Multiuser Detection for Multi-Rate CDMA in Multi-Path Fading Channels

Po-Wei Fu
Graduate Institute of Communication Engineering
National Taiwan University
Taipei, Taiwan, R.O.C.
powei@santos.ee.ntu.edu.tw

Kwang-Cheng Chen
Graduate Institute of Communication Engineering
National Taiwan University
Taipei, Taiwan, R.O.C.
chenkc@cc.ee.ntu.edu.tw

Abstract

Multi-code (MC) access and Variable-Spreading-Length (VSL) access are two widely applied realizations for multi-data-rate services in direct-sequence code-division multiple-access (DS/CDMA) systems. We study the behavior of these two multi-rate realizations operating in multi-path fading channels. Bounds of minimum probability of error based on maximum-likelihood sequence detection are developed for performance evaluations. Asymptotic multiuser efficiency is also analyzed as another performance index.

1. Introduction

Wireless communication technology proceeds a brand new evolution. In the future, services will not only focus on the traditional voice transmission and multi-media aspects will definitely play important roles as the main stream. One significant difference comes from various data rates in all kinds of media, including voice, image, and video etc. [1]. As code-division multiple-access (CDMA) technique being utilized for the third generation and future communication systems, fully realizing the capability of multi-data-rate communication in CDMA systems is thus an important subject. Among the proposed realizations for multi-rate CDMA system, Multi-Code (MC) and Variable-Spreading-Length (VSL) methods are the two most fundamental and widely applied schemes [1]. Different access schemes will affect the design of receiver and different communication environments will cause different results. In wireless CDMA systems, multi-path fading and multiple-access interference (MAI), which is induced by co-channel users, limit the performance. Some previous researches, such as [2], have discussed multi-rate CDMA systems in additive white Gaussian (AWGN) channels and VSL scheme is shown to outperform MC scheme by analyzing the performance of jointly optimal detection. Although MC scheme is more directly implemented in concept, the better correlation structure of the signature sequences in VSL scheme dominates the advantage. However, the behaviors of these two schemes in more practical environments are not yet studied and it is important to the design of efficient multiuser detectors in real applications. Multi-path fading effect is inevitable in wireless communications and it results in the phenomenon that the correlation of signature waveforms will exist even between the signals from different paths. In the following sections, the behavior of multi-rate CDMA systems in multi-path fading channels is studied by using the model of two-ray fading channels, and the result can be extended to the environments with more paths without loss of generality. We investigate the property of multiuser detection of these two access schemes, support the minimum probability of error and asymptotic multiuser efficiency (AME) by maximum likelihood design strategy as evaluations of performance, and finally analyze the results under different channel environments.

The remainder of this paper includes: The studied MC and VSL multi-rate CDMA system models are described in section 2. In section 3, we analyze the minimum probability of error and provide the bounds and asymptotic multiuser efficiency. Section 4 analyzes the two access methods from some observations and properties in multi-path fading channels. Section 5 provides the numerical examples. Conclusions are given in section 6.

2. Multi-Rate CDMA System Model

Multi-Code Access

In MC systems, it assigns more spreading codes to users in proportion to the ratio of the data rate over the basic data rate supporting in the system to transmit the data with higher rate. Transmit and receive them in parallel. The lengths of spreading codes are the same and thus the chip-rate is kept fixed. An effective user in MC systems is the one who uses an individual spreading sequence [3].

Variable-Spreading-Length Access

0-7803-6507-0/00/$10.00 ©2000 IEEE

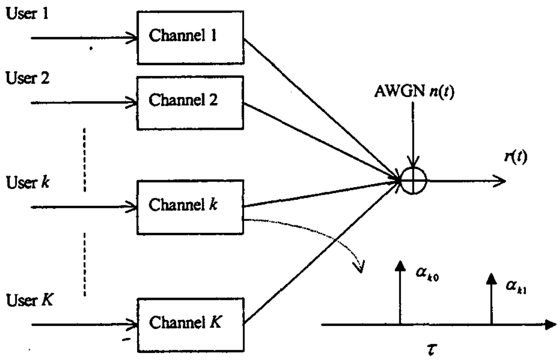

Fig. 1. The two-ray fading channel model

In VSL systems, transmitting the data of the user who has higher rate is realized by assigning shorter spreading codes, which also keeps the bandwidth fixed. The spreading factors in VSL systems are various from user to user by their data rates.

For simplicity, we consider an asynchronous dual-rate BPSK CDMA system in our analysis, where the high data rate is an integer multiple M of the low data rate. The users in the system are named as the high-rate users or the low-rate users corresponding to their transmission data rate. Assume there are K_1 low-rate users and K_2 high-rate users transmitting in our system and they are received asynchronously after passing multi-path fading channels. To focus on analyzing the multi-path fading effect on multi-rate transmission, a two-ray uncorrelated channel model is adopted without loss of generality. Each user is regarded as passing an individual two-ray fading channel (Fig. 1) and embedded in additive white Gaussian noise. The stochastic fading parameters α_{k0} and α_{k1} are not restricted to any specific distribution, but we consider slow Rayleigh fading environments in our analysis. Therefore, the received signal in the MC system can be represented as the following low-pass equivalent form:

$$r(t)=\sum_{k=1}^{K}\sum_{i}A_k b_k(i)S_k(t-iT_L-\xi_k)*h_k(t)+n(t) \qquad (1)$$

where A_k is the symbol amplitude, $b_k(i)$ is the ith symbol bit, and $S_k(t)$ is the normalized signature waveform of the kth effective user respectively. $K=K_1+MK_2$ and T_L is the symbol period of low-rate users. $h_k(t)$ represents the impulse response of the passed channel and ξ_k denotes the received timing offset in asynchronous cases. The AWGN is denoted as $n(t)$ with 2-sided power spectral density $No/2$. However, in VSL system:

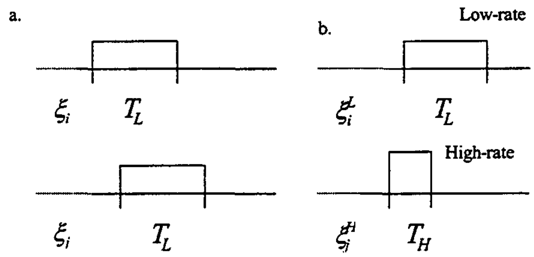

Fig. 2 Illustrations of signature waveforms of the generalized users
a. MC systems b. VSL systems

$$r(t)=\sum_{k=1}^{K_1}\sum_{i}A_k^L b_k^L(i)S_k^L(t-iT_L-\xi_k^L)*h_k^L(t)$$
$$+\sum_{k=1}^{K_2}\sum_{i}A_k^H b_k^H(i)S_k^H(t-iT_H-\xi_k^H)*h_k^H(t)+n(t) \qquad (2)$$

where the notations are used as well as those in MC case and the only difference is that the notations with upper description L denote low-rate users and H denote high-rate users.

To simplify the complexity of formulation and make the equation more tractable, we can alternatively view each transmitted symbol in the desired frame as a generalized user, no matter in MC systems or in VSL systems. That is, the number of the generalized users is equal to the total number of symbols transmitted in the desired frame. The nonzero support of the signature waveform of each generalized user is located corresponding to when the symbol is transmitted. The only difference between the MC system and the VSL system is the nonzero support duration of the signature waveforms, shown as Fig. 2. In VSL systems, the nonzero support duration of a high-rate generalized user is shorter than that of a low-rate generalized user; instead, the nonzero support durations are all the same in MC systems. Thus, we can rewrite (1) and (2) as the following for both systems:

$$r(t)=\sum_{k=1}^{K}A_k b_k S_k(t)*h_k(t)+n(t)$$
$$=\sum_{k=1}^{K}A_k b_k \alpha_{k0}S_k(t)+\sum_{k=1}^{K}A_k b_k \alpha_{k1}S_k(t-\tau)+n(t) \qquad (3)$$

which is a more compact form for analysis and the subscription k denotes the corresponding terms of the kth generalized user. K is the number of total generalized users. In the following section, we will derive the minimum probability of error for both multi-rate systems by maximum-likelihood sequence detection (MLSD).

3. Minimum Error Probability on MLSD

The available minimum error probability is an important property of interest while investigating the performance of both systems. Fortunately, from (3) we can analyze a multi-rate system conceptually like analyzing a single-rate system in multi-path fading channels. The minimum error probability of detection on a generalized user, however, is difficult to find a closed-form expression. We instead find the computable upper and lower bounds.

3.1 Maximum-Likelihood Sequence Detection

Let $\mathbf{B} = [b_1 b_2 \cdots b_K]^T$ denotes the vector consisting of the transmitted symbols of all generalized users. Assume the channel information and timing offsets have known in advance, the MLSD on $\mathbf{B}$ under certain channel realization is:

$$\hat{\mathbf{B}} = \arg\min_{\mathbf{B}} \int \left| r(t) - \sum_{k=1}^{K} A_k b_k \alpha_{k0} S_k(t) - \sum_{k=1}^{K} A_k b_k \alpha_{k1} S_k(t-\tau) \right|^2 dt$$
$$= \arg\min_{\mathbf{B}} \left\{ \mathbf{B}^T \boldsymbol{\alpha}_0 \mathbf{ARA} \boldsymbol{\alpha}_0^H \mathbf{B} + \mathbf{B}^T \boldsymbol{\alpha}_1 \mathbf{ARA} \boldsymbol{\alpha}_1^H \mathbf{B} - 2\,\mathrm{Re}\left[\mathbf{B}^T \boldsymbol{\alpha}_0^H \mathbf{AY} + \mathbf{B}^T \boldsymbol{\alpha}_1^H \mathbf{AY}^\tau - \mathbf{B}^T \boldsymbol{\alpha}_0 \mathbf{AR}^\tau \mathbf{A} \boldsymbol{\alpha}_1^H \mathbf{B} \right] \right\} \quad (4).$$

In the above equation, H denotes Hermition transpose operation, and the notations used are defined as follows:

$\mathbf{A} \equiv diag(A_1 \cdots A_K)$, $\boldsymbol{\alpha}_0 \equiv diag(\alpha_{10}\alpha_{20} \cdots \alpha_{k0})$,

$\boldsymbol{\alpha}_1 \equiv diag(\alpha_{11}\alpha_{21} \cdots \alpha_{k1})$, $\mathbf{Y} = [y_1 y_2 \cdots y_K]^T$,

$$\mathbf{R} = \begin{bmatrix} \rho_{11} & \cdots & \rho_{1K} \\ \vdots & \ddots & \vdots \\ \rho_{K1} & \cdots & \rho_{KK} \end{bmatrix}, \quad \mathbf{R}^\tau = \begin{bmatrix} \rho_{11}^\tau & \cdots & \rho_{1K}^\tau \\ \vdots & \ddots & \vdots \\ \rho_{K1}^\tau & \cdots & \rho_{KK}^\tau \end{bmatrix},$$

$\mathbf{Y}^\tau = [y_1^\tau y_2^\tau \cdots y_K^\tau]^T$, $y_k \equiv \int r(t) S_k(t) dt$,

$y_k^\tau \equiv \int r(t) S_k(t-\tau) dt$, $\rho_{ij} \equiv \int S_i(t) S_j(t) dt$,

$\rho_{ij}^\tau \equiv \int S_i(t) S_j(t-\tau) dt$.

The decision metric function is:

$$\Omega(\mathbf{B}|\boldsymbol{\alpha}) \equiv 2\,\mathrm{Re}\left[\mathbf{B}^T \boldsymbol{\alpha}_0^H \mathbf{AY} + \mathbf{B}^T \boldsymbol{\alpha}_1^H \mathbf{AY} - \mathbf{B}^T \boldsymbol{\alpha}_0 \mathbf{AR}^\tau \mathbf{A} \boldsymbol{\alpha}_1^H \mathbf{B} \right] - \mathbf{B}^T \boldsymbol{\alpha}_0 \mathbf{ARA} \boldsymbol{\alpha}_0^H \mathbf{B} - \mathbf{B}^T \boldsymbol{\alpha}_1 \mathbf{ARA} \boldsymbol{\alpha}_1^H \mathbf{B} \quad (5).$$

It implies that the decision statistics will be affected by not only the background noise but also the correlation matrixes $\mathbf{R}$ and $\mathbf{R}^\tau$. At (5), those terms involving matrix $\mathbf{R}$ represent the effect coming from each individual path and those with $\mathbf{R}^\tau$ exhibit the effect resulting from the interaction between different paths. Due to the different patterns of MC and VSL systems in $\mathbf{R}$ and $\mathbf{R}^\tau$, it results in different behaviors of the two multi-rate schemes in multi-path fading channels.

3.2 Minimum Probability of Error

Applying the concept of error vectors [4][5], the error vector $\boldsymbol{\varepsilon} = [\varepsilon_1 \varepsilon_2 \cdots \varepsilon_K]^T \equiv \frac{1}{2}(\mathbf{B} - \hat{\mathbf{B}})$ where $\varepsilon_i \in \{0, \pm 1\}$. The error probability of the generalized user k conditioned on $\boldsymbol{\alpha}$ is

$$P_{ek}|\boldsymbol{\alpha} = \Pr\{\hat{b}_k \neq b_k | \boldsymbol{\alpha}\} \le \sum_{\boldsymbol{\varepsilon} \in \mathbf{E}_k} \Pr\{\Omega(\mathbf{B} - 2\boldsymbol{\varepsilon}|\boldsymbol{\alpha})\ is\ \max, \boldsymbol{\varepsilon} \in \mathbf{A}_{ek}\} \le \sum_{\boldsymbol{\varepsilon} \in \mathbf{E}_k} 2^{-w(\boldsymbol{\varepsilon})} \Pr\{\Omega(\mathbf{B} - 2\boldsymbol{\varepsilon}|\boldsymbol{\alpha}) \ge \Omega(\mathbf{B}|\boldsymbol{\alpha})\} \quad (6)$$

where the error vector set $\mathbf{E}_k$ is defined as $\mathbf{E}_k = \{\boldsymbol{\varepsilon} \in \{0, \pm 1\}^K,\ \varepsilon_k \neq 0\}$ and $\mathbf{A}_{ek}$ is the admissible error vector set of user k [5]. The last inequality comes from the fact that if $\mathbf{B} - 2\boldsymbol{\varepsilon}$ is the most probable vector, $\mathbf{B} - 2\boldsymbol{\varepsilon}$ must be more probable than $\mathbf{B}$. It can be easily shown that $\Pr\{\Omega(\mathbf{B} - 2\boldsymbol{\varepsilon}|\boldsymbol{\alpha}) \ge \Omega(\mathbf{B}|\boldsymbol{\alpha})\}$ is independent with the transmitted symbol sequence in even multi-path channels, and more specifically

$$\Omega(\mathbf{B} - 2\boldsymbol{\varepsilon}|\boldsymbol{\alpha}) - \Omega(\mathbf{B}|\boldsymbol{\alpha}) = -4\boldsymbol{\varepsilon}^T \boldsymbol{\alpha}_0 \mathbf{ARA} \boldsymbol{\alpha}_0^H \boldsymbol{\varepsilon} - 4\boldsymbol{\varepsilon}^T \boldsymbol{\alpha}_1 \mathbf{ARA} \boldsymbol{\alpha}_1^H \boldsymbol{\varepsilon} - 4\,\mathrm{Re}\left\{ \boldsymbol{\varepsilon}^T \boldsymbol{\alpha}_0^H \mathbf{An} + \boldsymbol{\varepsilon}^T \boldsymbol{\alpha}_1^H \mathbf{An}^\tau + 2\boldsymbol{\varepsilon}^T \boldsymbol{\alpha}_0 \mathbf{AR}^\tau \mathbf{A} \boldsymbol{\alpha}_1^H \boldsymbol{\varepsilon} \right\} \quad (7)$$

which is Gaussian distributed with mean

$$= -4\boldsymbol{\varepsilon}^T \boldsymbol{\alpha}_0 \mathbf{ARA} \boldsymbol{\alpha}_0^H \boldsymbol{\varepsilon} - 4\boldsymbol{\varepsilon}^T \boldsymbol{\alpha}_1 \mathbf{ARA} \boldsymbol{\alpha}_1^H \boldsymbol{\varepsilon} - 8\,\mathrm{Re}\left\{ \boldsymbol{\varepsilon}^T \boldsymbol{\alpha}_0 \mathbf{AR}^\tau \mathbf{A} \boldsymbol{\alpha}_1^H \boldsymbol{\varepsilon} \right\}$$

and variance

$$= 4N_0 \boldsymbol{\varepsilon}^T \boldsymbol{\alpha}_0^H \mathbf{ARA} \boldsymbol{\alpha}_0 \boldsymbol{\varepsilon} + 4N_0 \boldsymbol{\varepsilon}^T \boldsymbol{\alpha}_1^H \mathbf{ARA} \boldsymbol{\alpha}_1 \boldsymbol{\varepsilon} + 8N_0\,\mathrm{Re}\left\{ \boldsymbol{\varepsilon}^T \boldsymbol{\alpha}_0^H \mathbf{AR}^\tau \mathbf{A} \boldsymbol{\alpha}_1 \boldsymbol{\varepsilon} \right\}$$

The error metric on each channel realization,

$$\|\mathbf{M}(\boldsymbol{\varepsilon}|\boldsymbol{\alpha})\|^2 \equiv \boldsymbol{\varepsilon}^T \boldsymbol{\alpha}_0 \mathbf{ARA} \boldsymbol{\alpha}_0^H \boldsymbol{\varepsilon} + \boldsymbol{\varepsilon}^T \boldsymbol{\alpha}_1 \mathbf{ARA} \boldsymbol{\alpha}_1^H \boldsymbol{\varepsilon} + 2\,\mathrm{Re}\left\{ \boldsymbol{\varepsilon}^T \boldsymbol{\alpha}_0 \mathbf{AR}^\tau \mathbf{A} \boldsymbol{\alpha}_1^H \boldsymbol{\varepsilon} \right\}$$

The probability

$$\Pr\{\Omega(\mathbf{B} - 2\boldsymbol{\varepsilon}|\boldsymbol{\alpha}) \ge \Omega(\mathbf{B}|\boldsymbol{\alpha})\} = Q\left(\frac{2\|\mathbf{M}(\boldsymbol{\varepsilon}|\boldsymbol{\alpha})\|}{\sqrt{N_0}} \right) \quad (9)$$

In fact, we can decompose the metric as a quadratic form by

$$\|\mathbf{M}(\boldsymbol{\varepsilon}|\boldsymbol{\alpha})\|^2 = [\boldsymbol{\varepsilon}^T \boldsymbol{\varepsilon}^T] \begin{bmatrix} \boldsymbol{\alpha}_0 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\alpha}_1 \end{bmatrix} \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{R}^\tau \\ \mathbf{R}^{\tau^H} & \mathbf{R} \end{bmatrix} \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha}_0^H & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\alpha}_1^H \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\varepsilon} \end{bmatrix} \quad (10)$$

which implies that we can always view it as a system of $2K$ asynchronous generalized users transmitting through single-path fading channels with only K unknown symbols. Therefore, we can apply the method of analysis in single-path fading channels [6] to derive the expectation value over various channel realizations. Decompose the nonnegative positive matrix $\begin{bmatrix} \mathbf{R} & \mathbf{R}^{\tau} \\ \mathbf{R}^{\tau^H} & \mathbf{R} \end{bmatrix} = \mathbf{G}^T\mathbf{G}$, and define $\mathbf{F}^T = \left[\boldsymbol{\varepsilon}^T \boldsymbol{\varepsilon}^T\right]\begin{bmatrix} \alpha_0 & \mathbf{0} \\ \mathbf{0} & \alpha_1 \end{bmatrix}\begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{bmatrix}\mathbf{G}^T$. It follows that $\|\mathbf{M}(\boldsymbol{\varepsilon}|\boldsymbol{\alpha})\| = \|\mathbf{F}\|$ and the upper bound of the kth generalized user in multi-path fading channels is:

$$P_{ek} \le E\left[\sum_{\boldsymbol{\varepsilon}\in E_k} 2^{-w(\boldsymbol{\varepsilon})} Q\left(\frac{2\|\mathbf{F}\|}{\sqrt{N_0}}\right)\right]$$
$$= \sum_{\boldsymbol{\varepsilon}\in F_k} 2^{-w(\boldsymbol{\varepsilon})} E\left[Q\left(\frac{2\|\mathbf{F}\|}{\sqrt{N_0}}\right)\right]$$
$$= \sum_{\boldsymbol{\varepsilon}\in F_k} 2^{-w(\boldsymbol{\varepsilon})} \sum_{i=1}^{2w(\boldsymbol{\varepsilon})} \frac{r_i}{2}\left(1 - \frac{1}{\sqrt{1+N_0/2\lambda_i}}\right) \quad (11)$$

where $w(\boldsymbol{\varepsilon})$ is the Hamming weight of $\boldsymbol{\varepsilon}$, λ_i is the ith distinct nonzero eignvalue of $E\left[\mathbf{F}\mathbf{F}^T\right]$ and $\gamma_i \equiv \prod_{i=1, i\neq j}^{2w(\varepsilon)} \frac{\lambda_i}{\lambda_i - \lambda_j}$ [7].

Note that to make the upper bound tighter without calculating redundant terms, the summation in the second equality is taken over the indecomposable error vector set $\mathbf{F}_k$ [5], which is first introduced in [5] for AWGN channel and modified in [6] for single-path fading channels. In multi-path fading channels, to make the summation and expectation operations exchangeable in (11), we modify it as the new definition without proof here for brief:

Definition:

For any error vector $\boldsymbol{\varepsilon} \in \mathbf{E}_k$, if it can be decomposed as $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_a + \boldsymbol{\varepsilon}_b$, where $\boldsymbol{\varepsilon}_a \in \mathbf{E}_k$, $\boldsymbol{\varepsilon}_b \in \mathbf{E}$, and satisfies $\mathrm{Re}\{\langle \mathbf{M}(\boldsymbol{\varepsilon}_a|\boldsymbol{\alpha}), \mathbf{M}(\boldsymbol{\varepsilon}_b|\boldsymbol{\alpha})\rangle\} \ge 0 \ \forall \boldsymbol{\alpha}$, then $\boldsymbol{\varepsilon}$ is a decomposable vector in $\mathbf{E}_k$; otherwise it is indecomposable.

In the above definition, the inner product $\langle \mathbf{M}(\boldsymbol{\varepsilon}_a|\boldsymbol{\alpha}), \mathbf{M}(\boldsymbol{\varepsilon}_b|\boldsymbol{\alpha})\rangle \equiv$

$$\left[\boldsymbol{\varepsilon}_a^T \boldsymbol{\varepsilon}_a^T\right]\begin{bmatrix} \alpha_0 & \mathbf{0} \\ \mathbf{0} & \alpha_1 \end{bmatrix}\begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{bmatrix}\begin{bmatrix} \mathbf{R} & \mathbf{R}^{\tau} \\ \mathbf{R}^{\tau^H} & \mathbf{R} \end{bmatrix}\begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{bmatrix}\begin{bmatrix} \alpha_0^H & \mathbf{0} \\ \mathbf{0} & \alpha_1^H \end{bmatrix}\begin{bmatrix} \boldsymbol{\varepsilon}_b \\ \boldsymbol{\varepsilon}_b \end{bmatrix}.$$

To derive the lower bound in minimum error probability, we assume that there is some side information given while detecting the symbols. Suppose $\boldsymbol{\varepsilon}^d$ is the error vector that makes $E\left[Q\left(\frac{2\mathbf{M}(\boldsymbol{\varepsilon}^d|\boldsymbol{\alpha})}{\sqrt{N_0}}\right)\right]$ largest of all, i.e. dominant error in the multi-path fading environment and the receiver only needs to detect if the dominant error has occurred. In such case, the error probability supports the lower bound as:

$$P_{ek} \ge E\left[\Pr\{\Omega(\mathbf{B} - \boldsymbol{\varepsilon}^d) - \Omega(\mathbf{B}) \ge 0\}\right]$$
$$= E\left[Q\left(\frac{2\mathbf{M}(\boldsymbol{\varepsilon}^d|\boldsymbol{\alpha})}{\sqrt{N_0}}\right)\right]$$
$$= \sum_{i=1}^{2w(\boldsymbol{\varepsilon})} \frac{r_i^d}{2}\left(1 - \frac{1}{\sqrt{1+N_0/2\lambda_i^d}}\right) \quad (12)$$

where the upper-script d denotes those parameters corresponding to the dominant error vector.

3.3 Asymptotic Multiuser Efficiency

AME measures the performance degradation of a receiver due to multiple access interference as additive channel noise diminishes. If only one user, high-rate or low-rate, is transmitting over the multi-path fading channel, the MLSD decision is made as:

$$\hat{\mathbf{B}} = \arg\min_{\mathbf{B}}\left\{A^2\left(|\alpha_0|^2 + |\alpha_1|^2\right)\mathbf{B}^T\mathbf{R}\mathbf{B} - 2A^2\,\mathrm{Re}\left\{\alpha_0^*\mathbf{B}^T\mathbf{Y} + \alpha_1^*\mathbf{B}^T\mathbf{Y}^{\tau} - \alpha_0\alpha_1^*\mathbf{B}^T\mathbf{R}^{\tau}\mathbf{B}\right\}\right\} \quad (13)$$

where the dimensions of $\mathbf{B}$, $\mathbf{Y}$, $\mathbf{R}$, $\mathbf{Y}^{\tau}$ and $\mathbf{R}^{\tau}$ correspond to the number of transmitted symbols in a detection frame and A is the symbol amplitude. Take the same analysis as above and approximate the error probability at high SNR [6] by the error vector with smallest hamming weight, the error probability of single user transmission in multi-path fading channel approaches

$$P_{es} = \frac{3N_0^2}{64A_k^4\lambda_1^s\lambda_2^s} + o\left(N_0^2\right) \quad (14)$$

where λ_1^s, λ_2^s are the eignvalues of $G\begin{bmatrix} E|\alpha_{k1}|^2 & 0 \\ 0 & E|\alpha_{k2}|^2 \end{bmatrix}G^T$.

On the other hand, the asymptotic error probability of detection in two-ray fading channel is

$$P_e = \left(\frac{N_0}{2}\right)^{2w(\varepsilon^d)} \frac{\left[4w(\varepsilon^d)-1\right]\left[4w(\varepsilon^d)-3\right]\cdots 1}{\left[2w(\varepsilon^d)\right]!\,2^{3w(\varepsilon^d)}} \left(\prod_{i=1}^{2w(\varepsilon)}\lambda_i\right) \quad (15)$$

Therefore, the AME is

$$\eta_k = \min_{\varepsilon\in E_k} \frac{\sqrt{3}}{8} 2^{5w(\varepsilon)/2} N_0^{1-w(\varepsilon)} \left(\frac{\left[2w(\varepsilon)\right]!}{\left[4w(\varepsilon)-1\right]\left[4w(\varepsilon)-3\right]\cdots 1}\right)^{1/2} \sqrt{\prod_{i=1}^{2w(\varepsilon)}\lambda_i} \Big/ \left(\sqrt{\lambda_1^s\lambda_2^s}A_k^2\right) \quad (16)$$

for both systems.

Compare the asymptotic P_e and P_{es}, because P_{es} is proportional to N_0^2, therefore, if the asymptotic P_e is only proportional to N_0, it implies the decaying rate of P_e is poor than P_{es} of single user transmission. It occurs when some eignvalues of $E[\mathbf{FF}^T]$ are zero. In single-path fading channels, [6] has proved that it occurs under high cross-correlation conditions. This result can also be proved true in multi-path fading environments. However, if the distribution of the asynchronous delays is uniformly, this situation can occur with probability zero [8]. In addition, the equations derived in this section could be easily extended to multiple-path cases by increasing the dimension of the number of generalized users in concept corresponding to the number of paths. Single-path fading channel is a special case.

4. Observations and Comparative Analysis of Access Methods

Even in multi-path channels, the shorter nonzero duration of the signature waveforms for high-rate users still supports advantage for VSL systems in the viewpoint of interference. From the analysis in the previous section, the error probability is determined by not only the inner-path correlation $\mathbf{R}$ but also the inter-path correlation $\mathbf{R}^{\tau}$. Specifically, the product of the eignvalues of $E[\mathbf{FF}^T]$ dominates the error probability, and the larger the product is the better it behaves. Comparing the structure of $\mathbf{R}$ and $\mathbf{R}^{\tau}$ in MC and VSL transmissions, there exist more nonzero terms in the correlation matrixes of MC transmission which results in smaller product of eignvalues for high-rate users. Therefore, the error probability of the high-rate users is smaller in VSL systems than in MC systems. As for the low-rate users, under the case that all the transmitted symbol energy are the same, the cross-correlation between a high-rate generalized user and a low-rate generalized user in VSL systems is generally smaller than the cross-correlation between two generalized users in MC systems. However, this difference is not obvious while asynchronous receiving and it results in the comparable error probabilities of low-rate users in both systems.

In multi-path channels, from the concept that each symbol can be viewed as a generalized user, the inter-symbol interference from the same original user can be also viewed as multiple access interference. Fewer nonzero terms implies less impact from interference, which is the same as in single-path channel [2]. Thus, the VSL system is still expected to outperform the MC system in detection error probability under multi-path fading environments due to the inherent better signal structure. Asynchronous receiving in a manner destructs the advantageous structure of VSL access scheme while compared with MC access scheme in the viewpoint of signal correlations.

On the other hand, investigating $\mathbf{R}^{\tau}$, if the path spread is not significant, the VSL system still has the advantage of fewer nonzero correlation terms than the MC system. If the path spread is longer than T_L, this difference is reduced. Extra paths just increase the number of equivalent generalized users and thus the VSL system is generally more robust to the multi-path effect than the MC system. Employing the information of all the paths in multi-path channels for detection in VSL systems is not as critical as in MC systems. Therefore, utilization of only one dominant path in the design of detector for complexity reduction is more appropriate in VSL systems, especially for detecting high-rate users.

5. Numerical Examples

To verify the analysis in the previous section, we give some numerical examples to illustrate the error probability and the AME of MC and VSL systems in multi-path fading channels and those in single-path fading channel are also illustrated as comparison. Suppose that there are two high-rate users and two low-rate users transmitting with equal energy per symbol in the system, and the delay offsets of all users are assumed uniformly distributed between zero and one symbol time. The impulse coefficients of the two-ray channel model are assumed uncorrelated and complex Gaussian distributed with zero mean and equal variance for all users. The path spread τ refers the two-ray fading propagation conditions in [9]. One of the two available data rate is twice as large as the other, and length-31 m-sequences plus one random bit are taken as the general spreading codes of the high-rate users in VSL systems. The spreading codes of all the users in MC systems and the low-rate users in VSL systems consist of a couple of length-31 m-sequences plus one random bit. All the subsequent curves are the result of an average over 100 different asynchronous delay realizations.

Fig. 3 shows the lower bounds on the BER upper bound of each system by considering the minimum-weight error events and the result of the same scenario in single-path fading channel is plotted in Fig. 4. From the results, the high-rate users in VSL systems get lower probability of error than the high-rate users in MC systems and the low-rate users in both systems are comparable in error probability. Among VSL systems, the error probability of high-rate users is smaller than that of the low rate users due to suffering less interference. The degradation of error probability is severer in MC systems than in VSL systems when the second path exists. In Fig. 5, the AME in multi-path fading channels is plotted versus the amplitude ratio of interferers over the desired user. The VSL system also performs better AME than the MC system. The AME in single-path fading channels is shown in Fig. 6. Compared with the result in AWGN environment [2], the severe AME degradation at low amplitude ratio region comes from asynchronous receiving in fading channels because some undesired timing offsets accompanying with fading

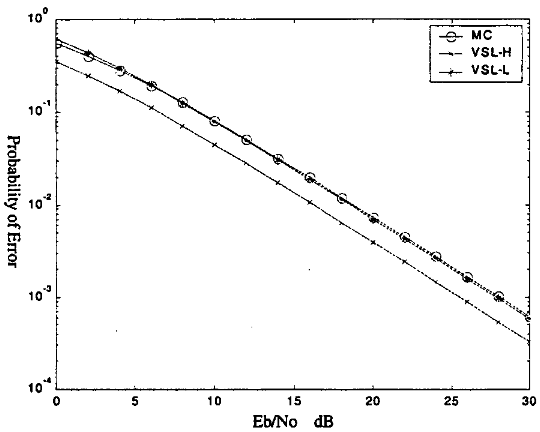

Fig.3 BER performance comparison of MC and VSL systems in multi-path fading channels. MC: user in MC systems, VSL-H: high-rate user in VSL systems VSL-L: low-rate user in VSL systems

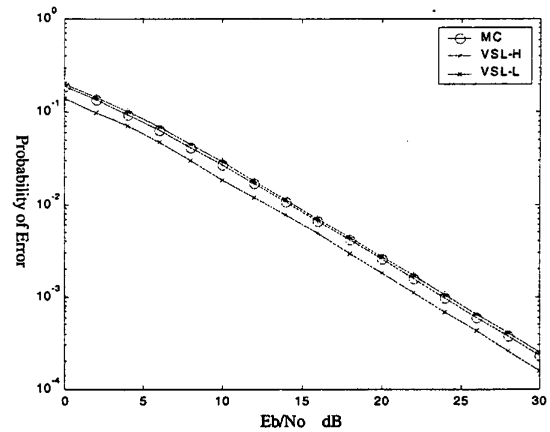

Fig.4 BER performance comparison of MC and VSL systems in single path fading channels. MC: user in MC systems VSL-H: high-rate user in VSL systems VSL-L: low-rate user in VSL systems

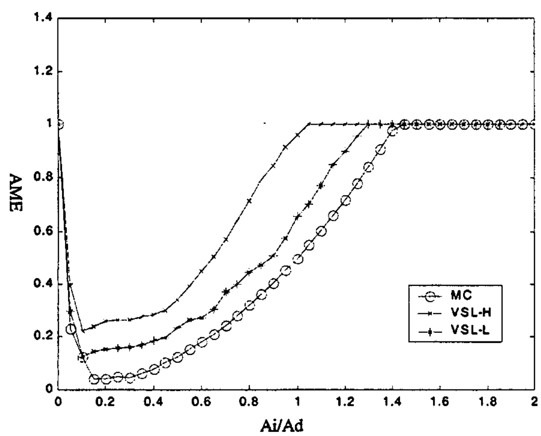

Fig.5 Asymptotic performance comparison of MC and VSL systems in multi-path fading channels. Ai/Ad: amplitude ratio of interferers over desired user

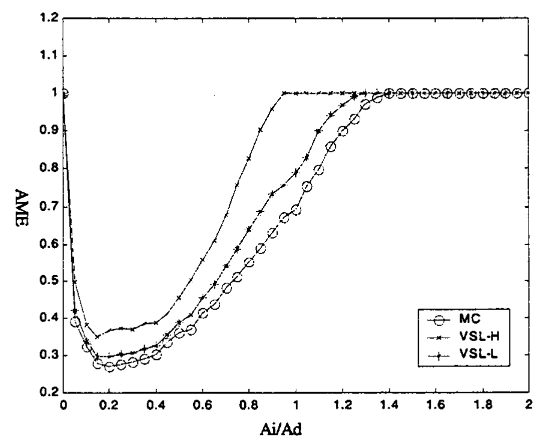

Fig.6 Asymptotic performance comparison of MC and VSL systems in single -path fading channel. Ai/Ad: amplitude ratio of interferers over desired user

conditions occurred.

6. Conclusion

Multi-rate multiuser detection in asynchronous multi-path fading channels can be always analyzed by viewing each symbol at each path as a generalized user transmitting in single-path channels without increasing the degree of freedom in hypothesis testing. The detection error probability is dominated by the patterns of both the correlation matrixes $\mathbf{R}$ and $\mathbf{R}^{\tau}$, which results in smaller error probability of VSL access scheme than MC access scheme for realizing multi-rate transmission and the robustness of VSL access scheme to multi-path effect is better than MC access scheme, especially for high-rate users. Although power-consuming and implementation complexity should be considered in practical applications, VSL access scheme is still suggested in asynchronous multi-rate systems if general code sets without special design are used.

Assigning orthogonal spreading codes for each effective user supports the MC systems an occasion to rival VSL systems in detection performance, though the orthogonal property is hard to maintain in real environments. Therefore, good design on spreading codes for low auto-correlation and cross-correlation is relatively recommended while choosing the MC access scheme. Detectors operated by utilizing only one dominant path are more appropriate in VSL systems for high-rate users because the remainder paths does not degrade the detection performance in VSL systems as much as in MC systems in multi-path environments.

References:

[1] T. Ottosson and A. Svensson, "Multi-rate schemes in DS/CDMA systems", *Proc. IEEE Vehic. Tech. Conf.*, pp. 1006-1010, 1995.

[2] U. Mitra, "Comparison of maximum-likelihood-based detection for two multi-rate access schemes for CDMA signals", *IEEE Trans. on Comm.*, Vol. 47, pp. 64-77, Jan. 1999.

[3] M. Juntti, "System concept comparisons for multirate CDMA with multiuser Detection", *Proc. IEEE Vehic. Tech. Conf.*, pp. 36-40, 1998.

[4] G. Forney, "Lower bounds on error probability in the presence of large inter-symbol interference", *IEEE Trans. On Comm.*, Vol. 20, pp. 76-77, Feb. 1972.

[5] S. Verdú, "Minimum probability of error for asynchronous Gaussian multiple-access channels", *IEEE Trans. on IT*, pp. 85-96, Jan. 1986.

[6] Z. Zvonar, D. Brady, "Multiuser detection in single-path fading channels", *IEEE Trans. On Comm.*, Vol. *42*, pp. 1729-1739, /Feb./March/Apr., 1994.

[7] J. Proakis, "*Digital communications*", third edition, McGraw-Hill, 1995.

[8] R. Lupas, S. Verdú, "Near-far resistance of multiuser detectors in asynchronous channels, " *IEEE Trans. On Comm.*, Vol.38, pp. 496-508, Apr., 1990.

[9] "TS 25.101 v3.1.0 (1999-12); UE Radio transmission and reception", *3GPP Technical Specification*, pp. 51, Dec. 1999.