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使用指向式天線之展頻無線通訊網路之最佳化設計
研究成果報告(精簡版)

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中文摘要：

在本計畫中，我們提出一個結合展頻通信技術與指向式天線技術的新世代無線網路。在此計畫中，我們以理論分析的方式來分析此新世代無線網路之效能。為了盡可能地切合實際的無線傳輸環境，我們所使用的無線通道模型包含了路徑衰減（Path Loss）、遮蔽效應（Shadowing Effect）、瑞雷衰落（Rayleigh fading）等常見之無線信號傳輸效應。在此實際的通道模型下，我們分析使用直接序列展頻技術（Direct-Sequence Spread Spectrum）以及跳頻展頻技術（Frequency-Hopped Spread Spectrum）之無線網路在任意的指向式天線之波束型態（Beam Pattern）下的系統效能。接著我們應用此理論分析，為此無線網路發展出一套系統參數設定之最佳化方法。我們可應用此方法尋找最佳化此無線網路傳輸量（Throughput）之使用者密度（Mobile Density）等系統參數。

目前在市面上已有眾多使用指向式天線之無線區域網路之產品，另外在無線都會網路的標準（WiMAX）制定中也已採用指向式天線技術。可以預見的，指向式天線技術與今日所廣泛採用之展頻技術的結合將是未來新世代無線網路技術的主要趨勢之一。本計畫可提供分析此類新世代網路所需之理論依據，並可廣泛的應用於此新世代無線網路之系統參數設定，以最佳化其系統效能。

關鍵詞：指向式天線、展頻、分碼多工存取、直接序列展頻、跳頻展頻、效能分析、路徑衰減、遮蔽、衰落、波束型態、最佳化、傳輸量、使用者密度、碼率。

英文摘要：

In this project, we consider a novel wireless network that combines spread spectrum/ CDMA communications and directional antenna technology. We have analyzed the system performance of a wireless network that applies both spread spectrum and directional antenna technologies. In particular, we have analyzed the performance of the wireless network using directional antennas with an antenna beam pattern of arbitrary shape under a realistic channel model which includes the effects of path loss, shadowing, and Rayleigh fading. The performance of both direct-sequence spread spectrum and frequency-hopped spread spectrum networks has been analyzed. The performance measure considered is the outage probability. The system throughput of the wireless network based on the outage probability analysis has also been derived. After analyzing the performance of the wireless networks, we have proposed a methodology to optimize the system design of the wireless networks. To show the effectiveness of the methodology, we have applied it to find the optimal mobile density that achieves the best system throughput.

Currently there are already commercial products that apply directional antenna technology to wireless LAN. Directional antenna technology is also adopted by the standardization of the next generation wireless MAN (WiMAX). The combination of the already widely accepted spread spectrum communications and the directional antenna technology will be an inevitable trend in the new generations of wireless networking. The research of this project provides the theoretical cornerstone as well as the system design optimization guideline for the new generations of wireless network.

關鍵詞：Directional antenna, spread spectrum, CDMA, direct-sequence spread spectrum, frequency-hopped spread spectrum, performance analysis, path loss, shadowing, Rayleigh fading, optimization, throughput, mobile density, code rate.

報告內容：

本計畫之主要執行成果已投稿至 *IEEE Transactions on Vehicular Technology*，第二版刻正審核中。英文版計畫成果報告內容（含前言、研究目的、文獻探討、研究方法、結果與討論、參考文獻）茲詳列如後頁所示。

Performance Analysis of CDMA Wireless Networks with Directional Antennas

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1 Introduction

The demand for wireless communication has been growing rapidly in the past two decades. As multimedia technology evolves, more and more users require larger bandwidth to support high data rate services such as exchanging multimedia files over wireless networks. However, wireless spectrum is limited and thus strictly regulated for various wireless communication systems. Any wireless network can only operate using a finite bandwidth. As a result, the number of users that can be accommodated by a wireless network, usually denoted as the user capacity, is restricted unless some other technology is applied.

Spread Spectrum/Code Division Multiple Access (CDMA) is one technology that can improve the user capacity. The development of CDMA dates from 1940s for US military communications. Later in the 1970s and 1980s, there are growing commercial interest of the technology. In the late 1980s and 1990s, Qualcomm developed and proposed a CDMA system. The system was later adopted by the Telecommunications Industry Association as Interim Standard 95 and began commercial operation in North America in 1996. One major advantage of CDMA compared to other multiple accessing schemes such as Time Division Multiple Access (TDMA) and Frequency Division Multiple Access (FDMA) is its great spectrum efficiency which can provide higher user capacity. CDMA is also robust against time fading due to the large bandwidth resulted from the spreading of the spectrum. As a result, CDMA technology is widely adopted in 3G systems such as Wideband CDMA (W-CDMA) and cdma2000.

Although CDMA technology can effectively increase the user capacity, it is still limited by the interference between the users. Ideally the interference between the users should be completely eliminated by the orthogonality of the spreading codes used by different users in the same cell. Nevertheless, due to the imperfect synchronization of the users and multipath fading, the orthogonality among the users can not be guaranteed. This results in the interference between the users and is called inter-user interference. The user capacity of a wireless network is greatly affected by the inter-user interference. In a wireless network, when a mobile transmits signal to another mobile, it also causes interference to the other mobiles in the same network. In particular, most systems use omnidirectional antennas; while the receiving mobile is located in a particular direction from the transmitting mobile, the omnidirectional antenna transmits signal to all directions with the same amount of power. This makes the interference problem even worse. To achieve a certain communication quality, the interference experienced in each communication link can not be too big. The larger the number of active mobiles in the network, the larger the interference experienced by each mobile. Therefore inter-user interference is the major obstacle of achieving high user capacity in a CDMA system. It is desirable to develop some technology which can reduce the inter-user interference effectively and thus improve the user capacity of CDMA systems to a even greater extent.

Recently, directional antennas have been considered to be a promising technology to reduce inter-user interference and improve the power efficiency for wireless networks [1, 2]. The concept of directional antennas is not new. It was applied in military radar systems during World War II. However, not until recent years did people start to consider applying directional antennas in modern wireless communications. There are several reasons for this. One is that the demand of wireless communications did not grow that rapidly until recent years, hence people did not worry about the user capacity problem that much. Another reason is that an advanced directional antenna needs to track the location of the target mobiles and interferers, and adaptively form its beam to achieve near optimal communication quality. This requires a huge amount of signal processing and computation complexity. The task was too demanding for the processors in the past, which were either too slow or too expensive to be applied. Nowadays we have many fast DSP processors and chips that are reasonably priced. This has led many to reconsider the use of directional antennas in wireless communications during the recent years [2].

By using directional antennas, both transmitting and receiving nodes have the ability to generate beam patterns with high transmission gain and reception gain in the direction of each other, and low gain elsewhere. The enhanced power efficiency enables to extend the battery life of a mobile which is crucial for wireless communication devices. The interference problem is also significantly reduced because now only nodes with receiving antennas pointing at the main beam of a transmitting node will be significantly interfered by the transmitting node. With the interference problem lightened, the system can have a higher user capacity than before.

Although it is promising to apply directional antenna technology to wireless communications, there are still many things that need to be considered to optimize the performance in different applications. For instance, if we apply directional antennas to a CDMA wireless network, there are system parameters of the network that need to be optimized to get the best system performance, such as the mobile density of the network or the code rate of the channel coding used. In order to optimally determine the system parameters, it is necessary for us to know how a system will perform under different settings of the system parameters. This is the major motivation of our project. We want to understand exactly how well does a CDMA wireless network with directional antennas perform and how to optimize the system design for such network.

In the literature, a few papers [3–6] have attempted to analyze the performance of wireless networks with directional antennas. However, the beam patterns of the antennas used in these papers were either omnidirectional or very simplified. These results can not be applied to analyze the system performance when a realistic beam pattern is considered. This motivates us to find an approach to analyze the network performance for an arbitrary beam pattern in a wireless channel. The major goal of our project is to analyze the system performance of a CDMA wireless network with directional antennas for arbitrary beam patterns over realistic channel models, and we want to apply the performance analysis to develop a methodology for the system design optimization of the CDMA wireless network. This will enable us to set the system parameter judiciously to optimize the system performance of the CDMA wireless network.

In this report, we first derive the outage probability of conventional wireless networks using arbitrary beam pattern under a realistic channel model, which includes the effects of path loss, shadowing, and Rayleigh fading. Slotted ALOHA [7] is used for the MAC protocol. By combining with performance analysis of bit-interleaved coded modulation and turbo codes in [8, 9], we can evaluate the performance of a wireless network with specific coding and modulation used in the physical layer. Later the analysis of conventional systems is generalized to find the outage probability of direct-sequence spread spectrum (DSSS) and frequency-hopped spread spectrum (FHSS) systems. The outage probability analysis is then applied to find the system throughput and the transport capacity [10] of the wireless networks. Using this as an example, we show that we can find the optimal system parameters for any system performance that can be characterized by the outage probability. This enables system designers to find the optimal configuration for such wireless networks. By the way, we prove that the network transport capacity of a wireless network using directional antennas is of order $\Theta(\sqrt{\lambda_M})$, where λ_M stands for the node density of the network. The finding conforms with the well known capacity of wireless networks derived by Gupta and Kumar in [10].

The remainder of this report is organized as follows. In Section 2, a brief introduction to the system model and the channel model is given. In Section 3, we derive the outage probability of a conventional wireless network using a simplified beam pattern. The analysis is then generalized to the case of an arbitrary beam pattern. In Section 4, we analyze the outage probability of DSSS and FHSS networks. In Section 5, the network throughput and transport capacity are both derived. In Section 6, numerical examples are given. Finally, we address the conclusions in Section 6.

2 Model Description

Consider a wireless network as shown in Fig. 1. We are interested in the performance of the communication link between two reference nodes Tx (transmitting node) and Rx (receiving node). We first assume all nodes use directional antennas for transmission and omnidirectional antennas for reception. Later we will generalize the analysis to the case where directional antennas are used for both transmission and reception. We assume all nodes use the same transmitting beam pattern. A typical beam pattern $g(\theta)$ is shown in Fig. 2, where $g(\theta)$ denotes the power gain of the transmitting antenna relative to an omnidirectional antenna in the direction of θ . For an omnidirectional antenna, $g(\theta) = 1$ for any θ. We make no assumption on how the directional antennas are formed, they can be either mechanically steered or electronically formed. The antennas are assumed to be devoid of any mutual correlation. Finally it is assumed that all nodes have the perfect knowledge of their own channels.

We use the same channel model as in [3], shown in Fig. 3. Assume that the distance between Tx and Rx is r_0, and Rx is in the direction of θ_0 from Tx. When Tx transmits power P_T, the received power at Rx is then $P_0 = P_T g(\theta_0)\, \zeta_0\, \xi_0^2$, where ζ_0 accounts for path loss and shadowing effect which is constant for at least a packet duration T. It is log-normal distributed with conditional probability density function (pdf)

$$f(\zeta_0 \mid r_0) = \frac{1}{\sqrt{2\pi}\sigma\zeta_0}\, e^{-\frac{1}{2}\frac{\left(\log\zeta_0 - \log(K r_0^{-\eta})\right)^2}{\sigma^2}}, \tag{1}$$

where $K r_0^{-\eta}$ denotes the average attenuation level due to path loss. The constant K denotes the average attenuation level measured at a reference distance from the transmitting node, and η is called path loss exponent which ranges from 2.0 to 6.0 depending on the type of channel [11]. In an urban area, η is around 2.7 to 3.5. In an obstructed environment or in buildings, where signal suffers more path loss, η is around 4.0 to 6.0. Also note that σ is usually denoted in dB. Typical values for σ are 5 to 10 dB.

The fading level ξ_0 is a random variable that accounts for the Rayleigh fading. If ξ_0 is Rayleigh distributed, then ξ_0^2 is exponentially distributed [12]. Hence, the conditional pdf of P_0 given ζ_0 is

$$f_0(P_0 \mid \zeta_0) = \frac{1}{P_T g(\theta_0)\zeta_0}\, e^{-\frac{P_0}{P_T g(\theta_0)\zeta_0}}, \tag{2}$$

and the conditional cumulative density function (CDF) is

$$F_0(P_0 \mid \zeta_0) = 1 - e^{-\frac{P_0}{P_T g(\theta_0)\zeta_0}}. \tag{3}$$

The channel between any interfering node and Rx follows the same model described above, except that the distance can be different. Also the signal from each interfering node to Rx is independently shadowed and independently Rayleigh faded. We also assume the signal power from an interfering node is constant throughout the packet duration.

The nodes are assumed to be randomly distributed on the two dimensional plane as a Poisson point process. Let λ_M denote the average node density within a circle of radius 1. For any disc of radius r on the plane, the number of nodes N_r in the disc has a Poisson distribution POI($\lambda_M r^2$), i.e., $Pr\{N_r = k\} = \frac{e^{-\lambda_M r^2}(\lambda_M r^2)^k}{k!}$. We assume slotted ALOHA is used for the MAC protocol. All nodes are synchronized and every packet is transmitted at the beginning of the time slot right after the packet is generated. Each time slot is of duration T which is the same as the packet duration. Each node generates data packets which is modeled by a Bernoulli process of rate p, i.e., packets are generated with probability p in each slot. Hence, given that there are N_r nodes in the network (excluding Tx and Rx), the number of nodes that actually interfere with Rx is randomly distributed as BIN(N_r, p).

3 Outage Probability Analysis of Conventional Wireless Networks

In this section, we consider a conventional wireless network without using CDMA. We analyze the performance of the link between Tx and Rx by deriving the outage probability of the link under slow fading. By slow fading we mean that the fading level of the targeted link between Tx and Rx is constant for at least one packet duration. This is usually the case when Tx, Rx, and the environment are all of low mobility. We first derive the outage probability for a simplified beam pattern of Tx as shown in Fig. 4 in Section 3.1. Then the analysis is generalized for any arbitrary shape of beam patterns used by Tx and Rx in Section 3.2. In this part of analysis, we assume that Tx is always perfectly aligned to Rx, i.e., $\theta = 0$ always points to the direction of Rx. The outage probability analysis is later generalized to CDMA cases in Section 4.

3.1 Simplified Beam Pattern

First consider the simplified beam pattern shown in Fig. 4. The beam pattern has L beams. Beam i, $i = 1, 2, \cdots, L$, has angular beam width δ_i (rad) and gain g_i with respect to the omnidirectional antenna of the same transmitting power. Without loss of generality, beam 1 is always the beam with the largest gain and is always pointing to Rx. The outage probability of the link between Tx and Rx is defined by

$$\phi_{L,r_0}(b) = P\{\text{Outage}\} \triangleq P\left\{\frac{P_0}{P_I + N} < b\right\}, \tag{4}$$

where P_0 is the received power, P_I the interference power, and N the noise power at Rx. If we denote the energy per symbol received at Rx (from Tx) by E_s and symbol duration by T_s, it is easy to see that $P_0 = E_s/T_s$ and $N = N_0/T_s$, where N_0 denotes the noise power density. It is assumed that the outage occurs whenever the signal-to-interference-and-noise ratio (SINR) at Rx is less than a certain threshold b which causes the failure of the channel coding and modulation to maintain the small packet error rate (PER) required by the data transmission. Hence, the SINR threshold b depends on the coding and modulation used by Tx. In a slow faded channel, the SINR should remain constant during the whole packet duration. When we determine the value of b for a specific coding and modulation scheme, we should refer to the performance of the coding and modulation scheme in an AWGN channel. The analyses in [8,9] enable us to plot the PER versus signal-to-noise ratio (SNR) for various coded modulation schemes and turbo codes in AWGN and different fading channels. If we model the interference as noise, we can determine the value of b easily from the PER analysis.

The key concept of computing $\phi_{L,r_0}(b)$ is to compute $\phi_{L,r_0}(b,a) = P\left\{\frac{P_0}{P_I(a)+N} < b\right\}$ first, where $P_I(a)$ only includes the interference power from those interferers located within a distance a from Rx. After taking the limit as a goes to infinity, we can obtain the outage probability $\phi_{L,r_0}(b) = \lim_{a\to\infty} \phi_{L,r_0}(b,a)$. Consider the nodes in the disc of radius a centered at Rx (Tx excluded). We classify them into L groups. The nodes in the i^{th} group all point at Rx with their beam i. Let $\boldsymbol{K_a} \triangleq (K_1, K_2, \cdots, K_L)$ denote the number of nodes in each group. The number of nodes in a disc of radius a has a Poisson distribution POI($\lambda_M a^2$). If we assume the direction of any node to its receiving node is uniformly distributed between 0 and 2π, then the probability of a node pointing at Rx with beam i is $\frac{\delta_i}{2\pi}$, which is proportional to the angular beam width δ_i. This implies K_i has a Poisson distribution POI($\frac{\lambda_M a^2 \delta_i}{2\pi}$). Since K_is' are independent,

$$P(\boldsymbol{K_a}) = \prod_{i=1}^{L} \frac{e^{\frac{-\lambda_M a^2 \delta_i}{2\pi}} (\frac{\lambda_M a^2 \delta_i}{2\pi})^{K_i}}{K_i!}. \tag{5}$$

Among the K_i nodes in the i^{th} group, the number of nodes that actually transmit packets and thus interfere with Rx, I_i, is of distribution BIN(K_i, p). Define $\boldsymbol{I_a} \triangleq (I_1, I_2, \cdots, I_L)$. Then

$$P(\boldsymbol{I_a} \mid \boldsymbol{K_a}) = \prod_{i=1}^{L} \binom{K_i}{I_i} p^{I_i} (1-p)^{K_i - I_i}. \tag{6}$$

To analyze the outage probability, number those I_i interferers in the i^{th} group from 1 to I_i. The distance from interferer j in group i to Rx is denoted by $r_{i,j}$. The pdf of $r_{i,j}$ given the interferer is in the disc of radius a centered at Rx is of the form

$$f_a(r_{i,j}) = \frac{2\pi r_{i,j}}{\pi a^2} = \frac{2 r_{i,j}}{a^2}. \tag{7}$$

For later use, we define vector $\boldsymbol{r_a} \triangleq [\boldsymbol{r_1}, \boldsymbol{r_2}, \cdots, \boldsymbol{r_L}]$, where $\boldsymbol{r_i} = (r_{i,1}, r_{i,2}, \cdots, r_{i,I_i})$, $i = 1, 2, \cdots, L$.

The path loss and shadowing at Rx experienced by the interference from interferer j in group i is denoted by $\zeta_{i,j}$. From (1), we have

$$f(\zeta_{i,j} \mid r_{i,j}) = \frac{1}{\sqrt{2\pi}\sigma \zeta_{i,j}} \; e^{-\frac{1}{2} \frac{\left(\log \zeta_{i,j} - \log(K r_{i,j}^{-\eta})\right)^2}{\sigma^2}}. \tag{8}$$

Define vector $\boldsymbol{\zeta_a} \triangleq [\boldsymbol{\zeta_1}, \boldsymbol{\zeta_2}, \cdots, \boldsymbol{\zeta_L}]$, where $\boldsymbol{\zeta_i} = (\zeta_{i,1}, \zeta_{i,2}, \cdots, \zeta_{i,I_i})$, $i = 1, 2, \cdots, L$.

The received interference power at Rx from interferer j in group i is denoted by $P_{i,j}$. The conditional distribution of $P_{i,j}$ given $\zeta_{i,j}$ is exponentially distributed as described in Section 2. The conditional pdf is

$$f(P_{i,j} \mid \zeta_{i,j}) = \frac{1}{P_T g_i \zeta_{i,j}} \; e^{-\frac{P_{i,j}}{P_T g_i \zeta_{i,j}}}. \tag{9}$$

Define vector $\boldsymbol{P_a} \triangleq [\boldsymbol{P_1}, \boldsymbol{P_2}, \cdots, \boldsymbol{P_L}]$, where $\boldsymbol{P_i} = (P_{i,1}, P_{i,2}, \cdots, P_{i,I_i})$, $i = 1, 2, \cdots, L$.

Finally recall that r_0, ζ_0, P_0 are the distance, path loss, and received power at Rx from Tx respectively. Since Tx is pointing at Rx with beam 1 which has a gain of g_1, the CDF of P_0 given ζ_0 in (3) becomes

$$F_0(P_0 \mid \zeta_0) = 1 - e^{-\frac{P_0}{P_T g_1 \zeta_0}}. \tag{10}$$

Before deriving the outage probability, we first give a high-level description about our derivation process. The probability $P\left\{\frac{P_0}{P_I(a)+N} < b\right\}$ is determined by b, a, and the realization of a random vector $\mathrm{x} \triangleq (\boldsymbol{K_a}, \boldsymbol{I_a}, \boldsymbol{r_a}, \boldsymbol{\zeta_a}, \boldsymbol{P_a}, r_0, \zeta_0, P_0)$. If we denote the pdf of $\boldsymbol{x}$ by $f(\boldsymbol{x})$, then $\phi_{L,r_0}(b,a) = P\left\{\frac{P_0}{P_I(a)+N} < b\right\}$ is simply

$$\phi_{L,r_0}(b,a) = \int_S f(\boldsymbol{x}) d\boldsymbol{x},$$

where $S = \{\boldsymbol{x} : \frac{P_0}{P_I(a)+N} < b\}$. The order of integration (with respect to the elements of $\boldsymbol{x}$) that we take is: P_0, $\boldsymbol{P_a}$, $\boldsymbol{r_a}$, $\boldsymbol{\zeta_a}$, $\boldsymbol{I_a}$, $\boldsymbol{K_a}$, and finally ζ_0. By first taking the integration with respect to P_0 and $\boldsymbol{P_a} = \{P_{i,j}\}$, we can obtain the conditional outage probability given $\boldsymbol{K_a}, \boldsymbol{I_a}, \boldsymbol{r_a}, \boldsymbol{\zeta_a}, r_0$, and ζ_0. It can be easily done by first applying the CDF of P_0 in (10) and then integrating with respect to the exponential pdf of the independently distributed $P_{i,j}$ in (9). This results in

$$\begin{aligned} \phi_{L,r_0}(b, a \mid \boldsymbol{K_a}, \boldsymbol{I_a}, \boldsymbol{r_a}, \boldsymbol{\zeta_a}, \zeta_0) &= P\{P_0 < bP_I(a) + bN \mid \boldsymbol{K_a}, \boldsymbol{I_a}, \boldsymbol{r_a}, \boldsymbol{\zeta_a}, \zeta_0\} \\ &= 1 - e^{-\frac{bN}{P_T g_1 \zeta_0}} \prod_{i=1}^{L} \prod_{j=1}^{I_i} \left[\frac{1}{1 + \frac{b g_i \zeta_{i,j}}{g_1 \zeta_0}}\right]. \end{aligned} \tag{11}$$

Next we take the expectation with respect to $\boldsymbol{r_a}$ and $\boldsymbol{\zeta_a}$, and substitute y for $\left(\log \zeta_{i,j} - \log\left(K r_{i,j}^{-\eta}\right)\right)$, x for $\left(\log \zeta_0 - \log\left(K r_0^{-\eta}\right)\right)$. We can thus obtain

$$\phi_{L,r_0}(b, a \mid \boldsymbol{K_a}, \boldsymbol{I_a}, \zeta_0) = 1 - e^{-\frac{bN}{P_T g_1 \zeta_0}} \prod_{i=1}^{L} \left[I_a(x, \frac{g_i}{g_1})\right]^{I_i}, \tag{12}$$

where

$$I_a(x,\psi) \triangleq \frac{1}{\sqrt{2\pi}\sigma a^2}\int_{-\infty}^{\infty} e^{-\frac{y^2}{2\sigma^2}}dy \int_0^a \frac{2rdr}{1+\psi\; b\; e^{y-x}\left(\left(\frac{r_0}{r}\right)^\eta\right)}\;. \tag{13}$$

Next by taking the expectation of (12) with respect to the log-normal pdf of ζ_0 in (1), we have

$$\phi_{L,r_0}(b,a \mid \boldsymbol{K_a},\boldsymbol{I_a}) = 1 - \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} e^{-\frac{be^{-x}}{\gamma_c}-\frac{x^2}{2\sigma^2}} \cdot \prod_{i=1}^{L}\left[I_a(x,\frac{g_i}{g_1})\right]^{I_i} dx, \tag{14}$$

where $\gamma_c = \frac{P_T K g_1}{N r_0^\eta} = \frac{E K g_1}{N_0 r_0^\eta}$ is the average received signal-to-noise ratio (SNR) at Rx from Tx, and recall that x is again used to substitute for $\left(\log\zeta_0 - \log\left(K r_0^{-\eta}\right)\right)$. Now take the expectation with respect to $\boldsymbol{K_a}$ and $\boldsymbol{I_a}$, we get

$$\phi_{L,r_0}(b,a) = 1 - \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} \exp\left\{\sum_{i=1}^{L}\frac{\lambda_M\; a^2\; \delta_i\; p}{2\pi}\cdot\left[I_a(x,\frac{g_i}{g_1})-1\right] - \frac{b\; e^{-x}}{\gamma_c} - \frac{x^2}{2\sigma^2}\right\}dx. \tag{15}$$

To get the outage probability, we need to take the limit of (15) as a goes to ∞. The limit of $a^2(I_a(x,\psi)-1)$ as a goes to ∞ is derived in [3] as

$$\lim_{a\to\infty} a^2(I_a(x,\psi)-1) = r_0^2\; e^{\frac{2\sigma^2}{\eta^2}}\;\frac{2\pi}{\eta}\;\csc\left(\frac{2\pi}{\eta}\right)\; b^{\frac{2}{\eta}}\;\psi^{\frac{2}{\eta}}\; e^{\frac{-2x}{\eta}}. \tag{16}$$

After some simplifications from (15), we can obtain the final form of the outage probability

$$\phi_{L,r_0}(b) = 1 - \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty}\exp\left\{-\Lambda(b,x)\cdot r_0^2\cdot\left[\sum_{i=1}^{L}\frac{\delta_i}{2\pi}\left(\frac{g_i}{g_1}\right)^{\frac{2}{\eta}}\right] - \frac{b\; e^{-x}}{\gamma_c} - \frac{x^2}{2\sigma^2}\right\}dx, \tag{17}$$

where

$$\Lambda(b,x) \triangleq \lambda_M p\; e^{\frac{2\sigma^2}{\eta^2}}\;\frac{2\pi}{\eta}\;\csc\left(\frac{2\pi}{\eta}\right)\; b^{\frac{2}{\eta}}\; e^{\frac{-2x}{\eta}}. \tag{18}$$

Though $\phi_{L,r_0}(b)$ is not in close-form, the integral can be easily computed through numerical integration.

3.2 Arbitrary Beam Pattern and Array Interference Factor

Now consider a more realistic beam pattern such as the one in Fig. 2. Note that Fig. 4 will converge to Fig. 2 as L goes to ∞ and δ_i goes to 0. Taking the limit of (17), we have the outage probability $\phi_{r_0}(b)$ of the realistic beam pattern

$$\begin{aligned}\phi_{r_0}(b) &= \lim_{\substack{L\to\infty\\ \delta_i\to 0}}\phi_{L,r_0}(b)\\ &= 1 - \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty}\exp\left\{-\Lambda(b,x)\cdot r_0^2\cdot\left[\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{g(\theta)}{g(\theta_0)}\right)^{\frac{2}{\eta}}d\theta\right] - \frac{b\; e^{-x}}{\gamma_c} - \frac{x^2}{2\sigma^2}\right\}dx.\end{aligned} \tag{19}$$

Notice that $\phi_{r_0}(b)$ depends on the beam pattern only through $\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{g(\theta)}{g(\theta_0)}\right)^{\frac{2}{\eta}}d\theta$. Hence we propose a new parameter, the array interference factor (AIF), for any beam pattern

$$\mathcal{A} \triangleq \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{g(\theta)}{g(\theta_0)}\right)^{\frac{2}{\eta}}d\theta, \tag{20}$$

which fully characterizes the performance of the beam pattern in random networks. For an omnidirectional antenna, $\mathcal{A}=1$. It easy to see that when $\mathcal{A}$ becomes large, the integrand of (19) becomes small, and thus $\phi_{r_0}(b)$ becomes large. Hence, $\phi_{r_0}(b)$ is an monotonic increasing function of AIF $\mathcal{A}$. This is due to the fact that any beam pattern with smaller AIF value will create (receive) less interference to (from) the system and thus have a better performance. With the introduction of AIF, now we can compare the performance of different beam patterns simply by comparing the AIF of each beam pattern.

For systems that use directional antennas for both transmission and reception, the outage probability is derived to be

$$\phi_{r_0}(b) = 1 - \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty}\exp\left\{-\Lambda(b,x)\cdot r_0^2\cdot[\mathcal{A}_t\cdot\mathcal{A}_r] - \frac{b\; e^{-x}}{\gamma_c} - \frac{x^2}{2\sigma^2}\right\}dx, \tag{21}$$

where $\mathcal{A}_t$ and $\mathcal{A}_r$ are the AIF values of the transmitting and receiving beam patterns respectively.

4 Outage Probability of CDMA Wireless Networks

Consider a DSSS network using directional antenna. Due to the imperfect synchronization, the spreading codes are not orthorgonal. While it is difficult to analyze such system, we can still have an approximate analysis of the outage probability if we assume $|\rho|$ the magnitude of the cross correlation coefficient between any two distinct spreading codes is almost the same for any choice of the spreading codes. Under such assumption, the approximate outage probability can be obtained as

$$\phi_{r_0}(b) = 1 - \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \exp\left\{ -|\rho|^{\frac{2}{\eta}} \cdot \Lambda(b,x) \cdot r_0^2 \cdot [\mathcal{A}_t \cdot \mathcal{A}_r] - \frac{b\, e^{-x}}{\gamma_c} - \frac{x^2}{2\sigma^2} \right\} dx. \tag{22}$$

For a FHSS network using directional antenna, the exact outage probability analysis is tractable. Assume the FHSS uses M frequency slots (subchannels) for frequency hopping. The probability for an active node to actually interfere the target link is then $\frac{1}{M}$. The outage probability of a FHSS system can be derived as

$$\phi_{r_0}(b) = 1 - \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \exp\left\{ -\frac{1}{M} \cdot \Lambda(b,x) \cdot r_0^2 \cdot [\mathcal{A}_t \cdot \mathcal{A}_r] - \frac{b\, e^{-x}}{\gamma_c} - \frac{x^2}{2\sigma^2} \right\} dx. \tag{23}$$

5 Analysis of Network Throughput and Transport Capacity

Network throughput and transport capacity are commonly used to measure the performance of wireless networks. Both are analyzed in this section for wireless networks with directional antennas.

5.1 Network Throughput

The network throughput of a wireless network is defined as the average number of successfully transmitted information bits per unit area per second per Hz. It can be approximated by

$$S(b) \approx \frac{\lambda_M p}{\pi} \cdot R \cdot \int_0^{r_{max}} f(r_0)\ \left(1 - \phi_{r_0}(b)\right)\ dr_0 \quad \left(\frac{\text{bits}}{\text{unit area} \cdot \text{sec} \cdot \text{Hz}} \right), \tag{24}$$

where $\frac{\lambda_M p}{\pi}$ accounts for the average number of simultaneous transmissions per unit area (1/unit area), R is the rate of the coded modulation used (bits/sec/Hz), and the integral $\int_0^{r_{max}} f(r_0)\ (1 - \phi_{r_0}(b))\ dr_0$ is the probability of successful transmission. We said (24) is an approximation because it assumes the successful transmissions of different nodes are independent events which is not true in reality since transmitting nodes interfere with each other. Also recall that $f(r_0)$ denotes the pdf of the distance r_0 between Tx and Rx which is determined by the MAC protocol and the network protocol. Since the values of r_0 of different Tx-Rx pairs in a network are not necessarily the same, we need to take the expectation of $\phi_{r_0}(b)$ with respect to r_0 when computing the system throughput.

The system throughput of a wireless network is affected by the node density of the network. When the density is high, the interference problem becomes more serious and thus the outage probability increases. This will affect the system throughput in (24). On the other hand, the outage probability is smaller if the density is low, but since not many nodes are transmitting, this might cause the network resources (bandwidth) to be wasted without producing much system throughput. Our outage probability analysis can be used to find the optimal node density for a wireless network. We consider two different $f(r_0)$. The first case is that the network is controlled where all Tx-Rx pairs in the network have the same distance between the two nodes (similar to the case of mesh network). Hence, $f(r_0) = \delta(r_0 - c)$ where c is a constant. From (24), the system throughput is approximately

$$S_1(b) \approx \frac{\lambda_M p}{\pi} \cdot (1 - \phi_c(b)) \cdot R, \tag{25}$$

where $\phi_c(b)$ can be computed through (21).

On the other hand, if the network allows a node to transmit signals to any node within a distance of r_{max}, then $f(r_0) = \frac{2r_0}{r_{max}^2}$. Substitute $\phi_{r_0}(b)$ in (24) by (21). After some manipulations, we can obtain the system throughput approximation as

$$S_2(b) \approx \frac{\lambda_M p\, R}{\sqrt{2\pi^3}\ \sigma\ r_{max}^2} \int_{-\infty}^{\infty} \frac{1 - \exp\left\{ -\Lambda(b,x) \cdot r_{max}^2 \cdot [\mathcal{A}_t \cdot \mathcal{A}_r] \right\}}{\Lambda(b,x) \cdot [\mathcal{A}_t \cdot \mathcal{A}_r]} \cdot \exp\left\{ -\frac{b\, e^{-x}}{\gamma_c} - \frac{x^2}{2\sigma^2} \right\} dx. \tag{26}$$

5.2 Transport Capacity

The transport capacity of a network characterizes the capability of the network to transport information toward the destination [10]. It can also be regarded as the accumulated sum of the link progress of the links taken place in a network area, where the link progress is defined as the product of hop distance and the local throughput of the link between a Tx-Rx node pair [3, 4]. The network transport capacity can be expressed as

$$T(b) \approx \frac{\lambda_M p}{\pi} \cdot R \cdot \int_0^{r_{max}} f(r_0) \cdot r_0 \cdot (1 - \phi_{r_0}(b)) \; dr_0 \quad \left(\frac{\text{bits}}{\text{unit area} \cdot \text{sec} \cdot \text{Hz}} \right), \tag{27}$$

Compared to (24), it is easy to see the major difference that distinguish the transport capacity from the throughput is the weighting of the hop distance r_0 in the integral. It is mentioned by Gupta and Kumar [10] that to achieve a high transport capacity, a node should communicate only with nearby nodes within a distance of order $\Theta(1/\sqrt{\lambda_M})$. In our notations, this translates to $r_{max} = \Theta(1/\sqrt{\lambda_M})$. If we simply just let $r_{max} = 1/\sqrt{\lambda_M}$, we have $f(r_0) = 2r_0/r_{max}^2 = 2\lambda_M r_0$. Then from (21) and (27), we have

$$T(b) \approx \frac{\sqrt{2}\lambda_M^2 pR}{\pi^{3/2}\sigma} \cdot \int_{-\infty}^{\infty} e^{-\frac{b\ e^{-x}}{\gamma_c} - \frac{x^2}{2\sigma^2}} \; dx \cdot \int_0^{\frac{1}{\sqrt{\lambda_M}}} r_0^{3-1} \left(\frac{1}{\sqrt{\lambda_M}} - x \right)^{1-1} e^{-\Lambda(b,x)\ \mathcal{A}_t \mathcal{A}_r \cdot r_0^2} \; dr_0 \tag{28}$$

Now apply the integration formula

$$\int_0^u x^{\nu-1} \; (u-x)^{\mu-1} \; e^{\beta x^n} \; dx = B(\mu, \nu) \; u^{\mu+\nu-1} \; {}_nF_n \left(\frac{\nu}{n}, \cdots, \frac{\nu+n-1}{n}; \frac{\mu+\nu}{n}, \cdots, \frac{\mu+\nu+n-1}{n}; \beta u^n \right) \tag{29}$$

to the inner integral of (28), where $B(\cdot, \cdot)$ denotes the beta function and ${}_nF_n(\cdots; \cdots; \cdot)$ denotes the generalized hypergeometric series [13]. We thus have

$$T(b) \approx \frac{\sqrt{2\lambda_M} pR}{\pi^{3/2}\sigma} \cdot \int_{-\infty}^{\infty} e^{-\frac{b\ e^{-x}}{\gamma_c} - \frac{x^2}{2\sigma^2}} \cdot \left[B(1,3) \; {}_2F_2 \left(\frac{3}{2}, 1; 2, \frac{5}{2}; -p \; e^{\frac{2\sigma^2}{\eta^2}} \frac{2\pi}{\eta} \; \csc\left(\frac{2\pi}{\eta} \right) b^{\frac{2}{\eta}} e^{\frac{-2x}{\eta}} \mathcal{A}_t \mathcal{A}_r \right) \right] dx \tag{30}$$

We can observe that there exists no λ_M in the integral of (30). Therefore the transport capacity of the network is of the same order as $\sqrt{\lambda_M}$. This matches the well known network transport capacity $\Theta(\sqrt{\lambda_M})$ derived by Gupta and Kumar in [10].

6 Numerical Experiments

In this section, several numerical examples are shown. Due to the limited space, we only show the FHSS cases here. The outage probability of FHSS systems with different numbers of frequency slots using BICM and beam pattern of Fig. 2 is plotted in Fig. 5. It is obvious from the figure that the system performance is significantly improved by the utilization of directional antenna and FHSS. We can also see that the performance gain increases as the number of FHSS frequency slots M increases since the interference is reduced as M grows.

In Fig. 6 we plot the network throughput of a FHSS system compared to a conventional system ($M = 1$). We can see for each setup, there indeed exists an optimal mobile density that can achieve the highest network throughput. This shows that we can apply our analysis work to find the optimal system design for the wireless networks. We can also see that with FHSS ($M = 64$), both the network throughput and the optimal node density are significantly improved.

7 Conclusions

In this project, the performance of CDMA networks with directional antennas of arbitrary beam pattern has been analyzed. By combining with the performance analysis of BICM and turbo codes, we have analyzed the performance of wireless networks with directional antennas for various coding and modulation schemes used. The characteristic value that determines the performance of a beam pattern has been found, which is useful for the performance comparison of different beam patterns in random networks. Finally our analysis is applied to find the throughput and the transport capacity of the wireless networks. It is shown in the numerical experiments that significant performance gain can be obtained by using directional antennas and spreading spectrum techniques. The performance analysis in this work provides system designers a way to characterize the system performance under different combinations of beam patterns and coding/modulation setups, as long as the performance can be expressed in terms of outage probability (for instance, the throughput). This is useful for the system designers to optimize the network performance.

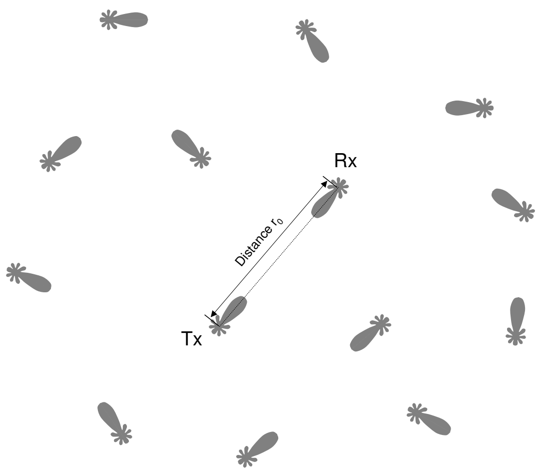

Figure 1: Wireless network using directional antennas.

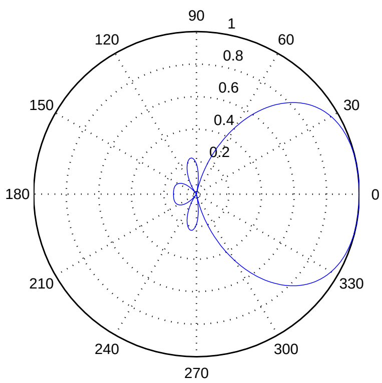

Figure 2: Beam pattern $g(\theta)$ (generated by 10 antenna elements, AIF: 0.5511 at $\eta = 4$ and 0.4545 at $\eta = 2.7$).

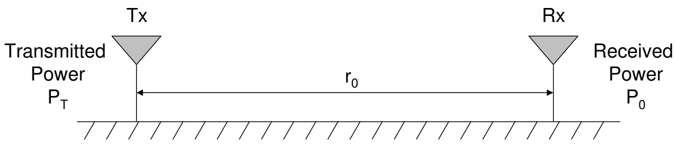

Figure 3: Channel between Tx and Rx.

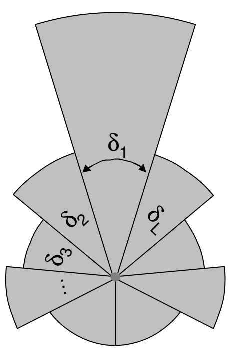

Figure 4: Simplified beam pattern.

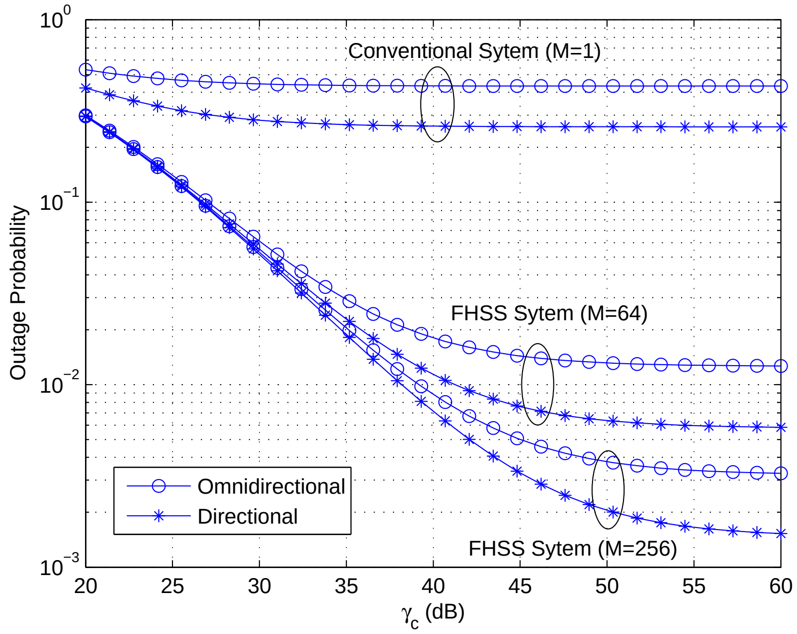

Figure 5: Outage probability vs. average received SNR γ_c of FHSS systems using (1, 5/7) convolutional coded 64-QAM in an urban area, number of frequency slots $M = 1, 64, 256$, $\lambda_M = 20.0$ nodes/km^2, $b = 13.5$ dB, $\sigma = 6$ dB, $\eta = 2.7$, $r_0 = 0.1$ km.

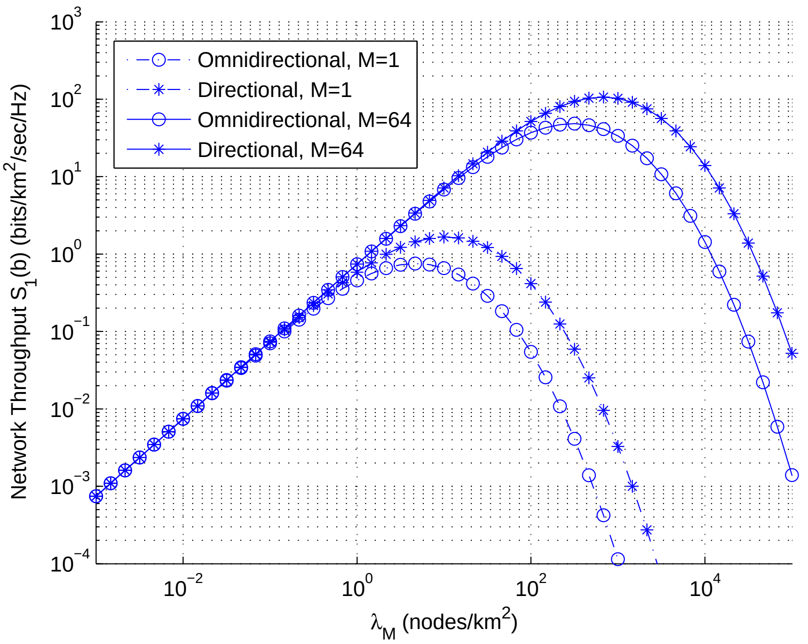

Figure 6: System throughput $S_1(b)$ vs. λ_M of FHSS systems using (1, 5/7) convolutional coded 64-QAM in an urban area, number of frequency slots $M = 1, 64$, $r_0 = c = 0.1$ km, $\gamma_c = 25$ dB, $\sigma = 6$ dB, $\eta = 2.7$, $p = 0.9$.

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計畫成果自評：

本計畫之研究成果與原計畫研究內容高度相符，預期之研究目標均已達成。主要研究成果列舉如下：

1. Outage probability analysis for wireless networks using directional antennas of arbitrary beam pattern.
2. Outage probability analysis for direct-sequence spread spectrum networks using directional antennas of arbitrary beam pattern.
3. Outage probability analysis for frequency-hopped spread spectrum networks using directional antennas of arbitrary beam pattern.
4. System throughput analysis for spread spectrum networks using directional antennas of arbitrary beam pattern.
5. Link throughput analysis for spread spectrum networks using directional antennas of arbitrary beam pattern.
6. System design optimization for spread spectrum networks using directional antennas of arbitrary beam pattern.

在文獻中，至今仍未有針對使用任意波束型態指向式天線之無線網路系統效能之完整分析，亦無針對展頻技術與指向式天線結合之無線網路之完整效能分析。本計畫之研究成果為此研究課題之先驅，因此學術價值甚高。另本計畫中所使用之無線通道模型包含了路徑衰減、遮蔽效應、瑞雷衰落等實際環境中常見之無線信號傳輸效應，在此實際通道模型下我們開發了可最佳化無線網路系統參數之技術。此技術將可廣泛應用於實際環境中無線網路系統效能之最佳化。因此本計畫成果亦有相當高之應用價值。

本計畫成果之主要內容已投稿至 *IEEE Transactions on Vehicular Technology*，修正版 (revised version) 刻正審核中。本計畫成果預計共可產出一至二篇期刊論文。

可供推廣之研發成果資料表

□ 可申請專利　■ 可技術移轉	日期：96 年 1 月 30 日
國科會補助計畫	計畫名稱：使用指向式天線之展頻無線通訊網路之最佳化設計 計畫主持人：葉丙成 計畫編號：NSC 95-2221-E-002-001　學門領域：電信
技術/創作名稱	使用指向式天線之展頻無線通訊網路之最佳化設計
發明人/創作人	葉丙成
技術說明	在本計畫中，我們提出一個結合展頻通信技術與指向式天線技術的新世代無線網路。所開發之技術包括在實際無線傳輸環境中使用直接序列展頻技術（Direct-Sequence Spread Spectrum）以及跳頻展頻技術（Frequency-Hopped Spread Spectrum）之無線網路在任意的指向式天線之波束型態（Beam Pattern）下的系統效能分析，以及建構於此分析之上的一套無線網路系統參數之最佳化之技術。此技術將可廣泛應用於使用指向式天線之無線網路系統上，以最佳化其系統效能。
可利用之產業及可開發之產品	無線網路裝置、網管系統軟體等相關產業
技術特點	1. 可應用於任意波束型態之指向式天線之無線網路系統 2. 可應用於使用直接序列展頻技術之無線網路系統 3. 可應用於使用跳頻展頻技術之無線網路系統
推廣及運用的價值	目前在市面上已有眾多使用指向式天線之無線區域網路之產品，另外在無線都會網路的標準（WiMAX）制定中也已採用指向式天線技術。可以預見的，指向式天線技術與今日早已廣泛採用之展頻技術之結合將是未來新世代無線網路所採用的主要技術之一。本技術可分析此類新世代網路之系統效能，並可廣泛的應用於此新世代無線網路之系統參數設定，以最佳化其系統效能。

※　1.每項研發成果請填寫一式二份，一份隨成果報告送繳本會，一份送　貴單位研發成果推廣單位（如技術移轉中心）。

※　**2.本項研發成果若尚未申請專利，請勿揭露可申請專利之主要內容。**

※　3.本表若不敷使用，請自行影印使用。