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# GSC-based adaptive beamforming with multiple-beam constraints under random array position errors☆

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**Abstract**

This paper deals with the problem of adaptive beamforming under random array position errors to provide multiple-beam constraints and suppress jammers simultaneously. Using a steering matrix with each column vector corresponding to the steering vector of a selective beam and a constraint vector with each entry equal to the gain of a selective beam, we construct the quiescent weight vector and the blocking matrix required by a generalized sidelobe canceller (GSC) to achieve a GSC-based adaptive array beamformer with multiple-beam constraints. For coping with the performance degradation due to random perturbations in array sensor positions, an iterative matrix reconstruction scheme in conjunction with derivative constraints is presented to alleviate the effect of random position errors. Simulation results show the effectiveness of the proposed technique.


*Keywords:* Generalized sidelobe canceller; Adaptive array; Multiple beams; Random position error

## 1. Introduction

In many applications, such as satellite communications [10], an antenna array must possess beamforming capability to receive more than one signal with specified gain requirements while suppressing all jammers. This purpose can be effectively achieved by using an antenna array with multiple-beam pattern [10,19]. In [19], an adaptive algorithm was proposed to find an adaptive weight close to a desired quiescent beam pattern under a unit norm constraint on the weight. However, the resulting problem to be solved is a nonlinear optimization problem and, hence, solving it requires a sophisticated procedure as shown in [19]. To tackle this problem, a technique based on Frost's algorithm [3] was recently presented in [8] for adaptive beamforming with multiple-beam constraints (MBC). Nevertheless, the existing methods for synthesizing an antenna array with multiple-beam pattern cannot deal with the situation where there exists random perturbations in array sensor positions.

Adaptive beamforming based on a generalized sidelobe canceller (GSC) has been widely considered because of its effectiveness and simplicity for achieving multiple linearly constrained beamforming [1,4] and partially adaptive beamforming [16–18,20]. However, many reports show that the GSC-based adaptive beamformers are usually very sensitive to the mismatches in

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steering angle [5] and weight vectors [21]. Recently, the problem of adaptive array signal processing under the situation with random perturbation in array sensor positions has been widely investigated in the literature [12–15].

In this paper, we present a technique for GSC-based adaptive beamforming with the capability of providing MBC in addition to jamming suppression. To satisfy the first goal, we formulate the problem as finding such a quiescent weight vector and a blocking matrix for a GSC that the array output power is minimized subject to MBC. It is shown that an analytical solution for the resulting optimization problem can be easily obtained. To achieve the second goal, an iterative matrix reconstruction scheme in conjunction with derivative constraints is presented to cure the performance deterioration due to random perturbations in array sensor positions. Simulation results demonstrate the effectiveness of the proposed technique.

This paper is organized as follows. In Section 2, the theory of adaptive array beamforming based on a GSC is briefly discussed. Section 3 presents the technique for GSC-based adaptive array beamforming with multiple-beam constraints. In Section 4, we present an iterative matrix reconstruction scheme in conjunction with derivative constraints to tackle the problem due to using an array with random sensor position errors. Several simulation examples are provided in Section 5 for showing the effectiveness of the proposed technique. A conclusion for the paper is given in Section 6.

## 2. Adaptive beamforming based on GSC

Consider a uniform linear array (ULA) with $M$ sensors and interelement spacing equal to $\lambda/2$, where $\lambda$ is the smallest signal wavelength of the signals with specified gain/null arrangements. Assume that $K$ narrow-band and far-field signals are impinging on the array from direction angles $\theta_i$, $i=1,2,\ldots,K$, off broadside. The signal received at the $m$th array sensor can be expressed as

$$x_m(t)=\sum_{i=1}^{K} s_i(t)a_m(\theta_i)+n_m(t),$$
$$m=1,2,\ldots,M, \tag{1}$$

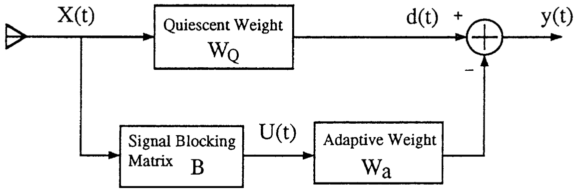


Fig. 1. The generalized sidelobe canceller (GSC) structure. $\mathbf{X}(t)$ denotes the received array data vector.

where $a_m(\theta_i)=\exp(j2\pi d_m \sin\theta_i/\lambda)$ and $d_m$ is the distance between the $m$th and the first array sensors, $s_i(t)$ is the complex waveform of the $i$th signal, and $n_m(t)$ is the spatially white noise with mean zero and variance $\sigma_n^2$ received at the $m$th array sensor. The corresponding data vector received by the array can be written as

$$\mathbf{x}(t)=\mathbf{A}\mathbf{s}(t)+\mathbf{n}(t), \tag{2}$$

where $\mathbf{A}=[\mathbf{a}(\theta_1)\ \mathbf{a}(\theta_2)\cdots\mathbf{a}(\theta_K)]$ with the direction vector of the $i$th signal given by $\mathbf{a}(\theta_i)=[a_1(\theta_i)\ a_2(\theta_i)\cdots a_M(\theta_i)]^{\mathrm{T}}$, the signal source vector is $\mathbf{s}(t)=[s_1(t)\ s_2(t)\cdots s_K(t)]^{\mathrm{T}}$, and the noise vector is $\mathbf{n}(t)=[n_1(t)\ n_2(t)\cdots n_M(t)]^{\mathrm{T}}$. The superscript T denotes the transpose operation. Assume that $\mathbf{s}(t)$ and $\mathbf{n}(t)$ are uncorrelated. Then the ensemble correlation matrix of $\mathbf{x}(t)$ is Toeplitz–Hermitian with size $M\times M$ and given by

$$\begin{aligned}\mathbf{R}_x&=[R_{ij}]=[R(i-j)]\\&=E\{\mathbf{x}(t)\mathbf{x}^{\mathrm{H}}(t)\}=\mathbf{A}\mathbf{S}\mathbf{A}^{\mathrm{H}}+\sigma_n^2\mathbf{I}\\&=\sum_{i=1}^{K}E\{|s_i(t)|^2\}\mathbf{a}(\theta_i)\mathbf{a}^{\mathrm{H}}(\theta_i)+\sigma_n^2\mathbf{I},\end{aligned} \tag{3}$$

where the superscript H denotes the complex conjugate transpose. $\mathbf{S}=E\{\mathbf{s}(t)\mathbf{s}^{\mathrm{H}}(t)\}$ has rank $K$ since the $K$ signals are not jointly correlated, i.e. they do not exhibit coherence among themselves.

Using the GSC structure as shown in Fig. 1, we can realize an adaptive array beamformer with linear constraints. The GSC-based adaptive array beamformer was proposed and shown to be effective in [4]. The operation performed by $\mathbf{B}$ is referred to as the signal blocking operation for removing the desired signals from the received array data. The advantages of the GSC structure are the easy implementation and the assessment of the performance degradation caused by steering or gain errors in array sensors. The output signal $d(t)$ of the upper branch is given by $d(t) = \mathbf{w}_q^H \mathbf{x}(t)$. The quiescent weight vector $\mathbf{w}_q$ is utilized to realize the constrained weight subspace and is chosen such that the output signal power $E[|d(t)|^2]$ is minimized subject to a set of $L$ linear constraints. Accordingly, $\mathbf{w}_q$ can be found from the following minimization problem:

$$\text{Minimize} \quad E[|d(t)|^2]$$
$$\text{Subject to} \quad \mathbf{C}^H \mathbf{w}_q = \mathbf{f}, \tag{4}$$

where $\mathbf{C}$ denotes the constraint matrix with size $M \times L$ and $\mathbf{f}$ is an $L \times 1$ response vector. Assume that the received data are composed of white noise only. Then $\mathbf{w}_q$ is given by

$$\mathbf{w}_q = \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{f}. \tag{5}$$

The sidelobe cancelling branch is utilized for the realization of the unconstrained weight subspace which is complementarily orthogonal to the column space of the constraint matrix $\mathbf{C}$. The adaptive weight vector $\mathbf{w}_a$ can be determined as follows. Since the output signal $y(t) = \mathbf{w}_q^H\mathbf{x}(t) - \mathbf{w}_a^H\mathbf{B}^H\mathbf{x}(t)$, where $\mathbf{B}$ is the so-called signal blocking matrix for blocking the desired signal from $\mathbf{x}(t)$, it follows that $\mathbf{B}$ must be chosen so that $\mathbf{C}^H\mathbf{B} = \mathbf{0}$. In this context, $\mathbf{w}_a$ is the optimal solution for the following minimization problem:

$$\text{Minimize} \quad E[|y(t)|^2]. \tag{6}$$

The solution of (6) can be easily determined as

$$\mathbf{w}_a = (\mathbf{B}^H\mathbf{R_x}\mathbf{B})^{-1}\mathbf{B}^H\mathbf{R}_x\mathbf{w}_q. \tag{7}$$

Accordingly, the overall weight vector for the GSC-based adaptive beamformer is given by $\mathbf{w} = \mathbf{w}_q - \mathbf{B}\mathbf{w}_a$.

## 3. GSC-based adaptive beamforming with MBC

Consider the application in a communication system where a plurality of signals must be received simultaneously. Based on the considered GSC structure, we can utilize an adaptive antenna array which possesses the capability to provide selective gain/null arrangements for different signal beams while suppressing all jammers. Let the ULA use a weight vector $\mathbf{w} = [w_1 \ \ w_2 \cdots w_M]^T$ for processing the received data vector $\mathbf{x}(t)$. Then the signal at the array output is given by $y(t) = \mathbf{w}^H\mathbf{x}(t)$. Assume that the selective gain/null requirements are specified by assigning a gain $c_j$ at the direction vector $\mathbf{a}(\theta_j)$ for $j = 1, 2, \ldots, P$, where $P$ denotes the number of signals with gain/null constraint. Consequently, the problem can be formulated by the following constrained optimization problem:

$$\begin{aligned} &\text{Minimize} \quad E\{|y(t)|^2\} = \mathbf{w}^H\mathbf{R}_x\mathbf{w} \\ &\text{Subject to} \quad \mathbf{G}^H\mathbf{w} = \mathbf{c}, \end{aligned} \tag{8}$$

where $\mathbf{G} = [\mathbf{a}(\theta_1) \ \ \mathbf{a}(\theta_2) \cdots \mathbf{a}(\theta_P)]$ denotes the $M \times P$ steering matrix and $\mathbf{c} = [c_1 \ \ c_2 \cdots c_P]^T$ the corresponding $P \times 1$ gain vector. From the theory of GSC-based adaptive beamforming described above, we can substitute $\mathbf{w} = \mathbf{w}_q - \mathbf{B}\mathbf{w}_a$ into (8) and reformulate (8) as the following equivalent optimization problem:

$$\text{Minimize} \quad (\mathbf{w}_q - \mathbf{B}\mathbf{w}_a)^H\mathbf{R}_x(\mathbf{w}_q - \mathbf{B}\mathbf{w}_a), \tag{9}$$

where the blocking matrix $\mathbf{B}$ must satisfy $\mathbf{G}^H\mathbf{B} = \mathbf{0}$. Then the solution for (9) can be found as follows:

$$\begin{aligned} &\mathbf{w}_q = \mathbf{G}(\mathbf{G}^H\mathbf{G})^{-1}\mathbf{c} \quad \text{and} \\ &\mathbf{w}_a = (\mathbf{B}^H\mathbf{R}_x\mathbf{B})^{-1}\mathbf{B}^H\mathbf{R}_x\mathbf{w}_q. \end{aligned} \tag{10}$$

It is shown by simulations that the proposed GSC-based adaptive array beamformer with MBC does possess the capability of receiving multiple signals as well as suppressing incoherent jammers.

However, the effectiveness of the proposed GSC-based adaptive beamformer will be deteriorated when there exists random perturbation in sensor positions. This is due to the fact that the eigenstructure of the correlation matrix $\mathbf{R}_x$ is destroyed by the random perturbation. To show the effect of random perturbations in sensor positions, let $\hat{\mathbf{u}}_m = [x_m, y_m]$ be the location of the $m$th array sensor as shown in Fig. 2 with

$$\begin{aligned} \hat{\mathbf{u}}_m &= \mathbf{u}_m + \Delta\mathbf{u}_m \\ &= [(m-1)d_x, 0] + [\Delta x_m, \Delta y_m], \end{aligned} \tag{11}$$

where $\mathbf{u}_m = [(m-1)d_x, 0]$ and $\Delta\mathbf{u}_m = [\Delta x_m, \Delta y_m]$, $\Delta x_m$ and $\Delta y_m$ are the position perturbations with the same variance $\sigma_e^2$. The signal received by the $m$th sensor

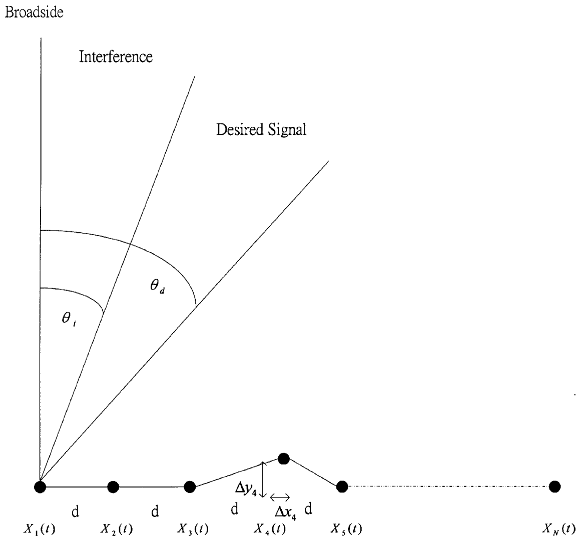


Fig. 2. Geometrical illustration of 1-D array with sensor position errors.

becomes

$$x_m(t) = \sum_{i=1}^{K} s_i(t)\hat{a}_m(\theta_i) + n_m(t),$$
$$m = 1, 2, \ldots, M, \tag{12}$$

where $\hat{a}_m(\theta_i) = \exp(j(2\pi/\lambda)\hat{\mathbf{u}}_m^{\mathrm{T}}\boldsymbol{\Theta}_i)$ and $\boldsymbol{\Theta}_i = [\sin\theta_i, \cos\theta_i]^{\mathrm{T}}$. Then, the resulting correlation matrix $\mathbf{R}_x$ can be expressed as

$$\mathbf{R}_x = E\{\mathbf{x}(t)\mathbf{x}^{\mathrm{H}}(t)\}$$
$$= \sum_{i=1}^{K} E\{|s_i(t)|^2\}\mathbf{P}_i\mathbf{a}(\theta_i)\mathbf{a}^{\mathrm{H}}(\theta_i)\mathbf{P}_i^{\mathrm{H}} + \sigma_n^2\mathbf{I}, \tag{13}$$

where

$$\mathbf{P}_i = \mathrm{diag}\left\{\exp\left(j\,\frac{2\pi}{\lambda}\,\Delta\mathbf{u}_1^{\mathrm{T}}\boldsymbol{\Theta}_i\right),\ \exp\left(j\,\frac{2\pi}{\lambda}\Delta\mathbf{u}_2^{\mathrm{T}}\boldsymbol{\Theta}_i\right),\right.$$
$$\left.\ldots, \exp\left(j\,\frac{2\pi}{\lambda}\,\Delta\mathbf{u}_M^{\mathrm{T}}\boldsymbol{\Theta}_i\right)\right\}. \tag{14}$$

Comparing (3) and (13), we note that the $\mathbf{P}_i$ in (14) represents the effect of sensor position perturbations. Moreover, (14) reveals that this effect depends on the source bearing $\theta_i$ and $[\Delta x_m, \Delta y_m], m = 1, 2, \ldots, M$. We also note from (3) that the correlation matrix $\mathbf{R}_x$ exhibits a Toeplitz structure and has $K$ significant eigenvalues greater than $\sigma_n^2$ when the $K$ signals are uncorrelated. However, the correlation matrix given by (13) loses this Toeplitz structure and its signal subspace is spanned by $\mathbf{P}_i\mathbf{a}(\theta_i)$ instead of $\mathbf{a}(\theta_i)$, $i = 1, 2, \ldots, K$. As a result, the array performance will be degraded in the presence of sensor position perturbations.

## 4. Solution to the random perturbation problem

### 4.1. An iterative matrix reconstruction scheme (IMRS)

To deal with the problem of random position perturbations, we shall consider an appropriate manner for restoring the desired eigenstructure of $\mathbf{R}_x$. After computing the correlation matrix $\mathbf{R}_x$ from (3), a reconstructed $M \times M$ matrix is given by

$$\hat{\mathbf{R}}_x = [\hat{R}_{ij}] = [\hat{R}(i-j)], \tag{15}$$

where

$$\hat{R}(-m) = \frac{1}{M-m}\sum_{i=1}^{M-m} R_{i(i+m)}, \quad 0 \leqslant m < M,$$

$$\hat{R}(m) = \hat{R}^*(-m), \tag{16}$$

where the superscript $*$ represents conjugate operation. (15) reveals that the resulting matrix $\hat{\mathbf{R}}_x$ is also Hermitian with size $M \times M$. In fact, the reconstruction scheme is similar to the Toeplitz approximation approach of [6] which was originally developed for bearing estimation in the coherent source environment.

In general, the reconstructed matrix $\hat{\mathbf{R}}_x$ would not have the desired eigenstructure property that its minimum eigenvalue has a multiplicity of $(M-K)$ unless the array size $M$ is infinite. Therefore, we propose an iterative algorithm to make the reconstructed matrix possess both the Toeplitz–Hermitian and the desired eigenstructure properties. First, the problem of reconstructing the desired eigenstructure from the estimated Toeplitz matrix $\hat{\mathbf{R}}_x$ is solved by performing the following minimization problem:

$$\mathrm{Minimize}_{\tilde{\mathbf{R}}_x \in S_{\mathrm{E}}}\ |\tilde{\mathbf{R}}_x - \hat{\mathbf{R}}_x|, \tag{17}$$

where $S_{\mathrm{E}}$ denotes the set of matrices that their $(M-K)$ smallest eigenvalues are positive and equal. The notation $|\mathbf{Q}|$ used in (17) represents $|\mathbf{Q}| = \left(\sum_{i=1}^{M}\sum_{j=1}^{M}|q_{ij}|^2\right)^{1/2}$ with the $M\times M$ matrix $\mathbf{Q}=[q_{ij}]$. The optimal solution for (17), denoted as $\tilde{\mathbf{R}}_{x\mathrm{o}}$, in the minimum metric distance sense is given by [9]

$$\tilde{\mathbf{R}}_{x\mathrm{o}} = \sum_{k=1}^{K}\lambda_k\mathbf{e}_k\mathbf{e}_k^{\mathrm{H}} + \lambda_{\mathrm{av}}\sum_{k=K+1}^{M}\mathbf{e}_k\mathbf{e}_k^{\mathrm{H}}, \tag{18}$$

where $\lambda_1 \geqslant \lambda_2 \geqslant \cdots \lambda_M$ and $\mathbf{e}_m$, $m=1,2,\ldots,M$, are the eigenvalues and the corresponding eigenvectors of $\hat{\mathbf{R}}_x$, respectively, and $\lambda_{\mathrm{av}}$ is the average of $\lambda_{P+1},\lambda_{P+2},\ldots,\lambda_M$. In practice, it is generally the case that the total number $K$ of signal sources is unknown for adaptive beamforming. Hence, we resort to a suboptimal solution given by

$$\tilde{\mathbf{R}}_{x\mathrm{s}} = \sum_{k=1}^{P}\lambda_k\mathbf{e}_k\mathbf{e}_k^{\mathrm{H}} + \lambda_{\mathrm{av}}\sum_{k=P+1}^{M}\mathbf{e}_k\mathbf{e}_k^{\mathrm{H}} \tag{19}$$

for (17). Moreover, we note that the nonlinear operations performed by (18) and (19) cannot guarantee a resulting matrix with a Toeplitz structure. On the other hand, the previous reconstruction scheme (Eqs. (15) and (16)) for obtaining $\hat{\mathbf{R}}_x$ from $\mathbf{R}_x$ may alter the eigenstructure of a matrix. Therefore, it cannot be ensured that the reconstructed matrix $\tilde{\mathbf{R}}_{x\mathrm{s}}$ possesses both the Toeplitz–Hermitian and the desired eigenstructure properties. However, the goal can be achieved using an iterative algorithm in which operations for obtaining $\hat{\mathbf{R}}_x$ and $\tilde{\mathbf{R}}_{x\mathrm{s}}$ are performed alternatively. Consequently, we summarize the proposed IMRS step by step as follows:

*Step* 1: Estimate $\mathbf{R}_x$ from the received signals. Then let the iteration number $i=0$ and $\tilde{\mathbf{R}}_{x\mathrm{s}}^{(0)}=\mathbf{R}_x$.

*Step* 2: Compute the matrix $\hat{\mathbf{R}}_x^{(i+1)}$ from $\tilde{\mathbf{R}}_{x\mathrm{s}}^{(i)}$ by using the operation of (15).

*Step* 3: Compute the matrix $\tilde{\mathbf{R}}_{x\mathrm{s}}^{(i+1)}$ from $\hat{\mathbf{R}}_x^{(i+1)}$ by using the operation of (19).

*Step* 4: If the matrix norm $|\tilde{\mathbf{R}}_{x\mathrm{s}}^{(i+1)} - \hat{\mathbf{R}}_x^{(i+1)}| > \varepsilon$, where $\varepsilon$ is a preset positive real number, then let $i=i+1$ and go to Step 2. Otherwise, go to the next step.

*Step* 5: Use the $\tilde{\mathbf{R}}_{x\mathrm{s}}^{(i+1)}$ to replace $\mathbf{R}_x$.

Finally, the proof regarding the convergence of the proposed iterative scheme is presented in the Appendix.

### 4.2. Derivative constraints

The concept of derivative constraints is proposed in [1] for dealing with the problem of adaptive beamforming in the presence of steering angle error. In general, steered-beam adaptive arrays are very sensitive to the mismatch between the direction vector of the desired signal and the steering vector [2,7]. By using derivative constraints on the main beam, the adaptive arrays can perform in a satisfactory manner without producing nulls within the main lobe region. From the direction vector of the $i$th signal given by $\mathbf{a}(\theta_i)=[a_1(\theta_i)\ a_2(\theta_i)\cdots a_M(\theta_i)]^{\mathrm{T}}$ with $a_m(\theta_i)=\exp(j2\pi d_m\sin\theta_i/\lambda)$ and $d_m$ equal to the distance between the $m$th and the first array sensors, we let $\phi_i = 2\pi d\sin\theta_i/\lambda$, where $d$ denotes the interelement spacing of the array sensors. With this new parameterization, the direction vector $\mathbf{a}(\theta_i)$ can be re-expressed as

$$\mathbf{a}(\phi_i) = [\exp(j(1-r)\phi_i), \exp(j(2-r)\phi_i), \ldots, \exp(j(M-r)\phi_i)]^{\mathrm{T}}, \tag{20}$$

where $r$ represents the location of the phase origin. Accordingly, the $l$th derivative constraint on the direction vector $\mathbf{a}(\phi_i)$ is implemented by taking the corresponding derivative vector as follows [1]:

$$\begin{aligned}\mathbf{a}_l(\phi_i) &= \left(\frac{\partial}{\partial j\phi_i}\right)^l \mathbf{a}(\phi_i)\\ &= [(1-r)^l\exp(j(1-r)\phi_i), (2-r)^l\\ &\quad\times\exp(j(2-r)\phi_i), \ldots, (M-r)^l\\ &\quad\times\exp(j(M-r)\phi_i)]^{\mathrm{T}}\end{aligned} \tag{21}$$

and setting the product of $\mathbf{w}^{\mathrm{H}}\mathbf{a}_l(\phi_i)$ to a gain value. For example, the gain value is set to zero for providing flatter beam/null response in the direction vector $\mathbf{a}(\phi_i)$ so that the array can possess the robust capabilities against steering errors.

By incorporating the IMRS and the derivative constraints, GSC-based adaptive beamforming with MBC and robustness to random perturbations in array positions can be achieved as follows. First, we perform the IMRS on the computed $\mathbf{R}_x$ to obtain $\tilde{\mathbf{R}}_{x\mathrm{s}}^{(i+1)}$ from Step 5 to replace $\mathbf{R}_x$. Then, a new $M\times P(l+1)$ constraint matrix $\mathbf{G}_{\mathrm{d}}$ is constructed from the steering matrix $\mathbf{G}$ by adding the $l$-order derivative constraints on each of the desired signal directions as follows:

$$\mathbf{G}_{\rm d} = [\mathbf{a}(\phi_1)\ \mathbf{a}(\phi_2) \ldots \mathbf{a}(\phi_P)\ \mathbf{a}_1(\phi_1)\ \mathbf{a}_1(\phi_2) \ldots \mathbf{a}_1(\phi_P)\ \mathbf{a}_2\ (\phi_1)\mathbf{a}_2(\phi_2) \ldots \mathbf{a}_2(\phi_P) \ldots \mathbf{a}_l(\phi_1)\ \mathbf{a}_l(\phi_2) \ldots \mathbf{a}_l(\phi_P)]. \quad (22)$$

Consequently, the $P(l+1) \times 1$ gain vector $\mathbf{c}_{\rm d}$ corresponding to $\mathbf{G}_{\rm d}$ is formed as follows:

$$\mathbf{c}_{\rm d} = [c_1\ c_2 \ldots c_P\ 0\ 0 \ldots 0]^{\rm T}. \quad (23)$$

The required signal blocking matrix $\mathbf{B}_{\rm d}$ must therefore satisfy $\mathbf{C}_{\rm d}^{\rm H}\mathbf{B}_{\rm d} = \mathbf{0}$.

## 5. Simulation results

In this section, several simulation examples performed on a PC with Pentium-IV CPU using Matlab programming language are presented for illustration and comparison. For all simulation examples, we use a ULA with 13 array sensors and the interelement spacing equal to half of the minimum wavelength $\lambda$ of the signals with specified gain/null requirements. There are four signals impinging on the array from $\theta_1$, $\theta_2$, $\theta_3$, and $\theta_4$ degrees, respectively, off array broadside. Moreover, the first two signals are assumed to be the desired signals with gains all equal to one. The other two signals are the jammers. All simulation results presented are obtained by averaging 25 independent runs with independent noise samples for each run. The value of $\varepsilon$ for terminating the iterative process is set to $10^{-6}$. The array performance is evaluated in terms of the resulting beam pattern and output signal-to-interference plus noise ratios (SINR). In practice, the ensemble correlation matrix $\mathbf{R}_x$ is not available. We resort to using the finite sample-size estimate $\bar{\mathbf{R}}_x$ (also called the sample correlation matrix) to replace $\mathbf{R}_x$ and performing the iterative scheme proposed in Section 4 on $\bar{\mathbf{R}}_x$ instead of $\mathbf{R}_x$ for simulations. 6000 data snapshots are used for computing the necessary sample correlation matrices related to the ensemble correlation matrices.

**Example 1.** Here, the desired signals and jammers with $[\theta_1, \theta_2, \theta_3, \theta_4] = [5, 25, 15, 40]$ degrees have a signal-to-noise power ratio (SNR) equal to 5 and 10 dB, respectively. The gain vector $\mathbf{c}$ is set to $[1\ \ 1]^{\rm T}$. Hence, $P$ is set to 2. The variance $\sigma_e^2$ of the random position perturbations is set to $0.05\lambda^2$. Fig. 3 shows the array performance of using the proposed GSC-based adaptive beamformer with MBC under the situations with (shown by the legend of *error*) and without (shown by the legend of *no error*) the random perturbations. Fig. 3 also depicts the array performance of using the proposed GSC-based adaptive beamformer with MBC and the proposed technique to deal with random perturbations (shown by the legend of *proposed*). The corresponding array output SINR for each of the three cases is plotted in Fig. 4. After utilizing 6000 data snapshots, the array output SINRs obtained for the cases without and with random perturbations are 15.77 and 2.59 dB, respectively. The array output SINR becomes 14.85 dB when applying the proposed technique. From these results, we note that the proposed technique can effectively deal with the problem of random position perturbations as well as speed up the convergence behavior of the array response.

**Example 2.** Here, the simulations of Example 1 are repeated except that the SNRs for the desired signals with $[\theta_1, \theta_2] = [-25, 25]$ degrees are set to 3 dB. Both the jammers with $[\theta_3, \theta_4] = [0, 40]$ degrees have SNR equal to 10 dB. The resulting array beam pattern and output SINR for each of the three cases are shown in Figs. 5 and 6, respectively. The array output SINRs obtained after using 6000 data snapshots are 13.32 and 2.69 dB for the cases without and with random perturbations, respectively. By applying the proposed technique, the array output SINR is increased to 13.12 dB. Again, the proposed technique provides effective capability for dealing with the considered problem.

## 6. Conclusion

This paper has presented a technique for adaptive beamforming using the generalized sidelobe canceller (GSC) structure with multiple signal gain/null specifications in addition to jammer

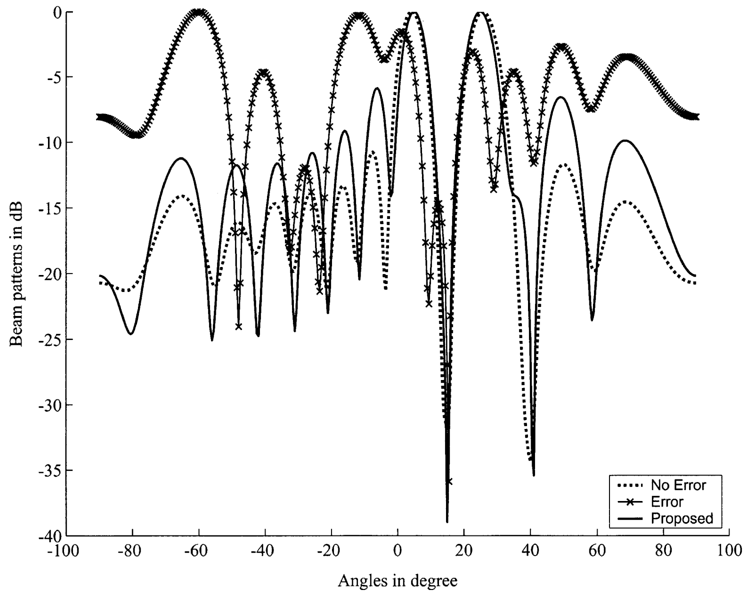


Fig. 3. The resulting array beam patterns for Example 1.

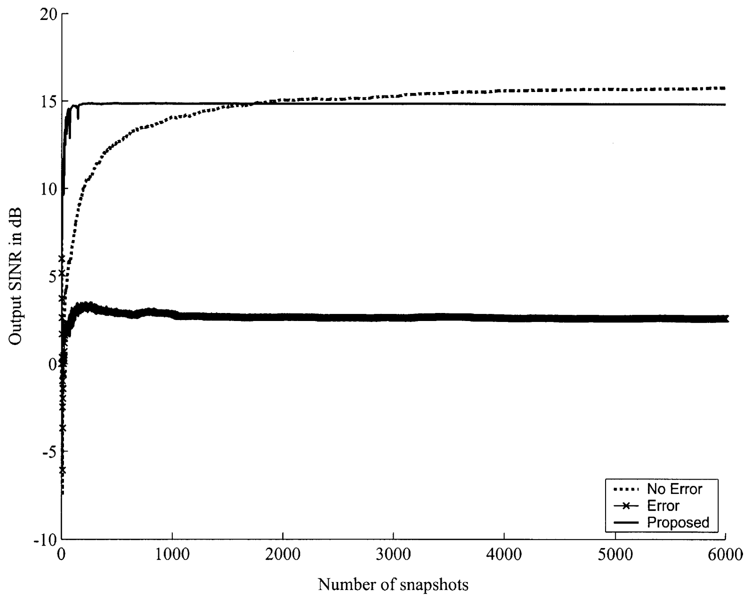


Fig. 4. The corresponding array output SINR versus number of snapshots for Example 1.

suppression. An iterative matrix reconstruction scheme in conjunction with derivative constraints has further been proposed to incorporating with the technique for dealing with the problem due to the random position perturbations of array sensors. The convergence property of the proposed

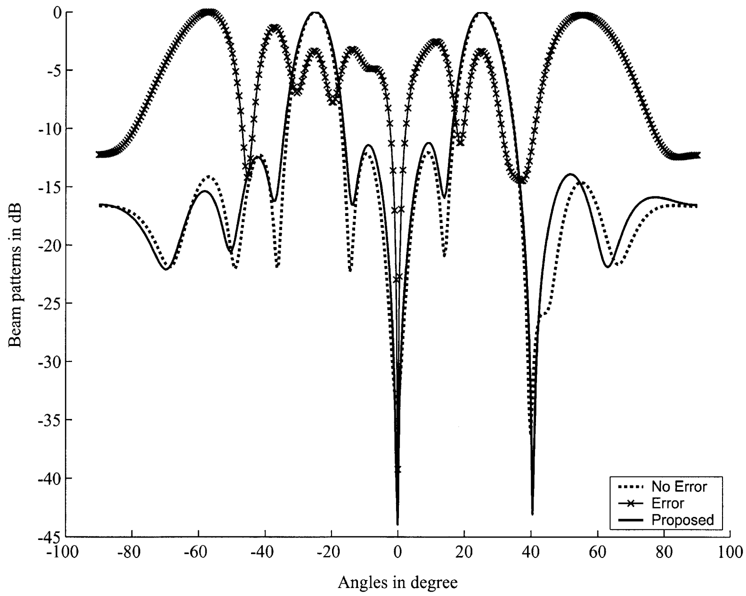


Fig. 5. The resulting array beam patterns for Example 2.

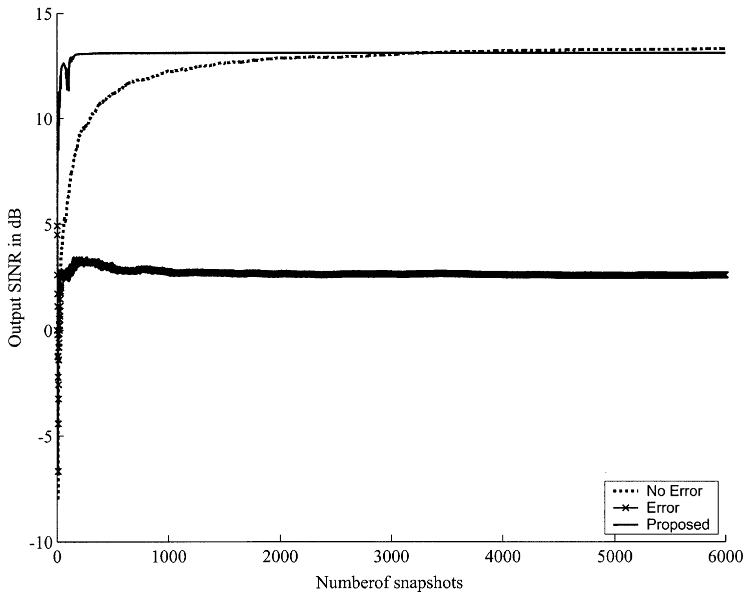


Fig. 6. The corresponding array output SINR versus number of snapshots for Example 2.

iterative scheme has also been provided. Simulation results have shown that the proposed technique can effectively cure the problem of GSC-based adaptive beamforming with multiple-beam constraints when random perturbations in array sensor positions exist.

**Appendix [8]**

Here, we prove the convergence of the IMRS. Given an arbitrary matrix as the initial point, denoted as $\tilde{\mathbf{R}}_{xs}^{(0)}$, the proposed iterative scheme generates a matrix sequence $S_{\mathrm{R}}=\{\tilde{\mathbf{R}}_{xs}^{(0)},\hat{\mathbf{R}}_{x}^{(1)},\tilde{\mathbf{R}}_{xs}^{(1)},\hat{\mathbf{R}}_{x}^{(2)},\ldots,\hat{\mathbf{R}}_{x}^{(i)},\tilde{\mathbf{R}}_{xs}^{(i)},\hat{\mathbf{R}}_{x}^{(i+1)},\tilde{\mathbf{R}}_{xs}^{(i+1)},\ldots\}$ in the following recursive manner: First, obtain $\hat{\mathbf{R}}_{x}^{(i+1)}$ from $\tilde{\mathbf{R}}_{xs}^{(i)}$ by using the operation shown by (15). Second, obtain $\tilde{\mathbf{R}}_{xs}^{(i+1)}$ from $\hat{\mathbf{R}}_{x}^{(i+1)}$ by using the operation shown by (19), for $i=0,1,2,\ldots$. Moreover, we observe that the operations shown by (16) and (19) are norm-reduced and constant trace operations because

$$|\hat{R}(-m)| \leqslant \frac{1}{M-m}\sum_{i=1}^{M-m}|R_{i(i+m)}|, \quad 0\leqslant m<M, \tag{A.1}$$

hence,

$$|\hat{R}(-m)|^2 \leqslant \frac{1}{M-m}\sum_{i=1}^{M-m}|R_{i(i+m)}|^2, \quad 0\leqslant m<M \tag{A.2}$$

and

$$\mathrm{trace}[\hat{\mathbf{R}}_x]=M\hat{R}(0)=\sum_{i=1}^{M}R_{ij}=\mathrm{trace}[\mathbf{R}_x] \tag{A.3}$$

and

$$|\tilde{\mathbf{R}}_{xs}|^2=\sum_{k=1}^{P}\lambda_k^2+(M-P)\lambda_{\mathrm{av}}^2 \leqslant \sum_{k=1}^{M}\lambda_k^2=|\mathbf{R}_x|^2 \tag{A.4}$$

and

$$\mathrm{trace}[\tilde{\mathbf{R}}_{xs}]=\sum_{k=1}^{P}\lambda_k+(M-P)\lambda_{\mathrm{av}} =\sum_{k=1}^{M}\lambda_k=\mathrm{trace}[\mathbf{R}_x]. \tag{A.5}$$

Since $\hat{\mathbf{R}}_{x}^{(i+1)}$ is the optimal solution of (17) when $\mathbf{R}_x=\tilde{\mathbf{R}}_{xs}^{(i)}$, we have

$$|\hat{\mathbf{R}}_{x}^{(i)}-\tilde{\mathbf{R}}_{xs}^{(i)}|\geqslant|\hat{\mathbf{R}}_{x}^{(i+1)}-\tilde{\mathbf{R}}_{xs}^{(i)}|. \tag{A.6}$$

Similarly, $\tilde{\mathbf{R}}_{xs}^{(i+1)}$ is obtained by the norm-reduced operation of (19) when $\mathbf{R}_x=\hat{\mathbf{R}}_{x}^{(i+1)}$, we have

$$|\hat{\mathbf{R}}_{x}^{(i+1)}-\tilde{\mathbf{R}}_{xs}^{(i)}|\geqslant|\hat{\mathbf{R}}_{x}^{(i+1)}-\tilde{\mathbf{R}}_{xs}^{(i+1)}|. \tag{A.7}$$

From (A.6) and (A.7), it follows that

$$|\hat{\mathbf{R}}_{x}^{(i)}-\tilde{\mathbf{R}}_{xs}^{(i)}|\geqslant|\hat{\mathbf{R}}_{x}^{(i+1)}-\tilde{\mathbf{R}}_{xs}^{(i+1)}|. \tag{A.8}$$

Next, define a real nonnegative sequence $\{d_i\}$ as

$$d_i=|\hat{\mathbf{R}}_{x}^{(i)}-\tilde{\mathbf{R}}_{xs}^{(i)}| \tag{A.9}$$

with $i=1,2,\ldots$. From (A.8) and (A.9), we note that the descending sequence $\{d_i\}$ must converge to some nonnegative constant $c$ [11]. If $c=0$, then $\hat{\mathbf{R}}_{x}^{(i)}=\tilde{\mathbf{R}}_{xs}^{(i)}$ as $i$ approaches $\infty$. This leads to the result that the matrix sequence $S_{\mathrm{R}}$ converges. On the other hand, if $c>0$, then we have from (A.6)–(A.9) that

$$\begin{aligned}|\hat{\mathbf{R}}_{x}^{(i)}-\tilde{\mathbf{R}}_{xs}^{(i)}|&=|\hat{\mathbf{R}}_{x}^{(i+1)}-\tilde{\mathbf{R}}_{xs}^{(i)}|\\&=|\hat{\mathbf{R}}_{x}^{(i+1)}-\tilde{\mathbf{R}}_{xs}^{(i+1)}|\end{aligned} \tag{A.10}$$

as $i$ approaches $\infty$. It follows from (A.10) that $\hat{\mathbf{R}}^{(i)}$ and $\hat{\mathbf{R}}^{(i+1)}$ are the solution of (17) when $\mathbf{R}_x=\tilde{\mathbf{R}}_{xs}^{(i)}$. Hence, $\hat{\mathbf{R}}_{x}^{(i)}=\hat{\mathbf{R}}_{x}^{(i+1)}$ since the solution for the minimization problem of (17) is unique. Therefore, the matrix subsequence $\{\hat{\mathbf{R}}_{x}^{(i)}\}$ converges. Similarly, the matrix subsequence $\{\tilde{\mathbf{R}}_{xs}^{(i)}\}$ also converges because $\tilde{\mathbf{R}}_{xs}^{(i)}$ is obtained from (19) when $\mathbf{R}_x=\hat{\mathbf{R}}_{x}^{(i)}=\hat{\mathbf{R}}_{x}^{(i+1)}$ and, hence, $\tilde{\mathbf{R}}_{xs}^{(i)}=\tilde{\mathbf{R}}_{xs}^{(i+1)}$. As a result, we would expect that the two subsequences converge to two different matrices since $c>0$. Moreover, based on the facts that $\tilde{\mathbf{R}}_{xs}^{(i)}\neq\hat{\mathbf{R}}_{x}^{(i+1)}$ and both the operations for obtaining $\hat{\mathbf{R}}_x$ and $\tilde{\mathbf{R}}_{xs}$ are norm-reduced operations, we have

$$|\tilde{\mathbf{R}}_{xs}^{(i)}|>|\hat{\mathbf{R}}_{x}^{(i+1)}|>|\tilde{\mathbf{R}}_{xs}^{(i+1)}|. \tag{A.11}$$

This leads to the result that $|\tilde{\mathbf{R}}_{xs}^{(i)}|>|\tilde{\mathbf{R}}_{xs}^{(i+1)}|$. However, this contradicts the result $\hat{\mathbf{R}}_{x}^{(i)}=\hat{\mathbf{R}}_{x}^{(i+1)}$ obtained from (A.10). Consequently, $c$ must be zero. Both the operations for obtaining $\hat{\mathbf{R}}_x$ and $\tilde{\mathbf{R}}_{xs}$ are constant trace operations; therefore, we find that $|\mathbf{R}_{\mathrm{S}}|>0$ for any $\mathbf{R}_{\mathrm{S}}\in S_{\mathrm{R}}$. Therefore, the proposed iterative matrix reconstruction scheme will not converge to the trivial solution, i.e. the null matrix. This completes the necessary proof.

## References


[1] K.M. Buckley, L.J. Griffiths, An adaptive generalized sidelobe canceller with derivative constraints, IEEE Trans. Antennas Propagat. 34 (March 1986) 311–319.

[2] R.T. Compton Jr., The effect of random steering vector errors in the Applebaum adaptive array, IEEE Trans. Aerospace Electronic Systems 18 (May 1982) 392–400.

[3] O.L. Frost, An algorithm for constrained adaptive array processing, Proc. IEEE 60 (August 1972) 926–935.

[4] L.J. Griffiths, C.W. Jim, An alternative approach for linearly constrained adaptive beamforming, IEEE Trans. Antennas Propagat. 30 (January 1982) 27–34.

[5] N.K. Jablon, Adaptive beamforming with the generalized sidelobe canceller in the presence of array imperfection, IEEE Trans. Antennas Propagat. 34 (August 1986) 996–1012.

[6] S.Y. Kung, C.K. Lo, R. Foka, A Toeplitz approximation approach to coherent source direction finding, Proceedings of the ICASSP, Tokyo, Japan, April 1986, pp. 193–196.

[7] C.-C. Lee, J.-H. Lee, Robust adaptive array beamforming under steering vector errors, IEEE Trans. Antennas Propagat. 45 (January 1997) 168–175.

[8] J.-H. Lee, T.-F. Hsu, Adaptive beamforming with multiple-beam constraints in the presence of coherent jammers, Signal Process. 80 (May 2000) 2475–2480.

[9] M. Max, T. Kailath, Detection of signals by information theoretic criteria, IEEE Trans. Acoust. Speech Signal Process. 33 (April 1985) 387–392.

[10] J.T. Mayhan, Area coverage adaptive nulling from geosynchronous satellites: phased arrays versus multiple-beam antennas, IEEE Trans. Antennas Propagat. 34 (March 1986) 410–419.

[11] M.H. Protter, C.B. Morrey, A First Course in Real Analysis, Springer, New York, 1977.

[12] A.M. Rao, D.L. Jones, Efficient signal detection in perturbed arrays, Proceedings of the tenth IEEE Workshop on Statistical Signal and Array Processing, Polono Manor, PA, 2000, pp. 429–433.

[13] A.M. Rao, D.L. Jones, Efficient quadratic detection in perturbed arrays via Fourier transform techniques, IEEE Trans. Signal Process. 49 (July 2001) 1269–1281.

[14] C.-M.S. See, B.-K. Poh, Parametric sensor array calibration using measured steering vectors of uncertain locations, IEEE Trans. Signal Process. 47 (April 1999) 1133–1137.

[15] W. Su, H. Gu, J. Ni, G. Liu, Performance analysis of MVDR algorithm in the presence of amplitude and phase errors, IEEE Trans. Antennas Propagat. 49 (December 2001) 1875–1877.

[16] K. Takao, K. Uchida, Beamspace partially adaptive antenna, IEE Proc., Part H, Microwaves Antennas Propagat. 136 (December 1989) 439–444.

[17] B.D. Van Veen, Eigenstructure based partially adaptive array design, IEEE Trans. Antennas Propagat. 36 (March 1988) 357–362.

[18] B.D. Van Veen, R.A. Roberts, Partially adaptive beamformer design via output power minimization, IEEE Trans. Acoust. Speech Signal Process. 35 (November 1987) 1524–1532.

[19] K.-B. Yu, Adaptive beamforming for satellite communication with selective earth coverage and jammer nulling capability, IEEE Trans. Signal Process. 44 (December 1996) 3162–3166.

[20] S.-J. Yu, J.-H. Lee, Design of partially adaptive beamformer based on information theoretical criteria, IEEE Trans. Antennas Propagat. 42 (May 1994) 676–689.

[21] S.-J. Yu, J.-H. Lee, Effect of random weight errors on the performance of partially adaptive array beamformers, Signal Processing 37 (1994) 365–380.