OPTICAL SWITCHING OF A NONLINEAR SLAB IN TOTAL INTERNAL REFLECTION STATE

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A study of a plane wave obliquely incident on a nonlinear slab of positive Kerr coefficient in total internal reflection (TIR) state is reported. Various bistable and multi-valued transmissions are investigated as the incident power varies. Especially, the mechanism of the resonant transmission is analyzed in detail, and an explanation of the invariance phenomenon in the first jump positions is proposed. Finally, a method for designing good bistable switchings is given.

1. Introduction

Nonlinear interface problems have received much attention recently because they are potentially useful as ultra-fast optical switches and logic elements, and the operating power can be greatly reduced [1,2]. A series of theoretical works on nonlinear interfaces in total internal reflection (TIR) state were published. Typical examples are the plane-wave theory [3] and the study of 2D gaussian beams under slowly-varying envelope approximation [4,5], which all predict the switching behavior of the transmittivity. Recently, Chen and Mills [6] have discussed the normal incidence of a plane wave on a nonlinear slab, in which bistable and resonant transmissions were found. The bistable and multi-stable phenomena have found applications in the fields such as optical switching [7,8], optical circuit, and optical memory [9], which are essential to the development of optical communication and optical computers [10].

In this paper, a plane wave obliquely incident on a nonlinear slab of positive Kerr coefficient in TIR state is studied. In section 2, general results will be presented and multi-valued actions of transmittivity will be discussed. In section 3, the existence and characteristics of the resonant transmission will be examined and formulae for calculating the resonant transmission directly will also be derived. In section 4, a unique phenomenon of the TIR state, invariance of the first jump position under the slab-length's variation, will be pointed out and computed results for the first jump will also be compared with the previous results for a nonlinear interface [3,4].

2. Mathematical formulation and general results

Let a $\hat{y}$-polarized plane wave

$$E_{\rm inc} = \hat{y} E_0 \exp[-ik_1(x\cos\psi + z\sin\psi)]$$

be obliquely incident on a nonlinear slab as shown in fig. 1, where d is the slab length and ψ is the incident angle. The symbol $\psi_{\rm cr} = \cos^{-1}(\sqrt{\epsilon_3/\epsilon_1})$ denotes the TIR angle of the interface between two linear media ϵ_1 and ϵ_3. Express the transmitted wave and the wave in the slab as

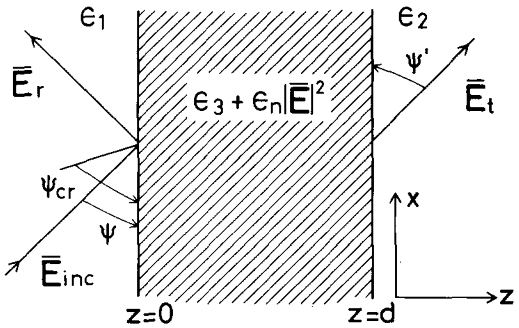

Fig. 1. Geometry of the nonlinear slab.

$$E_t = \hat{y} T E_0 \exp[-ik_2(x\cos\psi' + z\sin\psi')] ,$$

$$E = \hat{y} E_0 e(z) \exp[i\phi(z)] \exp(-ik_1 x\cos\psi) .$$

Then by the following the process of ref. [6], one has the relations for the wave fields

$$\int_{I(d')}^{I(z')} (AI + \delta^2 I^2 - \tfrac{1}{2}N - I^3 - W^2)^{-1/2}\, dI = \pm 2(d' - z') , \quad (1)$$

$$I(d') = W/(n_2 \sin\psi') = |T|^2 , \quad (2)$$

$$\tfrac{1}{2}N I^2(0) - (\delta^2 + n_1^2 \sin^2\psi)\; I(0) + [4n_1^2 \sin^2\psi + \delta^2 W/(n_2\sin\psi') - 2n_1 W\sin\psi - n_2 W \sin\psi' - NW^2/(2n_2^2\sin^2\psi')] = 0 . \quad (3)$$

Here new variables and parameters are introduced: $k_i \equiv \omega\sqrt{\mu_0\epsilon_i}$ $(i=0,\ 1,\ 2,\ 3)$, $z' \equiv k_0 z$, $d' \equiv k_0 d$, $n_i \equiv \sqrt{\epsilon_i/\epsilon_0}$, $\delta^2 n_1^2 \cos^2\psi - n_3^2$, $I(z') \equiv e^2(z')$, $N \equiv \epsilon_n E_0^2/\epsilon_0$, $\psi' \equiv \cos^{-1}(k_1 \cos\psi/k_2)$, while A and W are integration constants. Basically, only the relationship between the transmittivity $|T|^2$ and N is needed. From (1), $I(z')$ may be expressed in terms of jacobian elliptic functions [11] and (3) is the resultant nonlinear equation for W, which can be solved by the damped Newton method [12]. A nonlinear slab with positive coefficient in TIR state means $\epsilon_n > 0$ and $\delta^2 > 0$, so $I(z')$ can be expressed as

(i) $N \leqslant n_2^2 \sin^2\psi' + \delta^2$,

$$I(z') = \frac{I_1(I_2 - I_3) + I_3(I_1 - I_2)\mathrm{cn}^2(\cdot)}{(I_2 - I_3) + (I_1 - I_2)\mathrm{cn}^2(\cdot)}, \quad 0 \leqslant z' \leqslant d' ,$$

$$\mathrm{cn}(\cdot) \equiv \mathrm{cn}\left[(\tfrac{1}{2}N)^{1/2}(I_1 - I_3)^{1/2}(d' - z') \middle| \frac{I_1 - I_2}{I_1 - I_3} \right] . \quad (4a)$$

(ii) $N > n_2^2 \sin^2\psi' + \delta^2$,

$$I(z') = I_1 + (I_2 - I_1)\mathrm{cn}^2(\cdot), \quad 0 \leqslant z' \leqslant d' ,$$

$$\mathrm{cn}(\cdot) \equiv \mathrm{cn}\left[(\tfrac{1}{2}N)^{1/2}(I_2 - I_3)^{1/2}(d' - z') \middle| \frac{I_2 - I_1}{I_2 - I_3} \right] . \quad (4b)$$

In both cases,

$$I_1 \equiv N^{-1}\left\{ \left(\delta^2 - \frac{NW}{2n_2 \sin\omega'}\right) + \left[\left(\delta^2 - \frac{NW}{2n_2\sin\omega'}\right)^2 + 2n_2 NW \sin\omega' \right]^{1/2} \right\} ,$$

$$I_2 \equiv W/(n_2 \sin\omega') ,$$

$$I_3 \equiv N^{-1}\left\{ \left(\delta^2 - \frac{NW}{2n_2 \sin\omega'}\right) - \left[\left(\delta^2 - \frac{NW}{2n_2\sin\omega'}\right)^2 + 2n_2 NW \sin\omega' \right]^{1/2} \right\} .$$

Eq. (3) is solved for several cases, as illustrated in figs. 2–5. It should be remembered that if ϵ_n is fixed, then the variation of N is equivalent to the variation of incident optical power. Figs. 2 and 3 show typical examples for multi-valued transmissions. Note that resonant transmissions ($|T|^2 = 1$) occur at C, D, E, F (fig. 2) which also show periodicity and symmetry in field distributions (fig. 3). If N increases from zero, then a sudden switching of transmittivity takes place when passing through the first jump as defined in fig. 2. Certainly, there are more jumps if N is further increased. The first jump is emphasized here because it represents the transition from surface waves to oscillatory waves, as indicated by A–C in fig. 3. These important characteristics will be examined rigorously in later sections.

To gain a better insight to the multi-valued trans-

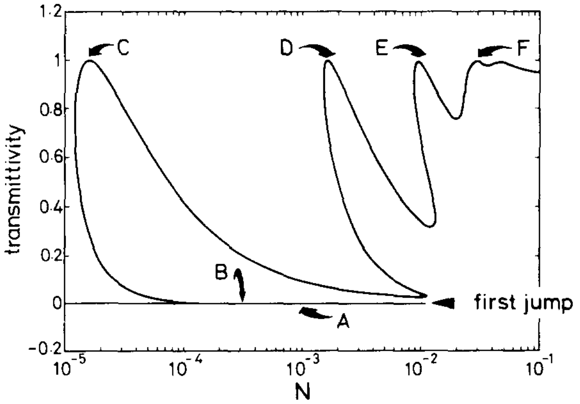

Fig. 2. Transmittivity $|T|^2$ versus N ($=\epsilon_n E_3^2/\epsilon_0$). $\epsilon_1 = \epsilon_2 = 1.93\,\epsilon_0$, $\epsilon_3 = 1.88\,\epsilon_0$, $d = 10\,\lambda_0$ (λ_0: free-space wavelength), $\psi = 7°$, $\psi_{cr} = 9.23°$.

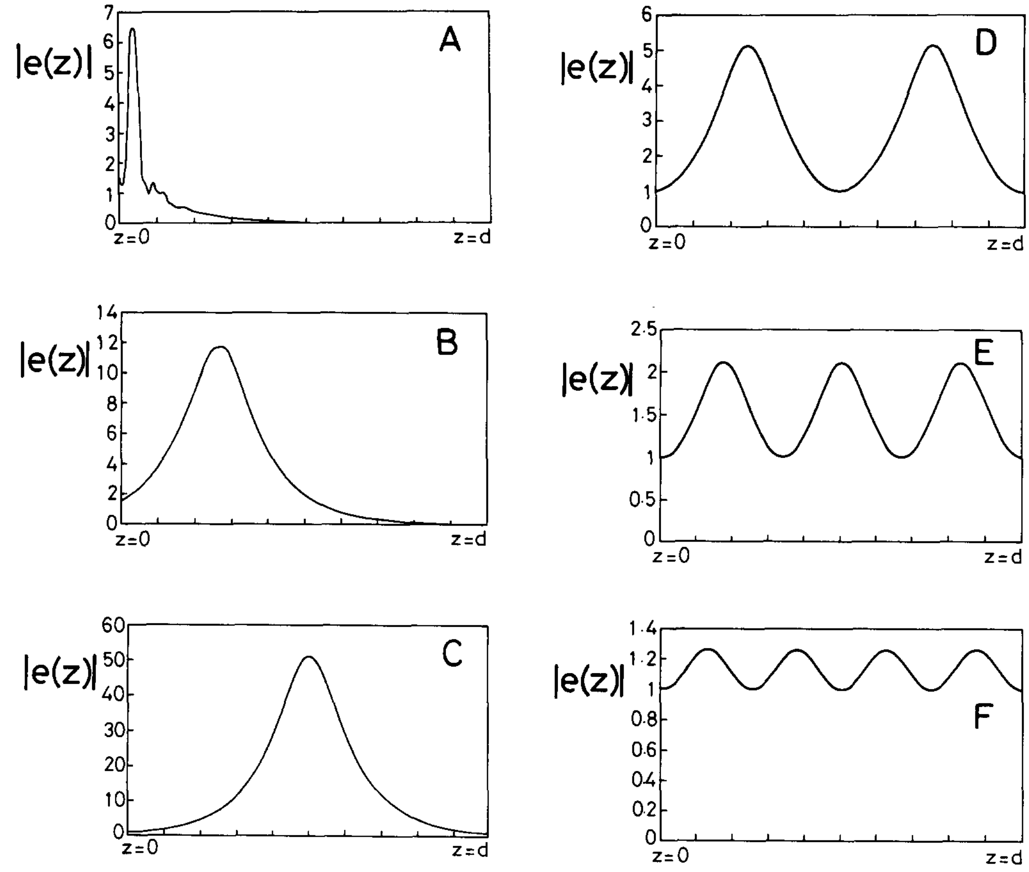

Fig. 3. Field-amplitude distributions $|e(z)|$ in the slab corresponding to the states defined in fig. 2.

missions, the cases with different slab-lengths and incident angles are considered, respectively, in fig. 4 and fig. 5. The multi-valued actions become notable as the slab-length increases or the incident angle decreases. In figs. 4 and 5, bistable transmission curves are also observed, which are very useful in optical switching. As will be shown in section 4, good bistable curves can be obtained in general situation. (Here good bistable curves mean the ones with very great contrast for switching within a narrow bistable region.) Note that the above conclusions are not limited to small incident angles which must be assumed in refs. [3–5]. As for numerical results, only symmetric configurations ($\epsilon_1=\epsilon_2$) are presented.

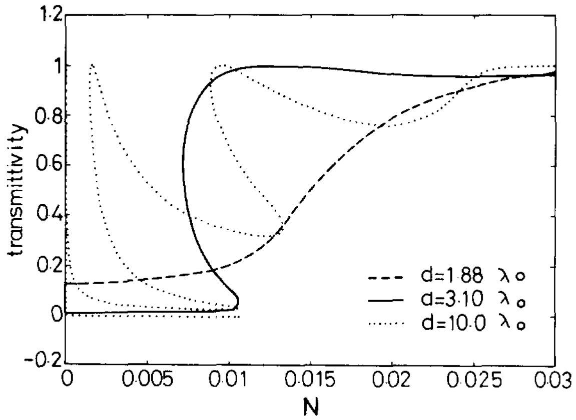

Fig. 4. Transmittivity $|T|^2$ versus N for three values of d. $\epsilon_1=\epsilon_2=1.93\,\epsilon_0$, $\epsilon_3=1.88\,\epsilon_0$, $\psi=7°$, $\psi_{cr}=9.23°$.

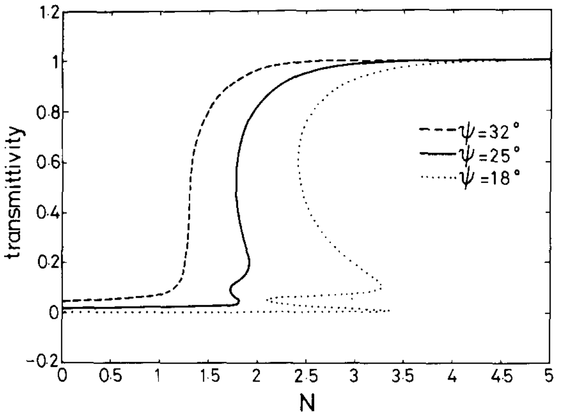

Fig. 5. Transmittivity $|T|^2$ versus N for three values of ψ. $\epsilon_1=\epsilon_2=10.1\,\epsilon_0$, $\epsilon_3=5\,\epsilon_0$, $d=0.243\,\lambda_0$, $\psi_{cr}=45.3°$.

3. Analysis of resonant transmissions

First, the necessary condition for the existence of resonant transmissions is derived. Putting $I(0)=1$ and $|T|^2=1$ in (3) gives

$$\epsilon_1=\epsilon_2\,, \tag{5}$$

which is the desired condition. Thus resonant transmissions never occur in asymmetric configurations. Next, with $\epsilon_1=\epsilon_2$, $I(0)=1$, and $|T|^2=2$ be substituted into (4), one has $\mathrm{cn}(\cdot)^2=1$. Then by the periodic nature of the jacobian cosine function $\mathrm{cn}(\cdot)$, one may derive the desired formulae for resonant transmissions,

(i) $N\leqslant n_2^2\sin^2\psi'+\delta^2$,

$$(\tfrac{1}{2}N)^{1/2}(I_1-I_3)^{1/2}d'=2nK\left(\frac{I_1-1}{I_1-I_3}\right)$$

$$n=1,2,3,\ldots,M\,. \tag{6a}$$

(ii) $N>n_2^2\sin^2\psi'+\delta^2$,

$$(\tfrac{1}{2}N)^{1/2}(1-I_3)^{1/2}d'=2nK\left(\frac{1-I_1}{1-I_3}\right),$$

$$n=(M+1),(M+2),(M+3),\ldots\,. \tag{6b}$$

Here $K(\cdot)$ is the elliptic integral of the first kind. Obviously, the periodicity and symmetry of the field distributions are simply the results of $\mathrm{cn}(\cdot)$. Calculations have also proved that eq. (6) is consistent with the numerical results computed from eq. (3). Fig. 3 is a typical example where C, D, E, F correspond to $n=1, 2, 3, 4$ in eq. (6). Further comparisons with ref. [6] show that the resonant transmission here is of different nature.

4. Analysis of the first jump

The first jump is defined, in fig. 2, as the state where the first switching action takes place when N increases from zero. The first jump position, occurring at $N=N_0$, has the intuitive meaning that, if $\psi_{tir}\equiv\sin^{-1}(\sqrt{(\epsilon_3+\epsilon_0N)/\epsilon_1})$, then N_0 is approximately the value of N for $\psi_{tir}=\psi$. Then it is found that the position of the first jump is almost invariant under variation of the slab length, an important phenomenon which can also be concluded from fig. 4. This phenomenon may be explained by the following consideration. Specifically, the wave field is essentially confined to the region around the nonlinear interface before the first jump (refer to fig. 6 for illustration), which is similar to the well-known concept of TIR in the linear theory. Thus the first jump should mainly depend on the physical condition of the interface at $z=0$ and is not influenced by the thickness of the slab. The wave field is not confined to the interface immediately after the first jump, which is also shown in fig. 6.

By the invariance argument, our first jump result for a nonlinear slab problem can be used to compute the one for a nonlinear interface problem merely by letting the slab-length approach infinity. The plane-

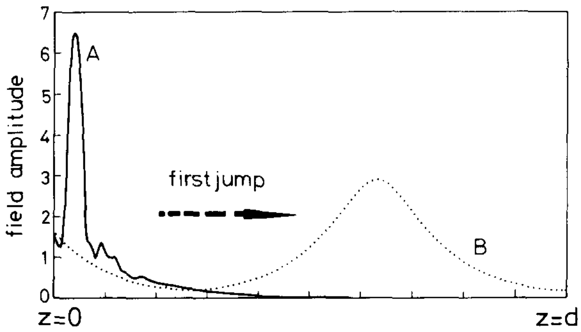

Fig. 6. Field-amplitude distributions $|e(z)|$ in the slab just before and after the first jump in fig. 2.

Table 1
Positions of first jump from three different approaches.

ψ	Our results	Ref. [3]	Ref. [4]
5°	0.0210	0.0212	0.0263
7°	0.0110	0.0106	0.0124
8°	0.0061	0.0062	

wave theory of a nonlinear interface was considered in ref. [3] and a 2D gaussian beam incident on a nonlinear interface was discussed in ref. [4]. Comparison of the positions of the first jump for three cases is given in table 1, where good agreement is observed. The mechanisms for multi-valued actions and resonant transmissions are different for these three cases by noting the different boundary conditions at infinity. But the first jump positions are mainly determined by the physical condition at the interface. This again explains the good agreement in the first jump positions for these three cases.

From the above consideration, a method for obtaining good bistable switchings is then proposed by selecting

$$\text{(position of first jump)}$$

$$\cong \text{(position of first resonant transmission)}\,,$$

where the first resonant transmission is calculated by eq. (6). Typical examples are $d=3.10\,\lambda_0$ in fig. 4 and $\psi=25°$ in fig. 5.

5. Conclusions

Two unique phenomena associated with TIR state, the resonant transmission and the first jump, have been analyzed in detail. A simple method has also been proposed for designing good bistable switchings. From a sufficient criterion for instability [13],

$$\mathrm{d}|T|^2/\mathrm{d}N<0 \quad \text{and} \quad |\mathrm{d}|T|^2/\mathrm{d}N|>|T|^2/N\,, \qquad (7)$$

it is easy to verify that the proposed bistable transmission curves have only two stable states [14], which means practical bistable switchings can exist. The stability problem of the multi-valued transmissions (fig. 2) is difficult to analyze and chaotic results of stability are expected. Applications of the multi-valued transmissions, such as multi-valued logic [15], are possible after the stability problem of the multi-valued transmissions is investigated in detail.

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