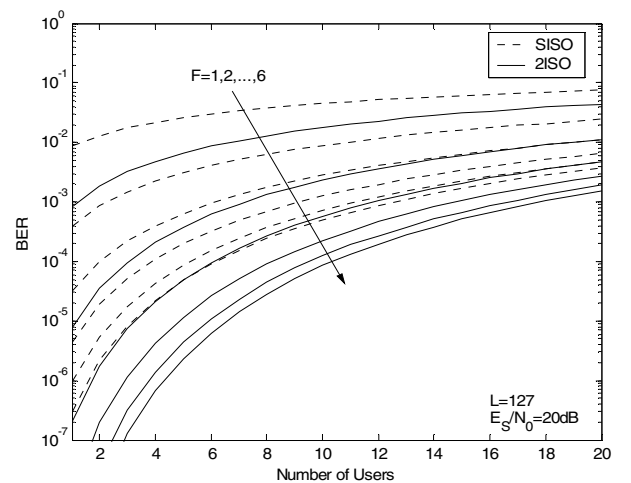
Fig. 2 BER versus SNR.

Fig. 3 BER versus L.

Fig. 4 BER versus K.