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Linear-phase quadrature mirror filters with coefficients -1, 0 and $+1$[1]

Ju-Hong Lee*, Tz-Chou Jung

Department of Electrical Engineering, National Taiwan University, Rm 517, 2nd Building, Taipei 106, Taiwan

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Abstract

This paper considers the linear-phase quadrature mirror filters (LP-QMFs) with coefficients constrained to the values of -1, 0 and $+1$. A new filter structure and a design technique based on a recently developed weighted least-squares (WLS) algorithm are presented. First, we design an LP-QMF with continuous coefficients using the WLS algorithm. This design process provides two favourable design results that the prototype LP analysis filter has least-squares stopband response and the resulting QMF bank shows quasi-equiripple reconstruction error behavior. In conjunction with the new proposed filter structure, we then present an efficient method to obtain a design solution with coefficients restricted to -1, 0 and $+1$ in the minimax sense. Several design examples demonstrating the effectiveness of the proposed technique are provided. © 1997 Elsevier Science B.V.

Zusammenfassung

Dieser Beitrag befaßt sich mit linearphasigen Quadratur-Spiegelfiltern (LP-QMF), deren Koeffizienten auf die Werte -1, 0 und $+1$ beschränkt sind. Eine neue Filterstruktur und eine Entwurfstechnik auf der Grundlage eines kürzlich entwickelten Kleinste-Quadrate Algorithmus mit Fehlergewichtung (WLS) werden vorgestellt. Zuerst entwerfen wir ein LP-QMF mit kontinuierlichen Koeffizienten und verwenden dazu den WLS-Algorithmus. Dieser Entwurfsprozeß liefert zwei günstige Ergebnisse in sofern, als das Prototyp-LP-Analysefilter ein Sperrbereichsverhalten mit Kleinste-Quadrate-Eigenschaft, die resultierende QMF-Bank einen Rekonstruktionsfrequenzgang mit Quasi-Equiripple-Fehlerverhalten aufweisen. In Verbindung mit der neu vorgeschlagenen Filterstruktur stellen wir dann eine effiziente Methode vor, mit der man eine Minimax-Entwurfslösung mit auf -1, 0 und $+1$ beschränkten Koeffizienten erhält. Etliche Entwurfsbeispiele werden vorgelegt, welche die Wirksamkeit der vorgeschlagenen Technik demonstrieren. © 1997 Elsevier Science B.V.

Résumé

Nous nous intéressons dans cet article aux filtres mirroirs en quadrature à phase linéaire (LP-QMF) de coefficients limités aux valeurs -1, 0 et $+1$. Une structure de filtre et une technique nouvelles basées sur un algorithme aux moindres carrés pondérés (WLS) récemment développé sont présentées. Tout d'abord, nous concevons un LP-QMF

*Corresponding author. Fax: 886 2 367 1909.

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à coefficients continus en utilisant l'algorithme WLS. Ce processus de conception produit deux résultats favorables dans le sens que le filtre d'analyse LP prototype a une réponse en bande coupée aux moindres carrés et que le banc QMF résultant a un comportement d'erreur de reconstruction à ondulation quasi constante. En conjonction avec la structure de filtre nouvelle, nous présentons ensuite une méthode efficace pour obtenir une solution de conception avec des coefficients restreints à -1, 0 et $+1$ au sens minimax. Plusieurs exemples de conception illustrant l'efficience de la technique proposés sont présentés.

Keywords: Quadrature mirror filter banks; Weighted least squares; Discrete optimization

1. Introduction

Quadrature mirror filter (QMF) banks find applications in many areas, especially in the subband coding of speech signals [2, 3] and the subband coding of images [6, 7]. Although QMF banks are useful for many applications, hardware implementation generally requires large and complicated digital circuits if they are realized with continuous coefficients. To achieve circuit complexity reduction or to speed up filtering operation implementation besides concern for the overall performance, it is preferable to design a linear-phase QMF (LP-QMF) bank with coefficients restricted to the values of -1, 0 and $+1$ only. However, there are practically no papers concerning the design of LP-QMFs with -1, 0 and $+1$ coefficients in the literature.

In this paper, we are concerned with the design and realization of LP two-band QMF banks with $-1, 0, +1$ coefficients. A new filter structure which consists of a transversal filter with tap coefficients constrained to $-1, 0, +1$ only and cascaded with an appropriate recursive network for constructing the prototype LP analysis filter is presented. Based on the weighted least-squares (WLS) algorithm recently developed in [4], a continuous-coefficient QMF bank with quasi-equiripple reconstruction error and least-squares stopband response for its prototype analysis filter is first designed. To obtain a design with -1, 0 and $+1$ coefficients which minimizes the peak reconstruction error and the squared stopband error, we then propose a new filter structure for realization. The coefficients -1, 0 and $+1$ are used in the oversampled domain and the design procedure leads to finely quantized coefficients. It is shown that very satisfactory LP-QMF banks with -1, 0 and $+1$ coefficients can be obtained using the proposed technique.

2. Formulation of the design problem

Fig. 1 shows the considered two-band QMF bank. From Fig. 1, it is easy to show that the input/output relationship in Z transform is given by

$$\hat{X}(z) = \tfrac{1}{2}[H_0(z)F_0(z) + H_1(z)F_1(z)]X(z) + \tfrac{1}{2}[H_0(-z)F_0(z) + H_1(-z)F_1(z)]X(-z). \quad (1)$$

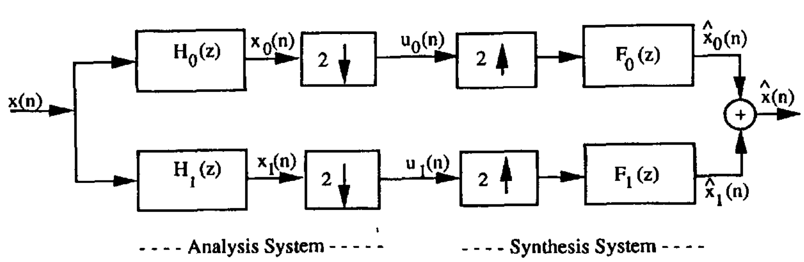

Fig. 1. The two-band QMF bank.

Let the synthesis filters $F_0(z)$ and $F_1(z)$ be equal to $H_1(-z)$ and $-H_0(-z)$, respectively, to eliminate the aliasing term. Based on the mirror-image symmetry about frequency $\omega = \pi/2$ for the analysis filters, i.e., $H_1(z) = H_0(-z)$, (1) becomes

$$\hat{X}(z) = \tfrac{1}{2}[H_0(z)H_0(z) - H_0(-z)H_0(-z)]X(z). \quad (2)$$

Next, let the low-pass filter $H_0(z)$ be an LP FIR filter with even length N, i.e., the inverse Z transform $h_0(n)$ of $H_0(z)$ is given by $h_0(n) = h_0(N-1-n)$ for $n = 0, 1, \dots, N/2-1$. Substituting the frequency response $H_0(e^{j\omega}) = |H_0 e^{j\omega})| e^{-j\omega(N-1)/2} = H_0(\omega)e^{-j\omega(N-1)/2}$ into (2) yields

$$\hat{X}(e^{j\omega}) = \frac{e^{-j\omega(N-1)}}{2}[|H_0(e^{j\omega})|^2 - (-1)^{N-1}|H_0(e^{j(\omega+\pi)}|^2]X(e^{j\omega}). \quad (3)$$

Therefore, the condition of perfect reconstruction implies that the QMF bank has an LP delay due to the term $e^{-j\omega(N-1)}$ and its magnitude response $T(\omega)$ must be unity, i.e.,

$$T(\omega) = |H_0(e^{j\omega})|^2 + |H_0(e^{j(\omega+\pi)})|^2 = H_0^2(\omega) + H_0^2(\omega+\pi) = 1 \quad \text{for all } \omega. \quad (4)$$

This requirement imposes constraints not only on the low-pass analysis filter $H_0(e^{j\omega})$, that it should be an ideal low-pass filter, but also on its behavior for all ω, that it should satisfy the flat reconstruction condition given in (4). Let the reconstruction error be defined as

$$e_r(\omega) = T(\omega) - 1 = [H_0^2(\omega) + H_0^2(\omega+\pi)] - 1. \quad (5)$$

Hence, the considered problem is to find $H_0(z)$ with coefficients -1, 0 and $+1$ such that the energy of its stopband response error and $e_r(\omega)$ are minimized simultaneously.

3. The proposed new filter structure

A new filter structure which consists of a transversal filter with tap coefficients constrained to -1, 0, $+1$ only and cascaded with an appropriate recursive network for constructing the prototype LP analysis filter is presented. Fig. 2 shows the block diagram for the new filter structure which is a modification of the one used for constructing an FIR low-pass filter presented in [1]. The input $x(n)$ is first multiplied by an appropriately chosen positive number Δ to change the range of $x(n)$. The purpose of oversampling the sequence by k is to keep the error due to the operation similar to the

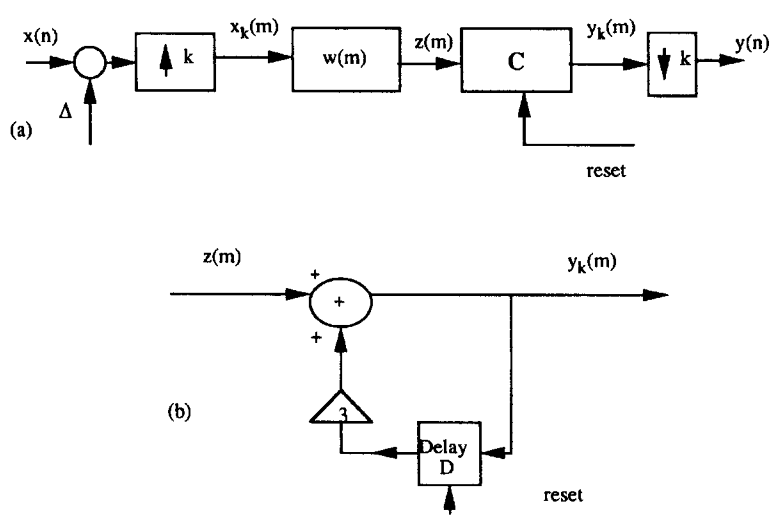

Fig. 2. (a) The proposed new filter structure. (b) The function block C.

delta modulation as shown by the function block C at an acceptable level. The impulse response $w(n)$ has a finite number of samples with values restricted to $-1, 0$ and $+1$. C represents a recursive network which performs an operation similar to the delta modulation with a step size fixed to one. The reset terminal receives a signal to clear the contents of the delay elements in C when $m = nk + 1$.

Let the resulting impulse response of the proposed filter structure be $h_d(n)$ with length N. $w(m)$ will have length N_k which is equal to $Nk + 1$. By setting $x(n)$ to an impulse sequence, the relationship between $y_k(m)$ and $w(m)$ can be found as follows:

$$y_k(0) = \Delta w(0), \quad y_k(m) = 0 + \Delta w(m)$$

$$\text{for } m = nk + 1, \ n = 1, 2, \ldots, N,$$

and

$$y_k(m) = 3y_k(m-1) + \Delta w(m),$$

$$\text{for } m \neq nk + 1, \ n = 1, 2, \ldots, N. \tag{6}$$

It is clear that imposing the reset operation in C is equivalent to putting a constraint for $y_k(m)$ as shown by the second equation in (6). Moreover, (6) can be rewritten as follows:

$$y_k(0) = \Delta w(0), \quad y_k(m) = \Delta \sum_{i = k\langle \frac{m-1}{k} \rangle + 1}^{m} w(i) 3^{m-i}$$

$$\text{for } m = 1, 2, \ldots, N_k - 1, \tag{7}$$

where $\langle x \rangle$ denotes the largest integer not greater than x. Moreover, the sum in (7) with these limits is due to the fact that in system C only the samples after the last reset have to be considered. Accordingly, the relationship between $y(n)$ which is equal to the impulse response $h_d(n)$ and $y_k(m)$ is given by

$$h_d(0) = y(0) = y_k(0) = \Delta w(0),$$

$$h_d(n) = y(n) = y_k(nk) = \Delta \sum_{m=(n-1)k+1}^{nk} w(m) 3^{nk-m}$$

$$\text{for } n = 1, 2, \ldots, N - 1. \tag{8}$$

Since $w(m)$ has values restricted to $-1, 0$ and $+1$ only for $m = 0, 1, \ldots, N_k - 1$ and $w(m) = 0$ for $m < 0$, thus we can rewrite (8) as follows:

$$h_d(n) = y(n) = y_k(nk) = \Delta \sum_{m=(n-1)k+1}^{nk} w(m) 3^{nk-m}$$

$$\text{for } n = 0, 1, 2, \ldots, N - 1. \tag{9}$$

From (9), we note that the value of $h_d(n)$ satisfies the following inequalities:

$$-\frac{3^k - 1}{2} \leqslant \frac{h_d(n)}{\Delta} \leqslant \frac{3^k - 1}{2}. \tag{10}$$

For any integer P within the range of $[-\frac{1}{2}(3^k - 1), \frac{1}{2}(3^k - 1)]$, it is easy to show that there exists a unique set of $w((n-1)k + 1)$, $w((n-1)k + 2), \ldots, w(nk)$ such that the integer P can be expressed as

$$P = \sum_{m=(n-1)k+1}^{nk} w(m) 3^{nk-m}. \tag{11}$$

4. The proposed WLS design method

4.1. Design of quasi-equiripple QMFs with continuous coefficients

From the design problem described in Section 2, the core work for designing an LP QMF bank is to design $H_0(e^{j\omega})$ with even length N such that those required conditions are satisfied. Let E be the overall error function given by

$$E = E_r + \alpha E_s. \tag{12}$$

where E_r and E_s denote the weighted energy of the reconstruction error with a weighting function $W(\omega)$ and the stopband energy related to $H_0(e^{j\omega})$ with a stopband edge $= \omega_s$. They are given by

$$E_r = \int_{\omega=0}^{\pi} W(\omega)[e_r(\omega)]^2 d\omega$$

and (13)

$$E_s = \int_{\omega=\omega_s}^{\pi} |H_0(e^{j\omega})|^2 \, d\omega,$$

respectively. It has been shown in [4] that minimizing (12) with an appropriately chosen $W(\omega)$ will provide the minimax reconstruction error design. However, solving the resulting minimization problem is not an easy task since (12) is a function of the fourth degree in the filter coefficients $h_0(n)$. To alleviate this difficulty, we resort to an iterative method to find the filter coefficients for minimizing the related error measure. Let $h_0^q(n)$ be the coefficients of $H_0(e^{j\omega})$ computed at the qth iteration. According to the properties of the QMF filters, we have the coefficients $h_1(n)$ of $H_1(e^{j\omega})$ at the qth iteration given by

$$h_1^q(n) = (-1)^n h_0^q(n) \tag{14}$$

and the coefficients $f_1(n) = -(-1)^n f_0(n)$, where $f_0(n)$ are the coefficients of $F_0(e^{j\omega})$. When $f_0(n)$ are very close to $h_0^q(n)$ or $F_0(e^{j\omega})$ is very close to $H_0^q(e^{j\omega})$, the aliasing term can be neglected. The magnitude of the QMF bank shown in (4) can then be approximately written as

$$T'(e^{j\omega}) = H_0^q(e^{j\omega})F_0(e^{j\omega}) + H_0^q(e^{j(\omega+\pi)})F_0(e^{j(\omega+\pi)}). \tag{15}$$

At the next iteration (i.e., the $(q+1)$th iteration), we want to find the coefficients $f_0(n)$ of $F_0(e^{j\omega})$ such that the following error function is minimized:

$$E' = \int_{\omega=0}^{\pi} W(\omega)[T'(e^{j\omega}) - 1]^2 \, d\omega + \alpha \int_{\omega=\omega_s}^{\pi} |F_0(e^{j\omega})|^2 \, d\omega. \tag{16}$$

Let $F_0(e^{j\omega}) = \sum_{n=0}^{N/2-1} 2 f_0(n) \cos(n - (N-1)/2)\omega$ and $\Omega = \{\omega_1, \omega_2, \ldots, \omega_k = \omega_s, \ldots, \omega_l\}$ be a dense grid of frequency linearly distributed in the range $[0, \pi]$. Next, we construct the following matrices:

$$\boldsymbol{Q} = (f_0(0), f_0(1), \ldots, f_0(N/2-1))^{\mathrm{T}},$$

$$\boldsymbol{H}(\Omega) = \operatorname{diag}(H_0^q(\omega_1), \ldots, H_0^q(\omega_k), \ldots, (H_0^q(\omega_l)),$$

$$\boldsymbol{U}_t(\Omega) = 2\begin{pmatrix} \cos\left(\frac{N-1}{2}\omega_1\right) & \cdots & \cos(0.5\omega_1) \\ & \vdots & \\ \cos\left(\frac{N-1}{2}\omega_k\right) & \cdots & \cos(0.5\omega_k) \\ & \vdots & \\ \cos\left(\frac{N-1}{2}\omega_l\right) & \cdots & \cos(0.5\omega_l) \end{pmatrix}, \tag{17}$$

$$\boldsymbol{U}_s = 2\begin{pmatrix} \cos\left(\frac{N-1}{2}\omega_k\right) & \cdots & \cos(0.5\omega_k) \\ & \vdots & \\ \cos\left(\frac{N-1}{2}\omega_l\right) & \cdots & \cos(0.5\omega_l) \end{pmatrix},$$

where $\boldsymbol{Q}$ is the coefficient vector with size $N/2 \times 1$. The superscript T denotes the transpose operation. Let

$$\boldsymbol{U} = \boldsymbol{H}(\Omega)\boldsymbol{U}_t(\Omega) + \boldsymbol{H}(\Omega+\pi)\boldsymbol{U}_t(\Omega+\pi), \tag{18}$$

where $\Omega + \pi$ denotes the set of $\{\omega_1 + \pi, \omega_2 + \pi, \ldots, \omega_k + \pi, \ldots, \omega_l + \pi\}$. Then (16) becomes

$$E'_q = (\boldsymbol{UQ} - \boldsymbol{I})^{\mathrm{T}}\hat{\boldsymbol{W}}(\boldsymbol{UQ} - \boldsymbol{I}) + \alpha(\boldsymbol{U}_s\boldsymbol{Q})^{\mathrm{T}}(\boldsymbol{U}_s\boldsymbol{Q}), \tag{19}$$

where $\boldsymbol{I}$ is an $l \times 1$ vector with elements $= 1$ and $\hat{\boldsymbol{W}} = \operatorname{diag}\{W(\omega_1), W(\omega_2), \ldots, W(\omega_k), \ldots, W(\omega_l)\}$. Clearly, minimizing (19) yields an analytical solution for (19), which is an approximation of the optimal solution for (16) and is given by

$$\boldsymbol{Q} = (\boldsymbol{U}^{\mathrm{T}}\hat{\boldsymbol{W}}\boldsymbol{U} + \alpha\boldsymbol{U}_s^{\mathrm{T}}\boldsymbol{U}_s)^{-1}\boldsymbol{U}^{\mathrm{T}}\hat{\boldsymbol{W}}\boldsymbol{I} = \boldsymbol{R}_0^{-1}\boldsymbol{U}^{\mathrm{T}}\hat{\boldsymbol{W}}\boldsymbol{I}, \tag{20}$$

where matrix $\boldsymbol{R}_0 = \boldsymbol{U}^{\mathrm{T}}\hat{\boldsymbol{W}}\boldsymbol{U} + \alpha\boldsymbol{U}_s^{\mathrm{T}}\boldsymbol{U}_s$. Using the coefficients $f_0^{q+1}(n)$ of $F_0(e^{j\omega})$ obtained from (20), we update the coefficients of $H_0(e^{j\omega})$ as follows:

$$h_0^{q+1}(n) = (1-\tau)h_0^q(n) + \tau f_0^{q+1}(n) \quad \text{for } n = 0, 1, \ldots, N/2 - 1. \tag{21}$$

The best value of the smoothing parameter τ $(0 < \tau < 1)$ is chosen experimentally.

When the iteration algorithm approaches the optimal solution, the difference between $h_0^{q+1}(n)$ and $h_0^q(n)$ will be very small. Hence, the coefficients of $F_0(e^{j\omega})$ will be very close to the coefficients of $H_0(e^{j\omega})$. Therefore, we use the following stopping criterion. If

$$\frac{|E_q - E_{q+1}|}{E_{q+1}} \leqslant \varepsilon \tag{22}$$

then the design process is terminated, where E_q denotes the value of E defined in (12) at the qth iteration and ε is a preset number. Moreover, using the proposed error measure given in (12) and the weight updating formula given in [4], the design technique is expected to obtain a QMF bank whose reconstruction error is approximately equiripple. Therefore, we first evaluate whether the resulting reconstruction error of the designed QMF bank is 'equiripple' enough. Let $\mathrm{Max}(V)$ and $\mathrm{Min}(V)$ denote the maximum and minimum values of the reconstruction error over all the extreme frequencies, respectively. Then the design process is stopped if

$$\frac{[\mathrm{Max}(V) - \mathrm{Min}(V)]}{\mathrm{Max}(V)} \leqslant \kappa \tag{23}$$

is satisfied, where κ is a preset positive constant. The design method is summarized as follows.

Step 1. Specify the required design parameters, such as the filter length N, the stopband edge frequency ω_s, the relative weight α, and the values of ε and κ.

Step 2. Select an appropriate initial guess $H_0^0(e^{j\omega})$ for $H_0(\omega)$ and set the iteration number $q = 0$. Set the initial value of the weighting matrix $\hat{\boldsymbol{W}}$ to an identity matrix.

Step 3. Compute the coefficients $f_0(n)$ of $F_0(\omega)$ at the qth iteration using the formula of (20).

Step 4. Compute the coefficients $h_0(n)$ of $H_0(\omega)$ at the $(q+1)$th iteration using the formula of (21).

Step 5. If the value of error function E_q satisfies a preset stopping criterion given in (22), then we go to Step 6. Otherwise, we set $q = q + 1$ and go to Step 3.

Step 6. If the stopping criterion specified in (23) is satisfied, then we terminate the design process. Otherwise, we adjust the frequency response weighting function $W(\omega)$ using the algorithm presented in [4] and update the corresponding weighting matrix $\hat{\boldsymbol{W}}$. Then, we go to Step 3.

4.2. Discrete optimization procedure

This procedure is a modification of that presented in [1] and briefly described as follows.

4.2.1. Constrained optimization

To find the optimal values for the remaining continuous filter coefficients subject to some filter coefficients taking on discrete values. We utilize the efficient LMS algorithm presented in [5] to reoptimize the remaining continuous coefficients when a chosen coefficient is fixed at a discrete value. Suppose that $\boldsymbol{h}^{(i)}$ represents the continuous coefficients vector excluding the ith coefficient $h_0(i)$ which is fixed at a discrete value. That is,

$$\begin{aligned}\boldsymbol{h}^{(i)} &= \{h_0(0), h_0(1), \ldots, h_0(i-1), h_0(i+1), \ldots,\\ &\quad h_0(N/2-1)\}^{\mathrm{T}}\\ &= \{h^{(i)}(0), h^{(i)}(1), \ldots, h^{(i)}(j), \ldots, h^{(i)}(N/2-2)\}^{\mathrm{T}}.\end{aligned} \tag{24}$$

Let $\boldsymbol{h}_1^{(i)}$ be the vector of (24) which minimizes the objective function of (19) when $h_0(i)$ is fixed at a discrete value. Let the difference between the optimal continuous value be $h_{0\mathrm{p}}(i)$ and the discrete value $h_{\mathrm{d}}(i)$ of the filter coefficient $h_0(i)$ is given by $\Delta h(i) = h_{\mathrm{d}}(i) - h_{0\mathrm{p}}(i)$. Based on the LMS algorithm of [5], we can obtain $\boldsymbol{h}_1^{(i)} = \boldsymbol{h}_0^{(i)} + (\boldsymbol{Q}^{(i)})\Delta h(i) = \{h_1^{(i)}(0), h_1^{(i)}(1), \ldots, h_1^{(i)}(j), \ldots, h_1^{(i)}(N/2-2)\}^{\mathrm{T}}$, where $\boldsymbol{h}_0^{(i)}$ denotes the coefficient vector $\boldsymbol{h}_0 = \{h_{0\mathrm{p}}(0), h_{0\mathrm{p}}(1), \ldots, h_{0\mathrm{p}}(N/2-1)\}^{\mathrm{T}}$ which is obtained from Section 4.1 with $h_{0\mathrm{p}}(i)$ omitted. $\boldsymbol{Q}^{(i)}$ is an $(N/2-1) \times 1$ column vector given by $\boldsymbol{Q}^{(i)} = \boldsymbol{R}_1^{-1}\boldsymbol{B}$, where $\boldsymbol{R}_1$ and $\boldsymbol{B}$ are obtained from the submatrices of $\boldsymbol{R}_0$ of (20). Let $\boldsymbol{R}_0$ be partitioned as follows:

$$\boldsymbol{R}_0 = \begin{bmatrix} A_1 & B_1 & A_2 \\ C_1^{\mathrm{T}} & D & C_2^{\mathrm{T}} \\ A_3 & B_2 & A_4 \end{bmatrix},$$

where $[C_1^{\mathrm{T}}\, D\, C_2^{\mathrm{T}}]$ is the ith row of $\boldsymbol{R}_0$, $[B_1^{\mathrm{T}}\, D\, B_2^{\mathrm{T}}]$ is the ith column of $\boldsymbol{R}_0$. Then

$$\boldsymbol{R}_1 = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \quad \text{and} \quad \boldsymbol{B} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}.$$

Next, we select $h_1^{(i)}(j)$ from $h_1^{(i)}$ and fix it at a discrete value $h_{\mathrm{d}}^{(i)}(j)$. Let $\boldsymbol{h}_2^{(i,j)}$ be the coefficient vector which minimizes (19) when $h(i)$ and $h^{(i)}(j)$ are fixed at discrete values. Let the difference between the optimal continuous value $h_{1\mathrm{p}}^{(i)}(j)$ and the discrete value $h_{\mathrm{d}}^{(i)}(j)$ of the coefficient $h^{(i)}(j)$ be $\Delta h^{(i)}(j) = h_{\mathrm{d}}^{(i)}(j) - h_{1\mathrm{p}}^{(i)}(j)$. Similarly, we can obtain $\boldsymbol{h}_2^{(i,j)} = \boldsymbol{h}_1^{(i,j)} + \boldsymbol{Q}^{(i,j)}\Delta h^{(i)}(j)$, where $\boldsymbol{h}_1^{(i,j)}$ represents the vector $\boldsymbol{h}_1^{(i)}$ with the coefficient $h_1^{(i)}(j)$ omitted. We obtain $\boldsymbol{Q}^{(i,j)}$ by following the same procedure as above with $\boldsymbol{R}_1$ instead of $\boldsymbol{R}_0$. We continue this process until all of the filter coefficients are chosen and fixed at discrete values. The required least-squares weighting function $W(\omega)$ for finding the WLS design solution is adjusted based on the obtained discrete filter coefficients using the systematic adjusting approach of [4].

4.2.2. *Tree search algorithm*

The algorithm used for performing the tree search is basically the same as that presented in [5]. But some modifications are made to enhance the capability of the proposed design method. After obtaining the optimal continuous coefficient design from (21), we choose a coefficient $h_0(i)$ and fix it at L discrete values in the vicinity of $h_{0p}(i)$. An optimization problem must be solved for each of the discrete values of $h_0(i)$ to find the corresponding $\boldsymbol{h}_1^{(i)}$. Based on the $\boldsymbol{h}_1^{(i)}$, L further optimization problems are produced when a second coefficient $h^{(i)}(j)$ is chosen and fixed at L discrete values. Therefore, L^2 optimization problems must be solved when $h_0(i)$ and $h^{(i)}(j)$ take on discrete values. To keep the required computation manageble, we select only L of the L^2 optimization problems for further discretizing the remaining coefficients. The criterion for selecting the L optimization problems is to choose the L problems which provide the smallest value of the weighted peak reconstruction error. Next, each of the L selected problems produces other L optimization problems when a third coefficient is chosen to take on L discrete values. The search process continues until all of the filter coefficients are discretized.

4.2.3. *Filter coefficients selection*

We present a criterion for dealing with the discrete coefficient constraint of (10). In general, the grid density decreases as the value of discrete coefficients increases. Thus, the effect of discretizing the small coefficient values is more easily compensated by the reoptimized values of the remaining coefficients. Hence, we discretize the coefficient with the largest relative sensitivity first at each tree stage. The relative sensitivity of a continuous coefficient $h(i)$ is defined as follows:

$$\text{Relative Sensitivity of } h(i) = \text{Max}\,|Q^{(i)}(j)|, \tag{25}$$

where $Q^{(i)}(j)$ is the jth element of the vector $\boldsymbol{Q}^{(i)}$. The proposed method is summarized as follows.

Step 1. Use the design method of Section 4.1 to find the optimal continuous coefficients $h_0(n)$ for the LP low-pass prototype filter $H_0(z)$.

Step 2. Choose 4 powers-of-two values in the vicinity of the maximum of $|h_0(n)|/[\frac{1}{2}(3^k-1)]$ as the values for the step size Δ.

Step 3. For a given Δ, perform the discrete optimization procedure described in Section 4.2 to find the corresponding discrete coefficients $h_d(n)$ $n=0,1,2,\dots,N/2-1$.

Step 4. Compute the reconstruction error $e_r(\omega_i)$, $i=1,2,\dots,K$, corresponding to $h_d(n)$ and adjust $\boldsymbol{U}$ using (18) and $\hat{\boldsymbol{W}}$ using the WLS algorithm of [4] based on $e_r(\omega_i)$. Then, recompute the new optimal continuous coefficients $h_0(n)$ based on the new matrices $\boldsymbol{U}$ and $\hat{\boldsymbol{W}}$.

Step 5. Repeat Step 3 and 4 until the peak value of $e_r(\omega_i)$ cannot be further reduced.

Step 6. Select the Δ which makes the peak reconstruction error smallest among the 4 powers-of-two values for Δ. Find the corresponding $w(m)$ by utilizing the relationship given by (9).

5. Computer simulations

For the following simulation examples, the number L for discretizing a chosen coefficient at each level of the tree search is set to 3.

Example 1. The design specifications are as follows. $H_0(z)$ has length $N=32$. The passband and stopband edge frequencies are $\omega_p=0.4\pi$ and $\omega_s=0.6\pi$, respectively. We set $\alpha=1.0$, $k=9$ and $\tau=0.5$. The resulting step size Δ and the discrete coefficients obtained are listed below: $\Delta=2^{-14}$,

$h_d(0)=h_d(31)=21\Delta$, $h_d(1)=h_d(30)=-41\Delta$,

$h_d(2)=h_d(29)=-29\Delta$, $h_d(3)=h_d(28)=98\Delta$,

$h_d(4)=h_d(27)=21\Delta$, $h_d(5)=h_d(26)=-193\Delta$,

$h_d(6)=h_d(25)=13\Delta$, $h_d(7)=h_d(24)=334\Delta$,

$h_d(8)=h_d(23)=-98\Delta$, $h_d(9)=h_d(22)=-544\Delta$,

$h_d(10)=h_d(21)=278\Delta$, $h_d(11)=h_d(20)=885\Delta$,

$h_d(12)=h_d(19)=-688\Delta$,

$h_d(13)=h_d(18)=-1633\Delta$,

$h_d(14)=h_d(17)=2143\Delta$, $h_d(15)=h_d(16)=7619\Delta$.

Fig. 3 shows the frequency responses including $H_0(\omega)$ and $T(\omega)$ for the designed two-band QMF

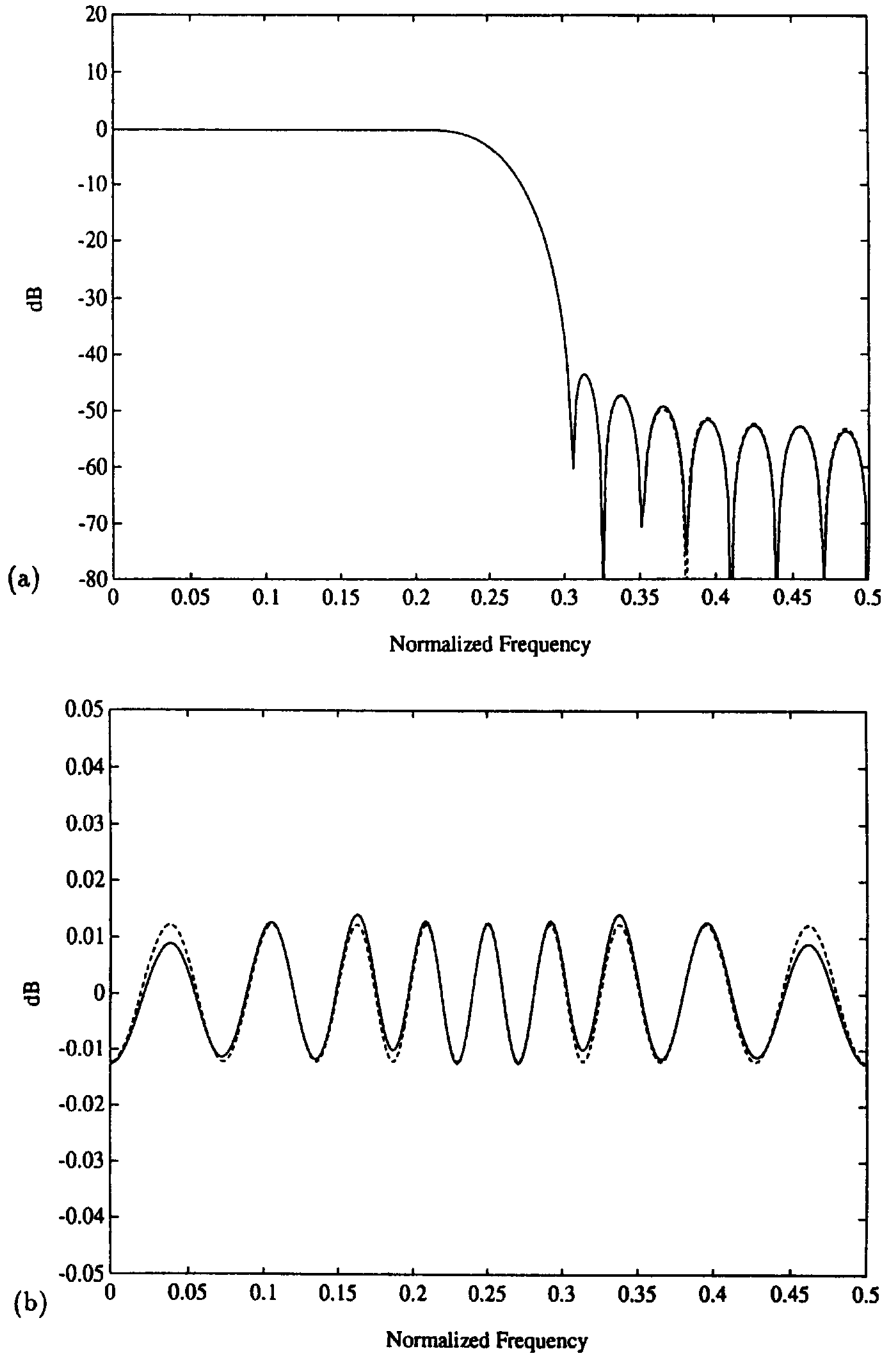

Fig. 3. The resulting frequency response of Example 1. (a) Magnitude response of the prototype filter. (b) Magnitude response of the two-band QMF bank (solid line: proposed design method, dash line: optimal continuous design).

Table 1
Significant simulation results of Example 1 for comparison

	The design with −1, 0, +1 filter coefficients	The design with optimal continuous coefficients
SEA (dB)	35.6815	36.3362
PRE (dB)	0.0134	0.0124

banks with $-1, 0$ and $+1$ coefficients and with the optimal continuous coefficients, respectively. We note that the designed QMF bank with continuous coefficients shows its quasi-equiripple characteristics. Table 1 lists several significant design results including the stopband edge attenuation (SEA) of the designed $H_0(e^{j\omega})$, the peak reconstruction error (PRE) of the designed two-band QMF bank. The SEA and PRE in dB are calculated by

$$\mathrm{SEA} = -20\log_{10}|H_0(\omega_s)| \quad \text{and}$$

$$\mathrm{PRE} = \text{Maximum of } |20\log_{10}T(\omega)| \quad \text{for } 0 \leqslant \omega \leqslant \pi.$$

From the simulation results, we observe that the proposed technique produces a two-band QMF bank with -1, 0 and $+1$ coefficients and performance satisfactorily close to the two-band continuous-coefficient QMF bank.

Example 2. The specifications used are as follows. The filter length N is equal to 80. The passband and stopband edge frequencies are $\omega_p = 0.45\pi$ and $\omega_s = 0.55\pi$, respectively. We set $\alpha = 2.0$, $k = 11$ and $\tau = 0.5$. The resulting step size Δ and discrete coefficients are listed below: $\Delta = 2^{-17}$,

$h_d(0) = h_d(79) = 20\Delta$, $h_d(1) = h_d(78) = -44\Delta$,

$h_d(2) = h_d(77) = -45\Delta$, $h_d(3) = h_d(76) = 61\Delta$,

$h_d(4) = h_d(75) = 51\Delta$, $h_d(5) = h_d(74) = -107\Delta$,

$h_d(6) = h_d(73) = -63\Delta$, $h_d(7) = h_d(72) = 165\Delta$,

$h_d(8) = h_d(71) = 71\Delta$, $h_d(9) = h_d(70) = -242\Delta$,

$h_d(10) = h_d(69) = -74\Delta$, $h_d(11) = h_d(68) = 339\Delta$,

$h_d(12) = h_d(67) = 66\Delta$, $h_d(13) = h_d(66) = -462\Delta$,

$h_d(14) = h_d(65) = -46\Delta$, $h_d(15) = h_d(64) = 611\Delta$,

$h_d(16) = h_d(63) = 5\Delta$, $h_d(17) = h_d(62) = -792\Delta$,

$h_d(18) = h_d(61) = 60\Delta$, $h_d(19) = h_d(60) = 1009\Delta$,

$h_d(20) = h_d(59) = -159\Delta$,

$h_d(21) = h_d(58) = -1268\Delta$,

$h_d(22) = h_d(57) = 303\Delta$,

$h_d(23) = h_d(56) = 1578\Delta$,

$h_d(24) = h_d(55) = -504\Delta$,

$h_d(25) = h_d(54) = -1952\Delta$,

$h_d(26) = h_d(53) = 786\Delta$,

$h_d(27) = h_d(52) = 2415\Delta$,

$h_d(28) = h_d(51) = -1183\Delta$,

$h_d(29) = h_d(50) = -3008\Delta$,

$h_d(30) = h_d(49) = 1758\Delta$,

$h_d(31) = h_d(48) = 3817\Delta$,

$h_d(32) = h_d(47) = -2639\Delta$,

$h_d(33) = h_d(46) = = -5033\Delta$,

$h_d(34) = h_d(45) = 4141\Delta$,

$h_d(35) = h_d(44) = 7195\Delta$,

$h_d(36) = h_d(43) = -7296\Delta$,

$h_d(37) = h_d(42) = -12580\Delta$,

$h_d(38) = h_d(41) = 18639\Delta$,

$h_d(39) = h_d(40) = 59913\Delta$.

Fig. 4 shows the frequency responses including $H_0(\omega)$ and $T(\omega)$ for the designed two-band QMF banks with -1, 0 and $+1$ coefficients and with the optimal continuous coefficients, respectively. We note that the designed QMF bank with continuous coefficients shows its almost equiripple characteristics. Table 2 lists the SEA of the designed $H_0(e^{j\omega})$, the PRE of the designed two-band QMF bank. Again, we observe from the simulation results that the proposed technique produces a two-band QMF bank with $-1, 0, +1$ coefficients

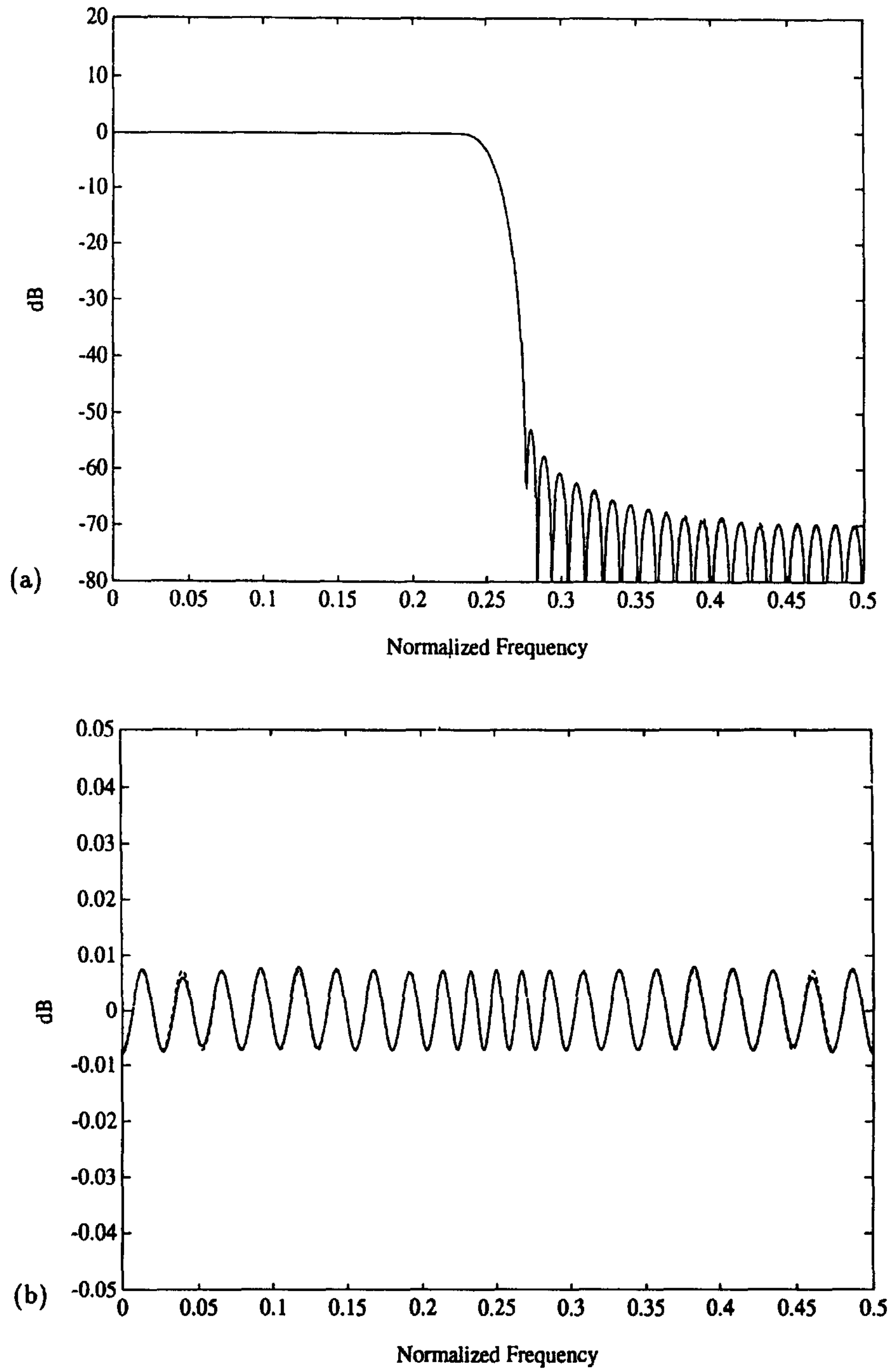

Fig. 4. The resulting frequency responses of Example 2. (a) Magnitude response of the prototype filter. (b) Magnitude response of the two-band QMF bank (solid line: proposed design method, dash line: optimal continuous design).

Table 2
Significant simulation results of Example 2 for comparison

	The design with $-1, 0, +1$ filter coefficients	The design with optimal continuous coefficients
SEA (dB)	46.9273	47.1025
PRE (dB)	0.00765	0.00729

and performance very close to the two-band continuous-coefficient QMF bank.

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