Performance of DPSK Signals With Quadratic Phase Noise

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Abstract—**Nonlinear phase noise induced by the interaction of the fiber Kerr effect and amplifier noises is a quadratic function of the electric field. When the dependence between the additive Gaussian noise and the quadratic phase noise is taken into account, the error probability for differential phase-shift keying signals is derived analytically. Depending on the number of fiber spans, the signal-to-noise ratio penalty is increased by up to 0.23 dB, due to the dependence between the Gaussian noise and the quadratic phase noise.**

Index Terms—**Error probability, fiber Kerr effects, nonlinear phase noise, phase modulation.**

I. INTRODUCTION

OTHER than the projection of additive Gaussian noise to the phase, phase noises from other sources can be considered as multiplicative noise that adds directly to the phase of the received signal. When the local oscillator is not locked perfectly into the signal, the noisy reference gives additive phase noise [1], [2]. Laser phase noise degrades coherent optical communication systems [3]–[5]. Those types of extra additive phase noise that add directly to the signal phase are independent of the additive Gaussian noise. In this paper, the additive phase noise is a quadratic function of the electric field. When the electric field is contaminated with additive Gaussian noise, although the quadratic phase noise is uncorrelated with the linear phase noise, both non-Gaussian distributed, the phase noise weakly depends on the additive Gaussian noise.

Differential phase-shift keying (DPSK) signals [6]–[16] have received renewed attention recently for long-haul or spectrally efficiency lightwave transmission systems. When optical amplifiers are used periodically to compensate the fiber loss, the interaction of optical amplifier noise and the fiber Kerr effect induced nonlinear phase noise, often called the Gordon–Mollenauer effect [17], or more precisely, nonlinear phase noise induced by self-phase modulation. Added directly into the signal phase, the Gordon–Mollenauer effect is a quadratic function of the electric field, and degrades the DPSK signal [11], [14], [17]–[23].

Previous studies found the variance or the corresponding Q-factor of the quadratic phase noise [11], [17], [24]–[27] or

the spectral broadening of the signal [14], [18], [28]. Recently, quadratic phase noise is found to be non-Gaussian distributed, both experimentally [20] and theoretically [29], [30]. As a non-Gaussian random variable, neither the variance nor the Q-factor is sufficient to completely characterize the phase noise. The probability density of quadratic phase noise is found in [30], and used in [23] to evaluate the error probability of the DPSK signal by assuming that quadratic phase noise and Gaussian noise are independent of each other. However, as shown in the simulation of [22] and [23], the dependence between Gaussian noise with quadratic phase noise increases the error probability.

Using the distributed assumption of an infinite number of fiber spans, the joint statistics of nonlinear phase noise and Gaussian noise is derived analytically by [19], [21], and [31]. The characteristic function of nonlinear phase noise becomes a very simple expression with the distributed assumption [29]. The error probability of the DPSK signal has been derived with [22] and without [21], [32] the assumption that nonlinear phase noise is independent of the Gaussian noise. Based on the distributed assumption, it is found that the dependence between linear and nonlinear phase noise increases both the error probability and signal-to-noise ratio (SNR) penalty [21], [32].

The distributed assumption is very accurate when the number of fiber spans is larger than 32 [21], [29]. For a typical fiber-span length of 80 km, a fiber link of 32 spans has a total distance of over 2500 km. Most terrestrial fiber systems have an overall distance of less than 1000 km, and the distributed assumption needs to be verified for small numbers of fiber spans. Recently, DPSK signals have been used in systems with small numbers of fiber spans [16], [33], [34]. Of course, the independence assumption can be used for either small [23] or large [22] numbers of fiber spans. However, as shown in [21] and [32], the independence assumption of [22] and [23] underestimates both the error probability and the required SNR, contradicting the principles of conservative system design.

In this paper, taking into account the dependence between the quadratic phase noise and Gaussian noise, the error probability of the DPSK signal is derived for a finite number of fiber spans, to our knowledge, for the first time. Comparing with the independence approximation of [23], the dependence between the quadratic phase noise and Gaussian noise increases the error probability of the system.

In the remaining parts of this paper, Section II gives the model of the quadratic phase noise, mostly following the approach of [30]; Section III derives the joint statistics of the additive Gaussian noise and the quadratic phase noise. Using the joint statistics, Section IV gives the exact error probability of DPSK

signals with quadratic phase noise, taking into account the dependence between the additive Gaussian noise and quadratic phase noise; Section V calculates the error probability and the SNR penalty of DPSK signals, and is compared with the independence approximation of [23]; and Section VI is the conclusion of the paper.

II. QUADRATIC NONLINEAR PHASE NOISE

For an N-span system, for simplicity and without loss of generality, the overall quadratic phase noise is [17], [25], [26], [30]

$$\Phi_{\mathrm{NL}} = |\vec{E}_0 + \vec{n}_1|^2 + |\vec{E}_0 + \vec{n}_1 + \vec{n}_2|^2 \\ + \cdots + |\vec{E}_0 + \vec{n}_1 + \cdots + \vec{n}_N|^2 \quad (1)$$

where $\vec{E}_0 = (A, 0)$ is a two-dimensional (2-D) vector as the baseband representation of the transmitted electric field, $\vec{n}_k$, $k = 1, \ldots, N$ are independent identically distributed (i.i.d.) zero-mean circular Gaussian random complex numbers as the optical amplifier noise introduced into the system at the kth fiber span. Both electric fields of $\vec{E}_0$ and amplifier noises of $\vec{n}_k$ in (1) can also be represented as a complex number. The variance of $\vec{n}_k$ is $E\{|\vec{n}_k|^2\} = 2\sigma_0^2$, $k = 1, \ldots, N$, where σ_0^2 is the noise variance per span per dimension. In (1), the constant factor of the product of the fiber nonlinear coefficient and the effective nonlinear length per span, γL_{eff}, is ignored for simplicity. Without affecting the SNR, both signal and noise in (1) can be scaled by the same ratio for a different mean nonlinear phase shift of $\langle \Phi_{\mathrm{NL}} \rangle = NA^2 + N(N+1)\sigma_0^2$, except for the case without quadratic phase noise of $\langle \Phi_{\mathrm{NL}} \rangle = 0$. After the scaling, the mean nonlinear phase shift is approximately equal to the product of the number of fiber spans and the launched power per span, especially for the usual case of large SNR with small noise.

In the linear regime, ignoring the fiber loss of the last span and the amplifier gain required to compensate it, the signal received after N spans is

$$\vec{E}_N = \vec{E}_0 + \vec{n}_1 + \vec{n}_2 + \cdots + \vec{n}_N \quad (2)$$

with a power of $P_N = |\vec{E}_N|^2$ and SNR of $\rho_s = A^2/(2N\sigma_0^2)$. In (1) and (2), the configuration of each fiber span is assumed to be identical with the same length and launched power.

In [30], using the method of [35] and [36], the characteristic function of the quadratic phase noise (1) is found to be

$$\Psi_{\Phi_{\mathrm{NL}}}(\nu) = \prod_{k=1}^{N} \frac{1}{1 - 2j\nu\sigma_0^2\lambda_k} \exp\left[\frac{j\nu A^2 \frac{(\vec{v}_k^T \vec{w})^2}{\lambda_k}}{1 - 2j\nu\sigma_0^2\lambda_k}\right] \quad (3)$$

where $\vec{w} = (N, N-1, \ldots, 2, 1)^T$, λ_k, $\vec{v}_k$, $k = 1, 2, \ldots, N$ are the eigenvalues and eigenvectors of the covariance matrix $\mathcal{C}$, respectively. The covariance matrix $\mathcal{C} = \mathcal{M}^T \mathcal{M}$ with

$$\mathcal{M} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix}. \quad (4)$$

The characteristic function of (3) is used to find the error probability of a DPSK signal in [23] based on the assumption that the quadratic phase noise of (1) is independent of the received electric field of (2).

III. JOINT STATISTICS OF GAUSSIAN NOISE AND QUADRATIC PHASE NOISE

To find the dependence between the quadratic phase noise and the received electric field, the joint characteristic function of

$$\Psi_{\Phi_{\mathrm{NL}}, \vec{E}_N}(\nu, \vec{\omega}) = E\left\{\exp(j\nu\Phi_{\mathrm{NL}} + j\vec{\omega} \cdot \vec{E}_N)\right\} \quad (5)$$

will be derived here, with Φ_{NL} and $\vec{E}_N$ given by (1) and (2), respectively.

Similar to [21] and [30], with $\vec{\omega} = (\omega_1, \omega_2)$ and $\vec{E}_N = (e_1, e_2)$, we obtain

$$j\nu\varphi_1 + j\omega_1 e_1 = j\nu NA^2 + j\omega_1 A + 2j\nu A\vec{w}^T\vec{x} \\ + j\omega_1\vec{w}_I^T\vec{x} + j\nu\vec{x}^T\mathcal{C}\vec{x} \quad (6)$$

where φ_1 is given by

$$\varphi_1 = |A+x_1|^2 + |A+x_1+x_2|^2 + \cdots + |A+x_1+\cdots+x_N|^2 \quad (7)$$

with $\vec{n}_i = (x_i, y_i)$, $i = 1, \ldots, N$, $\vec{w}_I = (1, 1, \ldots, 1)^T$, $j\omega_1 e_1 = j\omega_1(A + x_1 + x_2 + \cdots + x_N) = j\omega_1 A + j\omega_1\vec{w}_I^T\vec{x}$, and $\vec{x} = (x_1, x_2, \ldots, x_N)^T$.

Similar to [30], using the N-dimensional Gaussian probability density function (pdf) of $(2\pi\sigma_0^2)^{-(N/2)} \exp(-\vec{x}^T\vec{x}/2\sigma_0^2)$ for $\vec{x}$, we obtain

$$\Psi_{\varphi_1, e_1}(\nu, \omega_1) = \frac{e^{j\nu NA^2 + j\omega_1 A}}{(2\pi\sigma_0^2)^{\frac{N}{2}}} \\ \times \int \exp\left[2j\nu A\vec{w}^T\vec{x} + j\omega_1\vec{w}_I^T\vec{x} - \vec{x}^T\Gamma\vec{x}\right] d\vec{x} \quad (8)$$

or

$$\Psi_{\varphi_1, e_1}(\nu, \omega_1) = e^{j\nu NA^2 + j\omega_1 A}(2\sigma_0^2)^{-\frac{N}{2}} \det[\Gamma]^{-\frac{1}{2}} \\ \times \exp\left[-\left(\nu A\vec{w} + \frac{1}{2}\omega_1\vec{w}_I\right)^T \Gamma^{-1}\left(\nu A\vec{w} + \frac{1}{2}\omega_1\vec{w}_I\right)\right] \quad (9)$$

where $\Gamma = \mathcal{I}/(2\sigma_0^2) - j\nu\mathcal{C}$, and $\mathcal{I}$ is an $N \times N$ identity matrix. Similarly, using $A = 0$ in (9), we get

$$\Psi_{\varphi_2, e_2}(\nu, \omega_2) = \frac{\exp\left[-\frac{1}{4}\omega_2^2\vec{w}_I^T\Gamma^{-1}\vec{w}_I\right]}{(2\sigma_0^2)^{\frac{N}{2}} \det[\Gamma]^{\frac{1}{2}}} \quad (10)$$

where

$$\varphi_2 = y_1^2 + |y_1 + y_2|^2 + \cdots + |y_1 + \cdots + y_N|^2. \quad (11)$$

The joint characteristic function of

$$\Psi_{\Phi_{\mathrm{NL}}, \vec{E}_N}(\nu, \vec{\omega}) = \Psi_{\varphi_1, e_1}(\nu, \omega_1)\Psi_{\varphi_2, e_2}(\nu, \omega_2) \quad (12)$$

becomes

$$\Psi_{\Phi_{\mathrm{NL}}, \vec{E}_N}(\nu, \vec{\omega}) = \Psi_{\Phi_{\mathrm{NL}}}(\nu)\exp\left[j\omega_1 m_N(\nu) - \sigma_N^2(\nu)\frac{|\vec{\omega}|^2}{2}\right] \quad (13)$$

where

$$\Psi_{\Phi_{NL}}(\nu) = \frac{\exp\left[j\nu NA^2 - \nu^2 A^2 \vec{w}^T \Gamma^{-1} \vec{w}\right]}{(2\sigma_0^2)^N \det[\Gamma]} \tag{14}$$

$$m_N(\nu) = A + j\nu A \vec{w}^T \Gamma^{-1} \vec{w}_I \tag{15}$$

$$\sigma_N^2(\nu) = \frac{1}{2} \vec{w}_I^T \Gamma^{-1} \vec{w}_I. \tag{16}$$

Based on the eigenvalues and eigenvectors of the covariance matrix $\mathcal{C}$, the characteristic function of $\Psi_{\Phi_{NL}}(\nu)$ becomes that of (3), and

$$m_N(\nu) = A + 2j\nu\sigma_0^2 A \sum_{k=1}^{N} \frac{(\vec{v}_k^T \vec{w})(\vec{v}_k^T \vec{w}_I)}{1 - 2j\nu\sigma_0^2 \lambda_k}$$

$$= A \sum_{k=1}^{N} \frac{\frac{(\vec{v}_k^T \vec{w})(\vec{v}_k^T \vec{w}_I)}{\lambda_k}}{1 - 2j\nu\sigma_0^2 \lambda_k} \tag{17}$$

$$\sigma_N^2(\nu) = \sigma_0^2 \sum_{k=1}^{N} \frac{\left(\vec{v}_k^T \vec{w}_I\right)^2}{1 - 2j\nu\sigma_0^2 \lambda_k}. \tag{18}$$

The characteristic function of (13) is similar to the corresponding characteristic function with the distributed assumption [21]. If the number of spans N approaches infinity, the characteristic function should converge to that of [21].

Based on (13), we obtain

$$\mathcal{F}_{\vec{\omega}}^{-1}\left\{\Psi_{\Phi_{NL},\vec{E}_N}\right\} = \frac{\Psi_{\Phi_{NL}}(\nu)}{2\pi\sigma_N^2(\nu)} \exp\left(-\frac{|\vec{z} - \vec{\xi}_\nu|^2}{2\sigma_N^2(\nu)}\right) \tag{19}$$

with $\vec{\xi}_\nu = (m_N(\nu), 0)$, and $\mathcal{F}_{\vec{\omega}}^{-1}\{\cdot\}$ denotes the inverse Fourier transform with respect to $\vec{\omega}$. The partial characteristic function and pdf of (19) is similar to a 2-D Gaussian pdf with mean of $(m_N(\nu), 0)$ and variance of $\sigma_N^2(\nu)$. With the dependence on the quadratic phase noise, the variance of $\sigma_N^2(\nu)$ and the mean of $m_N(\nu)$ are both complex numbers depending on the "angular frequency" of ν. The marginal pdf of the received electric field $\vec{E}_N$ is a 2-D Gaussian distribution with variance of $\sigma_N^2(\nu)|_{\nu=0} = N\sigma_0^2$ and mean of $m_N(\nu)|_{\nu=0} = A$.

With normalization, the corresponding joint characteristic of (19) in [21] has

$$\sigma_\infty^2(\nu) = \frac{1}{2}\frac{\tan(\sqrt{j\nu})}{\sqrt{j\nu}} \text{ and } m_\infty(\nu) = \sec(\sqrt{j\nu})\sqrt{\rho_s} \tag{20}$$

when $N \to \infty$. Based on joint statistics of (19), similar to that of [21], [32], and [37], the exact error probability of the DPSK signal can be derived analytically, even for case with linearly compensated nonlinear phase noise [23]–[25], [27], [38]. As shown in [21], the optimal compensation curve of [26] and [27] can also be derived using (19).

IV. EXACT ERROR PROBABILITY

With nonlinear phase noise, assuming zero transmitted phase, the overall received phase is

$$\Phi_r = \Theta_n - \Phi_{NL} \tag{21}$$

where Θ_n is the phase of E_N (2). The received phase is confined to the range of $[-\pi, +\pi]$. The pdf of the received phase is a periodic function with a period of 2π. If the characteristic function of the received phase is $\Psi_{\Phi_r}(\nu)$, the pdf of the received phase has a Fourier series expansion of

$$p_{\Phi_r}(\theta) = \frac{1}{2\pi} + \frac{1}{\pi} \sum_{m=1}^{+\infty} \Re\left\{\Psi_{\Phi_r}(m)\exp(-jm\theta)\right\} \tag{22}$$

where $\Re\{\cdot\}$ denotes the real part of a complex number. In (22), we use the conjugate symmetry property of $\Psi_{\Phi_r}(-\nu) = \Psi_{\Phi_r}^*(\nu)$.

In order to derive the Fourier coefficient of $\Psi_{\Phi_r}(m)$, we need the joint characteristic function of Θ_n and Φ_{NL} at the integer "angular frequency" of $\nu = m$. Based on (19), using the same method as [19], [21], [32], and [39], we obtain

$$\Psi_{\Phi_{NL},\Theta_n}(\nu, m) = \frac{\sqrt{\pi}}{2}\Psi_{\Phi_{NL}}(\nu)\sqrt{\gamma(\nu)}e^{-\frac{\gamma(\nu)}{2}}$$

$$\times \left\{I_{\frac{m-1}{2}}\left[\frac{\gamma(\nu)}{2}\right] + I_{\frac{m-1}{2}}\left[\frac{\gamma(\nu)}{2}\right]\right\}, \quad m \geq 0 \tag{23}$$

where $\gamma(\nu) = (1/2)m_N(\nu)^2/\sigma_N^2(\nu)$ is the complex-valued frequency-dependent SNR parameter. When $\nu = 0$, it is obvious that $\gamma(\nu)|_{\nu=0} = \rho_s$.

From (21), the Fourier coefficient in (22) is $\Psi_{\Phi_r}(m) = \Psi_{\Phi_{NL},\Theta_n}(m, m)$. For the DPSK signal, the differential received phase is $\Delta\Phi_r = \Phi_r(t) - \Phi_r(t - T)$, in which the pdfs of $\Phi_r(t)$ and $\Phi_r(t - T)$ are the same as that of (22). The pdf of the differential received phase is the same as (22), with the Fourier coefficient equal to $|\Psi_{\Phi_r}(m)|^2$, i.e.,

$$p_{\Delta\Phi_r}(\theta) = \frac{1}{2\pi} + \frac{1}{\pi} \sum_{m=1}^{+\infty} |\Psi_{\Phi_r}(m)|^2 \cos(m\theta). \tag{24}$$

Similar to the procedure of [2], [3], [21], [23], [32], and [39]–[41], the error probability becomes

$$p_e = \frac{1}{2} - \frac{2}{\pi} \sum_{k=0}^{+\infty} \frac{(-1)^k}{2k+1} |\Psi_{\Phi_r}(2k+1)|^2 \tag{25}$$

or

$$p_e = \frac{1}{2} - \frac{1}{2} \sum_{k=0}^{\infty} \frac{(-1)^k |r_k e^{-r_k}|}{2k+1} \left|I_k\left(\frac{r_k}{2}\right) + I_{k+1}\left(\frac{r_k}{2}\right)\right|^2$$

$$\times |\Psi_{\Phi_{NL}}(2k+1)|^2 \tag{26}$$

where

$$r_k = \frac{m_N^2(2k+1)}{2\sigma_N^2(2k+1)} \tag{27}$$

analogous to the "angular frequency" depending on SNR as the ratio of complex power of $(1/2)m_N^2(\nu)$ to the noise variance of $\sigma_N^2(\nu)$.

The error-probability expression of (26) is almost the same as that in [21] and [32], but with a different parameter of (27). The error probability of (26) is also similar to the cases when additive phase noise is independent of Gaussian noise [2], [3], [23], [40], [41]. The frequency-dependent SNR is originated from the dependence between the additional phase noise and the Gaussian noise [19], [21], [32], [37].

V. NUMERICAL RESULTS

For DPSK signals with quadratic phase noise, Fig. 1 shows the exact error probability as a function of SNR ρ_s for the mean nonlinear phase shift of $\langle\Phi_{NL}\rangle = 0.5$ rad. Fig. 2 shows the
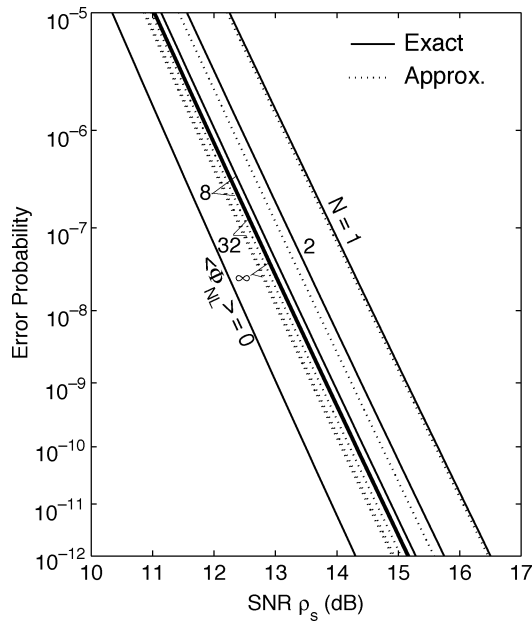
Fig. 1. Error probability of DPSK signal as a function of SNR for $N = 1, 2, 4, 8, 32$, and an infinite number of fiber spans and mean nonlinear phase shift of $\langle \Phi_{NL} \rangle = 0.5$ rad.
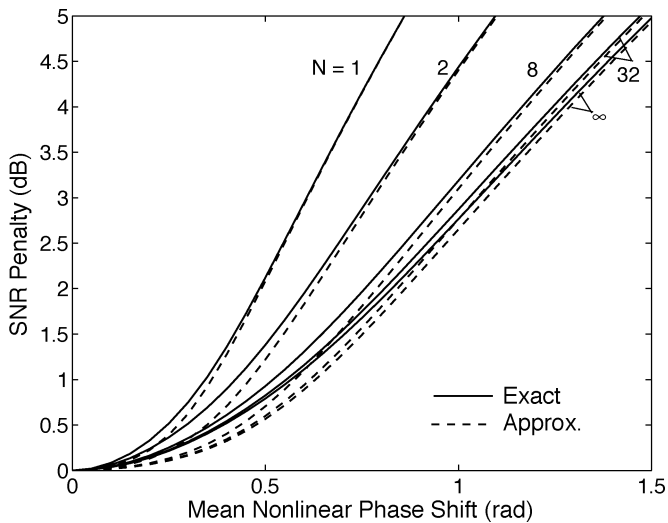

Fig. 2. SNR penalty versus mean nonlinear phase shift $\langle \Phi_{NL} \rangle$.

SNR penalty for an error probability of 10^{-9} as a function of the mean nonlinear phase shift $\langle \Phi_{NL} \rangle$. The SNR penalty is defined as the additional required SNR to achieve the same error probability of 10^{-9}. Both Figs. 1 and 2 are calculated using (26) and the independence approximation of [23]. The independence approximation of [23] underestimates both the error probability and SNR penalty of a DPSK signal with quadratic phase noise of (1). Both Figs. 1 and 2 also include the exact and approximated error probability for $N = \infty$ that are the distributed model from [32] and [22], respectively. The distributed model is applicable when the number of fiber spans is larger than 32. In Fig. 1, without quadratic phase noise of $\langle \Phi_{NL} \rangle = 0$, the error probability is $p_e = \exp(-\rho_s)/2$ [42]. The required SNR for systems without nonlinear phase noise of $\langle \Phi_{NL} \rangle = 0$ is $\rho_s = 20$ (13 dB) for an error probability of 10^{-9}.

From Figs. 1 and 2, for the same mean nonlinear phase shift of $\langle \Phi_{NL} \rangle$, the SNR penalty is larger for a smaller number of fiber spans. When the mean nonlinear phase shift is $\langle \Phi_{NL} \rangle = 0.56$ rad, the SNR penalty is about 1 dB with a large number ($N > 32$) of fiber spans, but up to a 3-dB SNR penalty for a small number ($N = 1, 2$) of fiber spans. For a 1-dB SNR penalty, the mean nonlinear phase shift is also reduced from 0.56 to 0.35 rad with a small number of fiber spans.

In [17], the optimal operating point is defined when the variance of the quadratic phase noise is approximately equal to the variance of the phase of Gaussian noise. In [22] and [23], the optimal operating point is calculated rigorously at the operation condition, in which the increase of launched power does not improve the system performance. The optimal operating point is reduced from 0.97 to 0.55 rad with the decrease in the number of fiber spans.

When the exact error probability is compared with the independence approximation of [23], the independence approximation is closer to the exact error probability for a small number of fiber spans. In all cases, the independence assumption of [22] and [23] underestimates the error probability of the system, contradicting the conservative principle of system design. The dependence between linear and nonlinear phase noise increases the SNR penalty up to 0.23 dB.

From the SNR penalty of Fig. 2, if a prior penalty of about 0.23 dB is added into the system, the independence assumption of [23] can be used to provide a conservative system-design guideline.

VI. CONCLUSION

For a system with a small number of fiber spans, the exact error probability of a DPSK signal with quadratic phase noise is derived analytically for the first time, when the dependence between linear and nonlinear phase noise is taken into account. For the same mean nonlinear phase shift, the error probability increases for a small number of fiber spans. The dependence between linear and nonlinear phase noises increases the error probability for DPSK signals. Depending on the number of fiber spans, the SNR penalty increases by up to 0.23 dB, due to the dependence between Gaussian noise and the quadratic phase noise.

For the same mean nonlinear phase shifts and SNR, the error probability of the system increases with the decrease in the number of fiber spans. As an example, the optimal operating point for a system with a large number of fiber spans ($N > 32$) is a mean nonlinear phase shift of about 1 rad that is reduced to about 0.55 rad for a system with a small number of fiber spans ($N = 1, 2$).

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