Comparison of Nonlinear Phase Noise and Intrachannel Four-Wave Mixing for RZ-DPSK Signals in Dispersive Transmission Systems

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***Abstract*—Self-phase-modulation-induced nonlinear phase noise is reduced with the increase of fiber dispersion but intrachannel four-wave mixing (IFWM) is increased with dispersion. Both degrading differential phase-shift keying signals, the standard deviation of nonlinear phase-noise-induced differential phase is about three times that from IFWM even in highly dispersive transmission systems.**

***Index Terms*—Differential phase-shift keying (DPSK), fiber nonlinearities, intrachannel four-wave mixing (IFWM), nonlinear phase noise.**

I. Introduction

RECENTLY, the differential phase-shift keying (DPSK) signal has been studied extensively for long-haul lightwave transmissions [1]–[4]. Mostly for 40-Gb/s systems, the DPSK signal has 3-dB receiver sensitivity improvement and provides better tolerance to fiber nonlinearities than ON–OFF keying. Most DPSK experiments use return-to-zero (RZ) pulse with phase modulated to each pulse.

The interaction of fiber Kerr effect with amplifier noise induces nonlinear phase noise [5]–[8], or more precisely, self-phase modulation (SPM)-induced nonlinear phase noise. Added directly to the signal phase, as shown later, nonlinear phase noise is the major degradation for DPSK signals.

When RZ pulses broaden by chromatic dispersion and overlap with each other, the pulse-to-pulse interaction gives intrachannel cross-phase modulation (IXPM) and four-wave-mixing (IFWM) [9], [10]. While IXPM from the signal does not degrade DPSK signals, IFWM adds ghost pulses to each DPSK RZ pulse [11]–[14].

For lossless fiber, both [15] and [16] studied nonlinear phase noise with chromatic dispersion for continuous-wave signal or soliton. For RZ-DPSK signals, the variance of the nonlinear phase noise is derived here analytically, to our knowledge, the first time. Comparing with the IFWM variance from [13] and [14], the phase noise standard deviation (STD) from nonlinear phase noise is about three times larger than that from IFWM even at highly dispersive transmission systems.

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II. Nonlinear Phase Noise for RZ Pulses

For a comparison to IFWM, nonlinear phase noise is evaluated based on the model of [11]–[14]. Assumed a Gaussian pulse with an initial $1/e$ pulsewidth of T_0, the kth pulse along the fiber is

$$u_k(z,t) = \frac{A_k T_0}{(T_0^2 - j\beta_2 z)^{1/2}} \exp\left[-\frac{(t-kT)^2}{2(T_0^2 - j\beta_2 z)}\right] \qquad (1)$$

where $A_k = \pm A_0$ is the pulse amplitude modulated by either 0 or π phases, β_2 is the coefficient of group-velocity dispersion, and T is the bit interval. The constant factor of fiber loss is ignored in (1) but includes afterward. Due to the fiber Kerr effect, from the model of [11] and [12], there is a nonlinear force of $j\gamma u_k u_l u_m^*$ from the overlap of the kth, lth, and mth pulses, where γ is the fiber nonlinear coefficient. The overall ghost pulse is equal to

$$j\gamma \int_0^L [u_k(z,t) u_l(z,t) u_m^*(z,t)] \otimes h_{-z}(t) e^{-\alpha z} dz \qquad (2)$$

where $\otimes$ denotes convolution, L is the fiber length, and α is the fiber attenuation coefficient. The impulse response of $h_{-z}(t)$ provides dispersion compensation for $h_z(t)$, where $h_z(t)$ is the impulse response for fiber chromatic dispersion with frequency response of $H_z(\omega) = \exp(j\beta_2 z \omega^2/2)$.

To be consistent with the model for IFWM of (2), for the pulse of $u_0(z,t)$, the SPM-induced nonlinear force including amplifier noise of $n(z,t)$ is equal to

$$j\gamma[u_0(z,t) + n(z,t)] \left|u_0(z,t) + n(z,t)\right|^2. \qquad (3)$$

For the signal, the nonlinear force is $j\gamma u_0 |u_0|^2$ or that of (2) with $k = l = m = 0$. The nonlinear force associated with nonlinear phase noise has two different terms of

$$2j\gamma |u_0(z,t)|^2 n(z,t) \quad \text{and} \quad j\gamma u_0^2(z,t) n^*(z,t) \qquad (4)$$

when all quadratic or higher order terms of the noise are ignored. For $2j\gamma|u_0(z,t)|^2 n(z,t)$, the nonlinear force corresponding to (2) is equal to

$$\Delta u_n(t) = 2j\gamma \int_0^L \left[|u_0(z,t)|^2 n(z,t)\right] \otimes h_{-z}(t) e^{-\alpha z} dz. \qquad (5)$$

At the input of the fiber, we assume that $E\{n(0,t+\tau) n^*(0,t)\} = 2\sigma_n^2 \delta(\tau)$ as a white noise, where σ_n^2 is the noise density per dimension. With fiber dispersion, $n(z,t) = n(0,t) \otimes h_z(t)$, $E\{n(z_1, t+\tau) n^*(z_2, t)\}$ has a Fourier transform of $2\sigma_n^2 e^{j\beta_2(z_1 - z_2)\omega^2/2}$. The temporal profile

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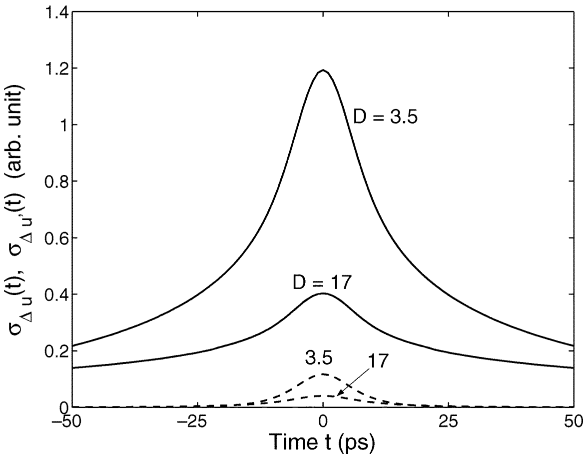

Fig. 1. Temporal distribution of nonlinear force due to the beating of signal with noise. The solid lines are $\sigma_{\Delta u_n}(t)$ and the dashed lines are $\sigma_{\Delta u'_n}(t)$.

of $\Delta u_n(t)$ can be represented by the variance of $\Delta u_n(t)$ as a function of time. Taking into account the noise dependence, with some algebra, we find that

$$\sigma^2_{\Delta u_n}(t) = E\left\{|\Delta u_n(t)|^2\right\} = \frac{4\gamma^2\sigma_n^2 T_0^2 A_0^4}{\pi} \times \int_{-\infty}^{+\infty} \left| \int_0^L \frac{\exp\left(-\frac{t^2+j\tau(z)^2\omega t+\beta_2^2 z^2\omega^2}{\tau(z)^2-2j\beta_2 z} - \alpha z\right)}{\sqrt{\tau(z)^2-2j\beta_2 z}} dz \right|^2 d\omega \quad (6)$$

where $\tau(z) = \sqrt{T_0^2 + \beta_2^2 z^2/T_0^2}$ is the pulsewidth of (1). Similarly, the variance profile corresponding to $j\gamma u_0^2(z,t)n^*(z,t)$ is

$$\sigma^2_{\Delta u'_n}(t) = \frac{\gamma^2\sigma_n^2 T_0^2 A_0^4}{\pi} \times \int_{-\infty}^{+\infty} \left| \int_0^L \frac{\exp\left[-\frac{(t-\beta_2 z\omega)(t-jT_0^2\omega)}{T_0^2+j\beta_2 z} - \alpha z\right]}{\sqrt{T_0^4+\beta_2^2 z^2}} dz \right|^2 d\omega. \quad (7)$$

Fig. 1 shows the temporal profile, both the STD of $\sigma_{\Delta u_n}(t)$ and $\sigma_{\Delta u'_n}(t)$ for typical fiber dispersion coefficients of $D = 17$ and 3.5 ps/km/nm. The initial launched pulse has an $1/e$ width of $T_0 = 5$ ps. The fiber link is $L = 100$ km with attenuation coefficient of $\alpha = 0.2$ dB/km. Fig. 1 shows that the nonlinear force of $\Delta u_n(t)$ due to the beating of $|u_0(z,t)|^2$ with $n(z,t)$ is far larger than that of $\Delta u'_n(t)$ due to the beating of $u^2(z,t)$ with $n^*(z,t)$. In terms of power, $\sigma^2_{\Delta u_n{}'}(t)$ is about 1% of $\sigma^2_{\Delta u_n}(t)$. The noise term of $\Delta u_n(t)$ also has more spreading over time than $\Delta u'_n(t)$. The beating of $u^2(z,t)$ with $n^*(z,t)$ can be ignored. Relatively, the spreading at large dispersion is larger than that in small dispersion.

The temporal profile of Fig. 1 is not able to estimate the dependence between the nonlinear phase noise at $t = 0$ and, for example, $t = T$, directly. As a trivial example for signals without chromatic dispersion and pulse distortion, the nonlinear force is proportional to $|u_0(0,t)|^2 n(0,t)$. As white noise, the noises of $n(0,t)$ at $t = 0$ and $t = T$ are independent of each other. In this trivial case, the profile corresponding to Fig. 1 is proportional to $|u_0(0,t)|^2$ but approaches infinity.

If the nonlinear force of $\Delta u_n(t)$ is passing through an optical filter with an impulse response of $h_o(t)$, the filter output at the time of mT is

$$\zeta_{0,m} = \int_{-\infty}^{+\infty} h_o(mT - t)\Delta u_n(t)dt. \quad (8)$$

The SPM phase noise from $\zeta_{0,0}$ is the noise generated by the beating of $|u_0(z,t)|^2$ with $n(z,t)$ and affect the DPSK pulse at $t = 0$. The term of $\zeta_{0,1}$ is IXPM phase noise from the beating of $|u_0(z,t)|^2$ with $n(z,t)$ and affect the DPSK pulse at $t = T$. Unlike deterministic SPM and IXPM from signal-to-signal beating, both SPM and IXPM phase noises degrade DPSK signals. Other than location, in general, $\zeta_{k,m}$ is statistically the same as $\zeta_{0,m-k}$. The term of $\zeta_{k,k}$ gives SPM phase noise and other terms of $\zeta_{k,m}$ with $k \neq m$ generates IXPM phase noise.

Followed the model of [13] and [14], the differential nonlinear phase noise from both SPM and IXPM phase noise is

$$\delta\phi_n = \frac{1}{A_0}\Im\left\{\sum_m \zeta_{m,0}\right\} - \frac{1}{A_1}\Im\left\{\sum_m \zeta_{m,1}\right\} \quad (9)$$

where $\Im\{\cdot\}$ denotes the imaginary part of a complex number. To give the output of A_0 and A_1, using the simplest assumption of Gaussian optical filter, the frequency response is $H_o(\omega) = \sqrt{1 + t_o^2/T_0^2}\exp(-t_o^2\omega^2/2)$, where t_o is the $1/e$ width of the impulse response of $h_o(t)$.

For simplicity, $A_0 = A_1$ is assumed for the same transmitted phase in consecutive symbols. Using the property that the real and imaginary parts of $\zeta_{k,m}$ are independent and identically distributed, the variance of $\delta\phi_n$ is

$$\sigma^2_{\delta\phi_n} = \frac{1}{A_0^2}\sum_{m_1}\sum_{m_2}\left(E\left\{\zeta_{m_1,0}\zeta^*_{m_2,0}\right\} - E\left\{\zeta_{m_1,0}\zeta^*_{m_2,1}\right\}\right). \quad (10)$$

Derived a function of $f_m(\omega)$ as

$$f_m(\omega) = 2\gamma|A_0|^2(T_0^2 + t_o^2)^{1/2} \times \int_0^L \frac{\exp\left\{-\frac{1}{2}t_o^2\omega^2 + \frac{\left[(t_o^2 - j\beta_2 z)\omega + jmT\right]^2}{\tau(z)^2 - 2j\beta_2 z + 2t_o^2} - \alpha z\right\}}{\sqrt{\tau(z)^2 - 2j\beta_2 z + 2t_o^2}} dz \quad (11)$$

we obtain

$$E\left\{\zeta_{m_1,0}\zeta^*_{m_2,0}\right\} = \frac{\sigma_n^2}{\pi}\int_{-\infty}^{+\infty} f_{m_1}(\omega)f^*_{m_2}(\omega)d\omega \quad (12)$$

and

$$E\left\{\zeta_{m_1,0}\zeta^*_{m_2,1}\right\} = \frac{\sigma_n^2}{\pi}\int_{-\infty}^{+\infty} f_{m_1}(\omega)f^*_{m_2-1}(\omega)e^{j\omega T}d\omega. \quad (13)$$

For an N-span system, the amplifier noise at the first span is the smallest and that in the last span is the largest. From [5] and [17], for large number of fiber spans with the identical span repeated one after another, the overall phase noise variance is $\sigma^2_{\Delta\phi_n} \approx N^3\sigma^2_{\delta\phi_n}/3$. The energy per pulse is $\sqrt{\pi}T_0|A_0|^2$ with a signal-to-noise ratio (SNR) of $\rho_s = \sqrt{\pi}T_0|A_0|^2/(2N\sigma_n^2)$. The mean nonlinear phase shift is $\langle\Phi_{\rm NL}\rangle = N\gamma L_{\rm eff}P_0 = N\gamma\sqrt{\pi}L_{\rm eff}|A_0|^2T_0/T$, where P_0 is the launched power and $L_{\rm eff} = (1 - e^{-\alpha L})/\alpha$ is the effective fiber length. The variance

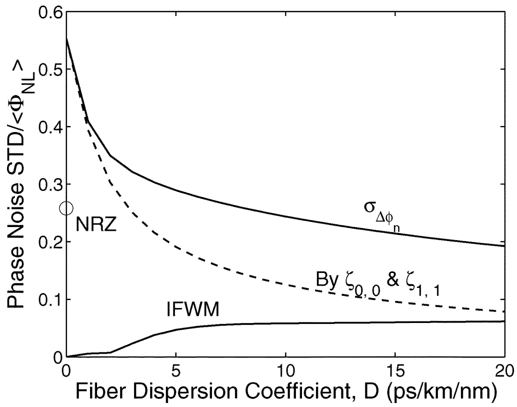

Fig. 2. Phase noise STD due to nonlinear phase noise and IFWM. The dashed line is SPM phase noise from $\zeta_{0,0}$ and $\zeta_{1,1}$ alone.

of nonlinear phase noise of $\sigma^2_{\Delta\phi_n}$ is proportional to $\langle\Phi_{\rm NL}\rangle^2/\rho_s$, similar to that in [5] and [17].

Fig. 2 shows the phase noise STD of $\sigma_{\Delta\phi_n}$ as a function of the fiber dispersion coefficient of the fiber link. For an SNR of $\rho_s = 20$ (13 dB), the STD of $\sigma_{\Delta\phi_n}$ is normalized with respect to the mean nonlinear phase shift of $\langle\Phi_{\rm NL}\rangle$. As in Fig. 1 with $T_0 = 5$ ps, Fig. 2 further assumes 40-Gb/s systems with $T = 25$ ps and an optical matched filter of $t_o = 5$ ps because larger bandwidth increases the nonlinear phase noise. Fig. 2 also shows the corresponding phase STD due to IFWM calculated by the method of [14]. For the optical matched filter, the STD from IFWM scales up by a factor of $\sqrt{3/2} = 1.22$ for the $\sqrt{3}$ times increase in the width of IFWM ghost pulse [11], [12], [14]. Fig. 2 also includes the corresponding result for nonreturn-to-zero (NRZ) signal at $D = 0$ [7], [17].

The IFWM-induced ghost pulses give a phase noise variance increase with fiber dispersion. With large fiber dispersion and significant pulse overlap, more terms induce ghost pulses and the overall contribution from IFWM increases slowly with fiber dispersion. From (1), fiber dispersion reduces the pulse amplitude but the increase of number of terms balances that out. For $D > 7$ ps/km/nm, the contribution from IFWM increases slowly with the increase of fiber dispersion.

The STD from nonlinear phase noise of $\sigma_{\Delta\phi_n}$ reduces with fiber dispersion. Even with large fiber dispersion, $\sigma_{\Delta\phi_n}$ from nonlinear phase noise is about three times larger than that from IFWM. Fig. 2 also shows the STD of $\Delta\phi_n$ with contribution from only SPM phase noise of $\zeta_{0,0}$ and $\zeta_{1,1}$. Ignored in [16], the contribution from IXPM phase noise of $\zeta_{m,k}, m \neq k$ at large dispersion is larger than that from SPM phase noise of $\zeta_{k,k}$. With an interesting implication, the STD of $\sigma_{\Delta\phi_n}$ closes to that for NRZ signal at large dispersion. The results of [7] and [8] are approximately correct for RZ pulses for systems with large dispersion.

Fig. 2 is equivalently for N identical fiber spans with $\langle\Phi_{\rm NL}\rangle = 1$ rad. For arbitrary link configuration, the integration of (11) can be replaced by N integrations for each span. Fig. 2 also assumes an optical matched filter. The function of $f_m(\omega)$ is valid for general Gaussian optical filter, but other filter types are possible, may require another layer of integration.

The nonlinear forces of (4) ignore all higher order terms, from [7] and [17], the noise-to-noise beating increases the phase-noise variance by the factor of $1/(4\rho_s)$ or 1.25% for Fig. 2.

III. Conclusion

The variance of nonlinear phase noise is derived analytically for RZ-DPSK signals in highly dispersive transmission systems. For an initial pulsewidth of $T_0 = 5$ ps, the phase noise STD from nonlinear phase noise is about three times larger than that from IFWM at large fiber dispersion of $D = 17$ ps/km/nm. Nonlinear phase noise typically degrades a DPSK signal more than IFWM ghost pulses.

References

[1] A. H. Gnauck *et al.*, "2.5 Tb/s (64 × 42.7 Gb/s) transmission over 40 × 100 km NZDSF using RZ-DPSK format and all-Raman-amplified spans," in *Optical Fiber Communication Conf. (OFC 2002)*, Anaheim, CA, 2002, Postdeadline paper FC2.

[2] C. Rasmussen *et al.*, "DWDM 40 G transmission over trans-Pacific distance (10 000 km) using CSRZ-DPSK, enhanced FEC and all-Raman amplified 100 km Ultra-Wave fiber spans," in *Optical Fiber Communication Conf. (OFC 2003)*, Atlanta, GA, 2003, Postdeadline paper PD18.

[3] J.-X. Cai *et al.*, "RZ-DPSK field trial over 13 100 km of installed non slope-matched submarine fibers," in *Optical Fiber Communication Conf. (OFC 2004)*, Anaheim, CA, 2004, Postdeadline paper PDP34.

[4] G. Charlet *et al.*, "WDM bit-to-bit alternate-polarization RZ-DPSK transmission at 40 × 42.7 Gbit/s over transpacific distance with large Q-factor margin," in *Eur. Conf. Optical Communication (ECOC 2004)*, Stockholm, Sweden, 2004, Postdeadline paper Th4.4.5.

[5] J. P. Gordon and L. F. Mollenauer, "Phase noise in photonic communications systems using linear amplifiers," *Opt. Lett.*, vol. 15, pp. 1351–1353, 1990.

[6] H. Kim and A. H. Gnauck, "Experimental investigation of the performance limitation of DPSK systems due to nonlinear phase noise," *IEEE Photon. Technol. Lett.*, vol. 15, no. 2, pp. 320–322, Feb. 2003.

[7] K.-P. Ho, "Impact of nonlinear phase noise to DPSK signals: A comparison of different models," *IEEE Photon. Technol. Lett.*, vol. 16, no. 5, pp. 1403–1405, May 2004.

[8] ——, "Statistical properties of nonlinear phase noise," in *Advances in Optics and Laser Research*, W. T. Arkin, Ed. Hauppauge, NY: Nova Science, 2003, vol. 3.

[9] I. Shake, H. Takara, K. Mori, S. Kawanishi, and Y. Yamabayashi, "Influence of inter-bit four-wave mixing in optical TDM transmission," *Electron. Lett.*, vol. 34, pp. 1600–1601, 1998.

[10] R.-J. Essiambre, B. Mikkelsen, and G. Raybon, "Intra-channel cross-phase modulation and four-wave mixing in high-speed TDM systems," *Electron. Lett.*, vol. 35, pp. 1576–1578, 1999.

[11] A. Mecozzi, C. B. Clausen, and M. Shtaif, "Analysis of intrachannel nonlinear effects in highly dispersed optical pulse transmission," *IEEE Photon. Technol. Lett.*, vol. 12, no. 4, pp. 292–294, Apr. 2000.

[12] A. Mecozzi, C. B. Clausen, M. Shtaif, S.-G. Park, and A. H. Gnauck, "Cancellation of timing and amplitude jitter in symmetric links using highly dispersed pulses," *IEEE Photon. Technol. Lett.*, vol. 13, no. 5, pp. 445–447, May 2001.

[13] X. Wei and X. Liu, "Analysis of intrachannel four-wave mixing in differential phase-shift keying transmission with large dispersion," *Opt. Lett.*, vol. 28, pp. 2300–2302, 2003.

[14] K.-P. Ho, "Error probability of DPSK signals with intrachannel four-wave-mixing in highly dispersive transmission systems," *IEEE Photon. Technol. Lett.*, vol. 17, no. 4, pp. 789–791, Apr. 2005.

[15] A. G. Green, P. P. Mitra, and L. G. L. Wegener, "Effect of chromatic dispersion on nonlinear phase noise," *Opt. Lett.*, vol. 28, pp. 2455–2457, 2003.

[16] C. J. McKinstrie and C. Xie, "Phase jitter in single-channel soliton systems with constant dispersion," *IEEE J. Sel. Topics Quantum Electron.*, vol. 8, no. 3, pp. 616–625, May/Jun. 2002.

[17] K.-P. Ho and J. M. Kahn, "Electronic compensation technique to mitigate nonlinear phase noise," *J. Lightw. Technol.*, vol. 22, no. 3, pp. 779–783, Mar. 2004.