# FREQUENCY-DOMAIN APPROACH TO DS-CDMA MULTIUSER DETECTION OVER FREQUENCY-SELECTIVE SLOWLY FADING CHANNELS

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**Abstract** - By extending the frequency-domain approach [1] to practical scenarios such as the frequency-selective slowly fading channels, we justify the value of our proposed frequency-domain approach to multiuser detection in the DS-CDMA communications systems. With this frequency-domain approach which maintains the frequency-domain orthogonality between different users, the DS-CDMA communication is MAI-free even in the frequency-selective slowly fading asynchronous channels. This frequency-domain approach not only saves the cost to maintain synchronous channels as in the practical communications systems employing the time-domain orthogonal codes (e.g. IS-95 and cdma2000), but also enjoys the pretty low complexity properties as in the conventional point-to-point communications, which holds many desirable properties attractive to the practical communications systems.

**Keywords** - Frequency-domain approach, multiuser detection, frequency-selective channels, and OFDM.

## 1. INTRODUCTION

Owing to its narrow-band interference suppression ability and its nature to mitigate channel fading impairment, direct sequence code division multiple access (DS-CDMA) communication has become one of the most important technology and has been extensively applied in many wireless communications systems, such as the CDMA cellular IS-95 and the third generation cellular (3G) systems known as IMT-2000. However, because users transmit information spread by their designed signature sequences over the same frequency band simultaneously, the wide-band interference from other users (known as multiple access interference (MAI)) might severely degrade the reception of the desired signal and therefore researchers all over the world have been working hard on mitigating MAI for the past decades.

Although multiuser detection guarantees the reliable detection as long as the employed signature waveforms satisfy the linear independence assumption [2], conventional detection employing orthogonal signature sequences prevails multiuser detection and is widely applied in practical communications systems such as the IS-95 and cdma2000. However, such a conventional detection scheme not only has to maintain synchronous channel but also degrades severely when the received timings between different users are not perfectly aligned [3], [4].

To solve this dilemma, a new spreading approach employing frequency-domain orthogonal signature sequences was proposed in [1]. This approach properly maintains frequency-domain orthogonality between transmitted signals from different users such that the time-domain shift which is equivalent to a scalar multiplication over frequency domain does not destroy the orthogonality in the asynchronous channels. Therefore, we do not have to maintain synchronous channels to achieve the near-optimum performance with complexity similar to conventional detection.

In this frequency-domain approach, the information bits of user $k$ with length $M$ is carried by the signal

$$s_k(t) = \sum_{n=0}^{N-1} p_k(n) \sum_{m=0}^{M-1} b_k(m)\Pi(t - mT_c - nT_M), \quad (1)$$

where $\{b_k(m)|m = 0, 1, \cdots, M-1\}$ is the information sequence, $\{p_k(n)|n = 0, 1, \cdots, N-1\}$ is the designed frequency-domain orthogonal signature sequence for user $k$, $\Pi(t)$ is the indicator function with support $[0, T_c)$, $T_c$ is the chip time, and $T_M \equiv MT_c$. This transmitted signal $s_k(t)$ is the consequence of chip interleaving over the conventional spread signal, or equivalently could be regarded as the roles exchange between the signature sequence and the information sequence. That is, the signature sequence is spread by the information sequence. Therefore, the complexity to implement this new spreading approach is similar to the conventional direct sequence spreading as shown in Figure 1 [1].

The received low-pass equivalent signal with $K$[1] users over additive white Gaussian noise (AWGN) channel is therefore

$$r_{LP}(t) = \sum_{k=1}^{K} \rho_k e^{j\theta_k} s_k(t - \tau_k) + z_{LP}(t), \quad (2)$$

where $z_{LP}(t)$ is the complex-valued Gaussian noise, $j \equiv \sqrt{-1}$, and $\rho_k$, $\theta_k$, and $\tau_k$ are the received amplitude, phase, and timing of user $k$ respectively[2]. By sampling $r_{LP}(t)$ at the

[1] In this paper, we suppose that the number of users is identical to the length of spreading sequence. That is, $K = N$, as we did in [1]. However, all the results are equally sustained as long as $K \leq N$.

[2] These parameters could be estimated easily with our proposed multiuser synchronizers in [5].

rate $1/T_s \equiv 2/T_c$, we have the sequence

$$r(n,m,q) \equiv r_{LP}(nT_M + mT_c + qT_s)$$
$$= \sum_{k=1}^{K} \rho_k e^{j\theta_k} p_k(n - \alpha_{k,m,q}) b_k((m - \beta_{k,q})_M) + z(n,m,q), \quad (3)$$

where $\alpha_{k,m,q} = \alpha$ if $-\alpha T_M \leq mT_c + qT_s - \tau_k < -(\alpha-1)T_M$; $\beta_{k,q} = \beta$ if $-\beta T_c \leq (qT_s - \tau_k) mod\ T_M < -(\beta-1)T_c$; $\alpha$ and $\beta$ are integers; $z(n,m,q) \equiv z_{LP}(nT_M + mT_c + qT_s)$; and $(\cdot)_M \equiv \cdot(mod\ M)$. Since the noiseless component in (3) which relates to $n$ is the signature sequence $p_k(n)$, taking $N$-point discrete Fourier transform (DFT) on (3) over $n$ gives

$$R(l,m,q) = \sum_{k=1}^{K} \rho_k e^{j\theta_k} P_k(l) e^{-j\frac{2\pi}{N} l \alpha_{k,m,q}} b_k((m - \beta_{k,q})_M) + Z(l,m,q), \quad (4)$$

where $P_k(l)$ and $Z(l,m,q)$ are the $N$-point DFT of $p_k(n)$ and $z(n,m,q)$ respectively. Therefore, when the employed signature sequences are

$$P_k(l) = \begin{cases} N, \text{if } k = l \\ 0, \text{otherwise} \end{cases}, \quad (5)$$

the DFT of the received signal is simplified to be

$$R(k,m,q) = N\rho_k e^{j(\theta_k - \frac{2\pi}{N}\alpha_{k,m,q})} b_k((m - \beta_{k,q})_M) + Z(k,m,q), \quad (6)$$

which successfully separates the received signal into $K$ received sequences $\{R(k,m,q) | m = 0,1,\cdots,M-1; q = 1,2\}$. Since the other transmitted signal than the one from user $k$ does not interfere with $R(k,m,q)$, conventional detection could be applied with near-optimum performance [1].

In this paper, we extend this frequency domain approach to practical scenarios such as the frequency-selective slowly fading channels. This frequency-domain approach, not only maintains its MAI-free property in the asynchronous channels, but also inherits the frequency-diversity from wideband transmission to combat fading channels. The rest of this paper is organized as follows: Section 2 extends the frequency-domain approach to frequency-selective slowly fading channels, where the multi-path components introduce inter-symbol interference (ISI) and thus the maximum likelihood sequence estimator is proposed. To further propose the feasible multiuser detectors, linear-complexity equalizers employing our proposed spreading approach are further introduced. In addition, we mathematical demonstrate the effectiveness of our proposed detectors. In Section 3, we conduct simulations to demonstrate the performance of our frequency-domain approach over practical fading channels. And we make the conclusions in Section 4.

## II. Frequency-Domain Approach to Multiuser Detection over Frequency-Selective Slowly Fading Channels

### A. Signal Model and Assumptions

Assume that the frequency-domain orthogonal signature sequences are employed and the information carried by the signature sequence is spread as in [1]. The transmitted signal from user $k$ is $s_k(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} p_k(n) b_k(m) \Pi(t - mT_c - nT_M)$, where $T \equiv NT_c$ is the symbol duration and $MT$ is the duration of a frame with $M$ symbols.

In addition, let $L$ denote the number of taps in the tapped delay line model of the frequency-selective fading channel [6]. The $K$ users' received low-pass equivalent signal in the frequency-selective slowly fading channel could be written as

$$r_{LP}(t) = \sum_{l=0}^{L-1} \sum_{k=1}^{K} c_{k,l} \rho_k e^{j\theta_k} s_k(t - \tau_k - \frac{l}{W}) + z_{LP}(t), \quad (7)$$

where $c_{k,l}$ is the tap weight coefficient in the $l$-th tap from user $k$; $W$ is the null-to-null bandwidth of the transmitted signal; and the other parameters are defined in the previous section. We further assume that these tap weight coefficients $\{c_{k,l} | k = 1,\cdots,K; l = 0,1,\cdots,L-1\}$ are statistically independent, continuous, and complex-valued random variables (e.g. the Gaussian random variables when considering the Rayleigh fading channels) and are time-invariant within the transmission, as we consider the slowly fading channels. In addition, we assume that the number of taps in the tapped delay line model for frequency-selective channels is smaller than twice of the number of symbols within a frame. That is, $L \leq 2M$. It is a reasonable assumption considering practical applications over fading channels with moderate delay spread, although this assumption can be released without difficulties.

Without loss of generality, we consider the binary phase shift keying (BPSK) modulation, as it is easy to extend the results to other linear modulation schemes such as M-ary phase shift keying (MPSK) and quadrature amplitude modulation (QAM). To ease the interpretation, we set the minimum received timing to be 0 and assume that $\tau_k + (L-1)\frac{T_c}{2} \in [0, dT_M)$ for $k = 1,2,\cdots,K$ and some positive integer $d$.

When considering continuous transmission with more than one frames, the guard interval (GI), which is the cyclic extension of frames with length larger than $dT_M$ should be inserted between transmitted frames in order to avoid the inter-frame interference due to frequency-selective channels. It is the similar rationale behind the orthogonal frequency-division multiplex (OFDM) systems with guard intervals to avoid intersymbol interference. However, unlike the case in OFDM systems such as IEEE 802.11a to insert the GI every symbol, we only need to insert a GI every frame which implies smaller overhead and therefore better efficiency than OFDM systems.

### B. Demodulation over Multi-path Fading Channels

By sampling the received low-pass equivalent signal with rate $1/T_s \equiv 2/T_c = W$ in the frequency-selective slowly fading channel according to the Nyquist criterion, we have

$$\begin{aligned} r(n,m,q) &\equiv r_{LP}(nT_M + mT_c + qT_s) \\ &= \sum_{l=0}^{L-1}\sum_{k=1}^{K} c_{k,l}\rho_k e^{j\theta_k} p_k(n-\alpha_{k,l,m,q}) b_k((m-\beta_{k,l,q})_M) \\ &+ z(n,m,q), \end{aligned} \tag{8}$$

where

$$\alpha_{k,l,m,q} = \alpha \in \mathcal{Z} \tag{9}$$

$$\text{if } -\alpha T_M \le mT_c + qT_s - \tau_k - lT_s < -(\alpha-1)T_M;$$

$$\beta_{k,l,q} = \beta \in \mathcal{Z} \tag{10}$$

$$\text{if } -\beta T_c \le (qT_s - lT_s - \tau_k) \bmod T_M < -(\beta-1)T_c.$$

In order to utilize the designed frequency-domain orthogonal signature sequence to eliminate the MAI, we take $N$-point DFT on (8) over $n$ and have,

$$\begin{aligned} R(i,m,q) &\equiv \sum_{n=0}^{N-1} r(n,m,q)e^{j\frac{2\pi}{N}ni} \\ &= \sum_{l=0}^{L-1}\sum_{k=1}^{K} c_{k,l}\rho_k e^{j\theta_k} P_k(i) e^{-j\frac{2\pi}{N}i\alpha_{k,l,m,q}} b_k((m-\beta_{k,l,q})_M) \\ &+ Z(i,m,q), \end{aligned} \tag{11}$$

where $Z(i,m,q) \equiv \sum_{n=0}^{N-1} z(n,m,q)e^{j\frac{2\pi}{N}ni}$ and $P_k(i)$ is the $N$-point DFT of $p_k(n)$.

Since we employ the frequency-domain orthogonal signature sequence $p_k(n) = e^{j\frac{2\pi}{N}kn}$ whose DFT is as in (5), the DFT of the received low-pass equivalent signal is simplified to be

$$\begin{aligned} &R(k,m,q) \\ &= \sum_{l=0}^{L-1} c_{k,l}\rho_k e^{j(\theta_k - \frac{2\pi}{N}k\alpha_{k,l,m,q})} N b_k((m-\beta_{k,l,q})_M) \\ &+ Z(k,m,q). \end{aligned} \tag{12}$$

In (12), the received signal from user $k$ is not interfered by other users. However, it is interfered by its inter-symbol interference (ISI) due to multi-path effects. The maximum likelihood sequence estimation is

$$\begin{aligned} &(\hat{b}_k(0), \hat{b}_k(1), \cdots, \hat{b}_k(M-1)) \\ &= \arg \min_{\substack{\bar{b}_k(m)\in\{\pm 1\};\\ m=0,1,\cdots,M-1}} \sum_{m=0}^{M-1}\sum_{q=0}^{1} |R(k,m,q) \\ &- \sum_{l=0}^{L-1} c_{k,l}\rho_k e^{j(\theta_k-\frac{2\pi}{N}k\alpha_{k,l,m,q})} N\bar{b}_k((m-\beta_{k,l,q})_M)|^2, \end{aligned} \tag{13}$$

whose complexity grows exponentially to the length of information bits $M$ as it requires $2^M$ branches of match filters. This maximum likelihood sequence estimator is the optimum detector in the sense of minimizing the probability of detection error, as long as the prior probability of the information sequence $\{b_k(m)|m = 0,1,\cdots,M-1\}$ is uniformly distributed. This optimum detector not only combats the fading impairment by utilizing the frequency diversity which comes from the frequency-selective channel, but also mitigates the inter-symbol interference derived from the multi-path effect.

To propose the detector with feasible complexity, we consider the linear-complexity solution to resolve the ISI. In light of the success of our previous work on linear-complexity multiuser synchronization and detection [7], [8], we shall linearly combine the sufficient statistics to mitigate the ISI. To make a clear presentation, let $n_k$, $m_k$, and $q_k$ denote three non-negative integers with $n_k < N$, $m_k < M$, and $q_k < 2$ such that $\tau_k \in (n_kT_M + m_kT_c + q_kT_s, n_kT_M + m_kT_c + (q_k+1)T_s]$. The sufficient statistics can be written as

$$\begin{aligned} y_k((m-m_k)_M) &\equiv \sum_{q=q_k+1}^{q_k+2} R(k,m,q)e^{-j\theta_k} \\ &= \sum_{l=0}^{M-1} \hat{c}_{k,l}\rho_k N b_k((m-m_k-l)_M) + \sum_{q=q_k+1}^{q_k+2} Z(k,m,q)e^{-j\theta_k} \\ &= \rho_k N \underline{c}_{k,m-m_k}\underline{b}_k + \sum_{q=q_k+1}^{q_k+2} Z(k,m,q)e^{-j\theta_k}, \end{aligned} \tag{14}$$

for $m = 0,1,\cdots,M-1$, where $\hat{c}_{k,l} = 0$ for $l \ge M$; $\hat{c}_{k,l} \equiv c_{k,2(l-1)_M+1}e^{-j\frac{2\pi}{N}k\alpha_{k,2(l-1)_M+1,m,q_k+1}} + c_{k,2l}\sum_{q=q_k+1}^{q_k+2} e^{-j\frac{2\pi}{N}k\alpha_{k,2l,m,q}} + c_{k,2l+1}e^{-j\frac{2\pi}{N}k\alpha_{k,2l+1,m,q_k+2}}$; $\underline{c}_{k,m}$ is a $1 \times M$ row vector whose $l$-th entry is $\hat{c}_{k,(m-l)_M}$; and $\underline{b}_k$ is a $M \times 1$ column vector with the $l$-th entry being $b_k(l)$.

Therefore, by letting $\underline{y}_k = [y_k(0), y_k(1), \cdots, y_k(M-1)]^t$, where the superscript $^t$ denotes the transpose operation, we have

$$\underline{y}_k = N\rho_k \mathbf{C}_k \begin{bmatrix} b_k(0) \\ b_k(1) \\ \vdots \\ b_k(M-1) \end{bmatrix} + \begin{bmatrix} \sum_{q=q_k+1}^{q_k+2} Z(k,(m_k+0)_M,q)e^{-j\theta_k} \\ \sum_{q=q_k+1}^{q_k+2} Z(k,(m_k+1)_M,q)e^{-j\theta_k} \\ \vdots \\ \sum_{q=q_k+1}^{q_k+2} Z(k,(m_k+M-1)_M,q)e^{-j\theta_k} \end{bmatrix}, \tag{15}$$

where $\mathbf{C}_k$ is a $M \times M$ square matrix whose $l$-th row vector is $\underline{c}_{k,m}$. In fact, $\mathbf{C}_k$ is an invertible matrix with probability 1, since the tap weight coefficients are independent continuous random variables. Therefore, by linearly combining the sufficient statistics $y_k(m)$ for $m = 0,1,\cdots,M-1$, the inter-symbol interference can be removed. That is,

$$\hat{b}_k(m) = arg\ Re\{\sum_{n=0}^{M-1} d_{k,m}(n)y_k(n)\}, \qquad (16)$$

where $\sum_{n=0}^{M-1} d_{k,m}(n)y_k(n)$ is an estimate of $b_k(m)$ and $\{d_{k,m}(n)|n = 0, 1, \cdots, M-1\}$ are the designed linear combination coefficients.

These linear combination coefficients can be designed according to the "zero-forcing" criterion or the minimum mean square error criterion as in the linear equalization. That is, we can design $d_{k,m}(n)$ to be the entry of $(N\rho_k \mathbf{C}_k)^{-1}$ in its $m$-th row and $n$-th column (zero-forcing criterion) or design $d_{k,m}(n)$ to minimize the mean square error between $b_k(m)$ and $\sum_{n=0}^{M-1} d_{k,m}(n)y_k(n)$ (minimum mean square error criterion).

The block diagram of the detection scheme in the frequency-selective slowly fading channel is plotted as in Figure 2 which linearly combines the $M$ sufficient statistics $\{y_k(m)|m = 0, 1, \cdots, M-1\}$ for each information bits with different combination coefficients.

## III. Numerical Analysis

We adopt the similar frequency-selective slowly fading channel model considered in IEEE wireless local area networks (WLAN) [9]. This popular channel model simulates the frequency-selective slowly fading channel as a tapped delay line with spacing $T_s$. In addition, the tap coefficients are independent complex-valued Gaussian random variables with zero mean and exponentially decayed variance.

Without loss of generality, we consider the BPSK modulated signal with 5 active users. That is, $K = 5$. In addition, the number of symbols within a frame is $M = 7$ and the length of the signature sequence is $N = 8$.

In the first experiment, the optimal performance derived from maximum likelihood sequence estimation is shown in Figure 3. It can be seen that the optimal detection not only mitigates the ISI derived from frequency-selective fading channels but also utilizes the frequency-diversity inherited from frequency-selective channels to combat channel fading impairment.

On the other hand, Figure 4 justifies the effectiveness of the linear decorrelating equalization based on "zero-forcing" criterion. The ISI introduced by frequency-selective channels is completely eliminated with feasible complexity which grows linearly to $M$. That is, only $M$ branches of correlation is required to efficiently estimate the received signals.

## IV. Conclusions

By utilizing the frequency-domain orthogonality, the multipath effect which comes from the frequency-selective fading channel does not destroy the orthogonality between different users. Therefore, the MAI does not interfere with the desired signal even in the practical multi-path fading channels and the conventional low-complexity and near-optimum demodulation is realizable.

In this paper, we first extended the frequency-domain approach to the frequency-selective slowly fading channels. Since the transmitted signal from different users are orthogonal over frequency domain, the multiuser detection is simplified into a bank of single user detection problems as in the point-to-point communications. In order to mitigate the ISI caused by the multi-path effect, both the maximum likelihood sequence estimator (LMSE) which is optimum in the sense of minimizing probability of error and the linear-complexity equalizers were further introduced.

As demonstrated from the conducted simulations, both the LMSE detector and the linear-complexity equalizers successfully mitigate the ISI introduced from the frequency-selective channels. However, unlike the LMSE to efficiently utilize the frequency diversity inherited from frequency-selective channels to combat the channel fading impairment, the linear equalizer with "zero-forcing" criterion does not utilize the frequency diversity. The development on the detection with feasible complexity and efficiently utilizing the frequency diversity is desired in the future work.

On the other hand, by letting $M = 1$, the received noiseless signal in the AWGN channel is

$$r_{noiseless}(t) = \sum_{k=1}^{K} \rho_k e^{j\theta_k} b_k \sum_{n=0}^{N-1} p_k(n)\Pi(t - nT_c - \tau_k). \qquad (17)$$

By sampling $r_{noiseless}(t)$ at rate $1/T_s = 2/T_c$ and letting $\tau_k = 0$ and $\rho_k e^{j\theta_k} = 1$ for all $k$, we have

$$\begin{aligned} r_{noiseless}(n, q) &\equiv r_{noiseless}(nT_c + qT_s) \\ &= \sum_{k=1}^{K} b_k e^{j\frac{2\pi}{N}kn}, \end{aligned} \qquad (18)$$

which is similar to a OFDM symbol with $K$ sub-carriers. That is, our proposed frequency-domain approach degenerates to the OFDM system when the $K$ users' information bits are transmitted from a transmitter (e.g. the base station) and $M = 1$. In this degenerated case with $M = 1$, the equalization process to mitigate ISI is simpler. However, we need to insert guard interval every symbol in order to mitigate the ISI which implies less efficiency. In addition, this degenerated case shows less resistance to narrow-band interference comparing to the general frequency-domain approach which inherits the interference rejection properties from direct sequence spread spectrum systems. It is mainly because that the frequency-domain approach is a direct sequence spread spectrum system where the sequence $p_k(n)$ is spread by the random information sequence $b_k(m)$.

With these desired properties held by the frequency-domain approach, including its saving to maintain synchronous channels, its low-complexity as in the conventional point-to-point communications with near-optimum performance, its MAI-free property even in the multi-path fading channels, its efficiency with less overhead on GIs, and its resistance to narrow-band interference, it is very much attractive to the practical communications systems.

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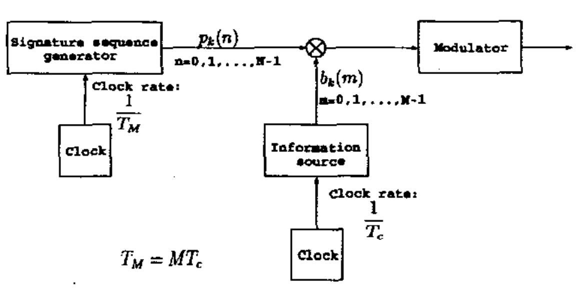


Fig. 1. Block diagram of the newly proposed spreading.

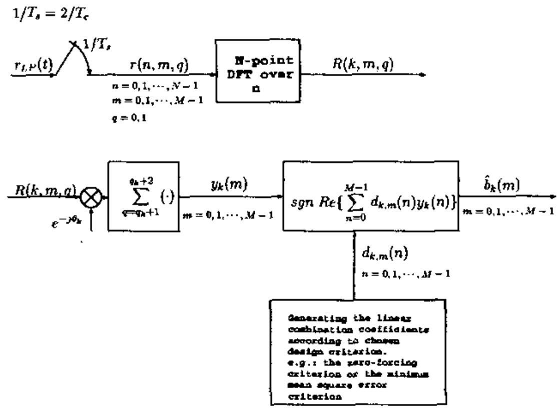


Fig. 2. Block diagram of frequency-domain approach to multiuser detection over frequency-selective slowly fading channels.

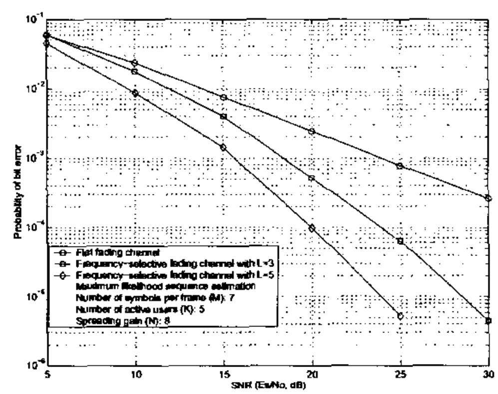


Fig. 3. Performance of optimal decoding based on maximum likelihood sequence estimation.

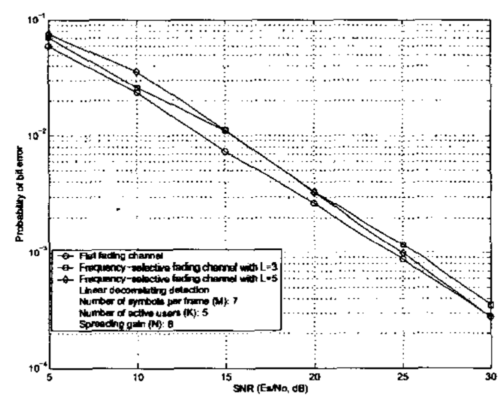


Fig. 4. Performance of suboptimal "zero-forcing" decoding with linear complexity.