Performance Evaluation of Range Estimation via Angle Measurements

Chieh-Hao Yang[1](楊介豪), Chia-Ming Wang[2](王嘉銘), Chin-Jung Liao[3](廖金榮)
Yi-Ping Hung[1](洪一平), Kuo-Chin Fan[2](范國清)

1. Department of Computer Science and Information Engineering, National Taiwan University, Taipei, Taiwan, R.O.C.
2. Department of Computer Science and Information Engineering, National Central University, Chungli, Taiwan, R.O.C.
3. Chung-Shan Institute of Science and Technology, Taoyuan, Taiwan, R.O.C.

Tel: (02)2362-5336 Ext. 433 , Fax: (02)2362-8167
e-mail: hung@csie.ntu.edu.tw[1], chia@fox1.csie.ncu.edu.tw[2]

Abstract

In this paper, we introduce a simulation package to evaluate the performance of 3D range estimation via angle measurements. First, we compare different coordinates modeling of conventional 2-dimensonal bearings-only tracking (BOT) algorithms and design two methods for estimating 3D range. Second, we are able to adjust the observer and target motion with a number of parameters and observe the ranging results with different motion trajectories. Third, we include a more practical observer trajectory model called "arc-and-straight" in our simulation. Experimental results will demonstrate the ranging algorithm performance.

1 Introduction

Estimating 3D range in passive EO systems is a useful task in computer vision applications. Some of the past literatures utilize optical flow information to obtain the 3D range [11][12], but if the target is far from us, it may only include a few pixels in images so that optical flow is difficult to be estimated accurately. Therefore, using optical flow information to estimate range of faraway targets might be inappropriate. In addition, since EO systems can measure the angles (bearings) of observer and target with good precisions [10], the potential accuracy of passive ranging algorithms via angle measurements is significantly improved, particularly in the presence of closely spaced target. Since "angle" is a better measurement than optical flow, the conventional 2-dimensinonal bearings-only tracking algorithms [1][3][7][8] may be suitable for 3-dimensional range estimation.

We formulate the conventional bearings-only tracking problem as follow. Consider the general 2-dimensonal environments, an observer and a target are confined to a plane. The situation is shown in Fig. 1, where it is desired to estimate the relative position and velocity parameters of target and observer from noise-corrupted passive measurements of the angle β.

We denote the target position and velocity as r_{Tx}, r_{Ty}, v_{Tx}, v_{Ty}, where the observer position and velocity as r_{Ox}, r_{Oy}, v_{Ox}, v_{Oy}. β_m is the measured angle referenced to y-axis. In this problem, we suppose that the target is moving with constant velocity, and the observer is moving with arbitrary motion. What we want to estimate is the relative position and velocity $[r_x, r_y, v_x, v_y] = [r_{Ox}, r_{Oy}, v_{Ox}, v_{Oy}] - [r_{Tx}, r_{Ty}, v_{Tx}, v_{Ty}]$. All of these parameters are functions of time t.

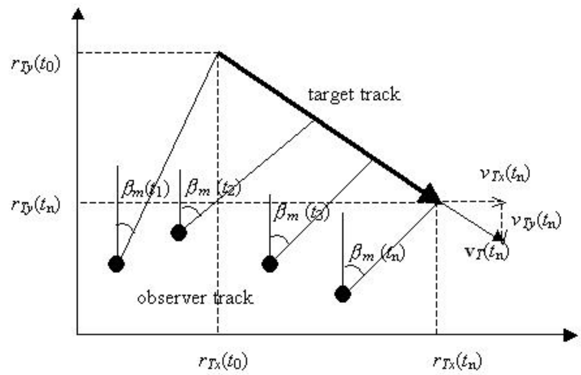


Figure 1. The target-observer model in 2-dimensional plane

Several important techniques have been proposed to 2D bearings-only tracking with varying results. The differences in methods involve the modeling of the process and the selection of the estimation algorithm. Since this process in inherently nonlinear, Lindgren ,Gong and Aidala used Cartesian coordinate to develop a state dynamic model in discrete and continuous viewpoints [1][8], and formulate this problem as a pseudo-linear manner. Then the linear estimation theory such as Kalman filter could be applied [5]. Nevertheless, studies also showed that erratic behavior of Kalman filter may be obvious especially when formulated in Cartesian coordinates [1][2].

Studies by Aidala and Hammel showed that formulating the bearings only tracking in polar coordinates leads to an extended Kalman filter, which is both stable and asymptotically unbiased [7]. In the past research, 2D BOT problem in polar coordinates modeling has already been used in simulating 3D range estimation algorithm [10], but it didn't focus on the relationship of observer motion and the system performance. Observer trajectory may effectively influence the range estimation results, and it is the major topic we want to research.

In this paper, we design a simulation package to evaluate the performance of 3-dimensonal ranging algorithm based on 2-dimensional bearings-only tracking. We design two 3D ranging method and different observer moving trajectories to evaluate the performance. In section 2, we introduce some famous 2D bearings-only tracking algorithms and our 3D ranging methods. Section 3 describes our simulation system, and experimental results are shown in section 4. Some conclusions are discussed in section 5.

2 Algorithms descriptions of 2D Bearings-Only Tracking and 3D Range Estimation

2.1 2D BOT problem: Modeling using Cartesian coordinates

Consider the geometry depicted in Fig.1, if constant target velocity is assumed, the equations of motion for this two-dimensional configuration may be written as

$$\begin{aligned}\dot{\mathbf{r}}(t) &= \mathbf{v}(t)\\ \dot{\mathbf{v}}(t) &= \mathbf{a}(t) = -\mathbf{a}_O(t)\end{aligned} \tag{1}$$

where $\mathbf{r}(t) = [r_x(t), r_y(t)]^T$ denotes the relative position of target and observer, $\mathbf{v}(t) = [v_x(t), v_y(t)]^T$ denotes the relative velocity, $\mathbf{a}(t) = [a_x(t), a_y(t)]^T$ denotes the relative acceleration, and $\mathbf{a}_O(t) = [a_{Ox}(t), a_{Oy}(t)]^T$ denotes the observer acceleration.

Subsequent integration from some time t_0 to t ($t_0 < t$)of equation set (1) yields

$$\begin{aligned}\mathbf{r}(t) &= \mathbf{r}(t_0) + (t-t_0)\mathbf{v}(t_0) - \int_{t_0}^{t}(t-\tau)\mathbf{a}_O(\tau)d\tau\\ \mathbf{v}(t) &= \mathbf{v}(t_0) - \int_{t_0}^{t}\mathbf{a}_O(\tau)d\tau\end{aligned} \tag{2}$$

We can rewrite (2) in matrix form as follow:

$$\begin{bmatrix}\mathbf{v}(t)\\ \mathbf{r}(t)\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ t-t_0 & 0 & 1 & 0\\ 0 & t-t_0 & 0 & 1\end{bmatrix}\begin{bmatrix}\mathbf{v}(t_0)\\ \mathbf{r}(t_0)\end{bmatrix} - \begin{bmatrix}\int_{t_0}^{t}\mathbf{a}_O(\tau)d\tau\\ \int_{t_0}^{t}(t-\tau)\mathbf{a}_O(\tau)d\tau\end{bmatrix} \tag{3}$$

If we define the state vector $\mathbf{x}(t) = [r_x(t) \quad r_y(t) \quad v_x(t) \quad v_y(t)]^T$, (3) becomes

$$\mathbf{x}(t) = \mathbf{\Phi}(t,t_0)\mathbf{x}(t_0) - \mathbf{w}_O(t,t_0) \tag{4}$$

where

$$\mathbf{\Phi}(t,t_0) = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ t-t_0 & 0 & 1 & 0\\ 0 & t-t_0 & 0 & 1\end{bmatrix}$$ is the state transition matrix,

and

$$\mathbf{w}_O(t,t_0) = \begin{bmatrix}\int_{t_0}^{t}\mathbf{a}_O(\tau)d\tau\\ \int_{t_0}^{t}(t-\tau)\mathbf{a}_O(\tau)d\tau\end{bmatrix}$$ is the deterministic input. (4)

is the state equation of Cartesian coordinates of 2D BOT.

We then denote the true bearings and measured bearings as $\beta(t)$ and $\beta_m(t)$. According to Fig.1, they satisfy the relation as follows:

$$\begin{aligned}\beta_m(t) &= \beta(t) + \eta(t)\\ &= \tan^{-1}\left[\frac{r_x(t)}{r_y(t)}\right] + \eta(t)\\ &= h[\mathbf{x}(t)] + \eta(t)\end{aligned} \tag{5}$$

where $\eta(t)$ is a zero mean white noise process with variance $\sigma^2(t)$.

Equation (5) is the measurement equation. Having (4) and (5), we are able to adopt the extended Kalman filter (EKF) to calculate the best estimation of $\mathbf{x}(t)$. In fact, the non-linearity of the measurement equation will lead to some abnormal behavior in the BOT problem, and will cause the filter instability. To prevent this problem, Aidala [1] proposed an algorithm called Pseudo-linear estimation (PLE). In this algorithm, the equation (5) was algebraically manipulated to yield a new measurement equation

$$0 = \hat{\mathbf{H}}(t)\mathbf{x}(t) + d(t)\sin\eta(t) \tag{6}$$

where

$$\begin{aligned}\hat{\mathbf{H}}(t) &= [0 \quad 0 \quad \cos\beta_m(t) \quad -\sin\beta_m(t)]\\ d(t) &= \sqrt{r_x^2(t) + r_y^2(t)}\\ &= r_x(t)\sin\beta(t) + r_y(t)\cos\beta(t)\end{aligned}$$

Here, the non-linearity has been embedded in the measurement noise. If

$$\begin{aligned}\varepsilon(t) &= d(t)\sin\eta(t)\\ &= \text{effective measurement noise at time } t\end{aligned} \tag{7}$$

and $\eta(t)$ is Gaussian distributed, it can be shown [1] that $\varepsilon(t)$ has the following statistics:

$$\begin{aligned}E[\varepsilon(t)] &= 0\\ E[\varepsilon^2(t)] &= [d^2(t)/2][1-\exp\{-2\sigma^2(t)\}]\\ &\approx d^2(t)\sigma^2(t),\ \sigma^2(t) \ll 1\end{aligned} \tag{8}$$

Equations (4), (6) and (7) may now be combined to provide a pseudo-linear model of the bearings-only tracking problem. In this model, the Kalman filter (KF) can be used for optimal estimation.

2.2 2D BOT problem: Modeling using polar coordinates

Some other researches have shown that using the Cartesian coordinates and extended Kalman filter would exhibits unstable behavior characteristics in bearings-only tracking problem. Although pseudo-linear model can prevent this problem, it generates biased estimations whenever noisy measurements are processed [2]. In contrast, formulating the TMA estimation problem in modified polar coordinates leads to an extended Kalman filter which is both stable and asymptotically unbiased. The pertinent equations of state and measurement are formulated in modified polar (MP) coordinates, while the algorithm itself is configured as an extended Kalman filter. This coordinates system was shown to be well-suited for bearings-only tracking in several related literatures [7].

The MP state vector is comprised of the following components: bearing, bearing-rate, range rate divided by range, and the reciprocal of range. That is,

$$\mathbf{y}(t)=\begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \\ y_4(t) \end{bmatrix}=\begin{bmatrix} \dot{\beta}(t) \\ \dot{d}(t)/d(t) \\ \beta(t) \\ 1/d(t) \end{bmatrix} \quad (9)$$

If $\mathbf{y}(t)$ denoted the MP state vector, then $\mathbf{x}(t)$ and $\mathbf{y}(t)$ will be related at all instants of time by the nonlinear one-to-one transformations

$$\mathbf{x}(t) = f_x[\mathbf{y}(t)] \quad (10)$$

$$\mathbf{y}(t) = f_y[\mathbf{x}(t)] \quad (11)$$

Letting $t=t_0$ in (10), and substituting the result into (4) to eliminate $\mathbf{x}(t_0)$, yields

$$\mathbf{x}(t) = \mathbf{\Phi}(t,t_0) f_x[\mathbf{x}(t_0)] - \mathbf{w}_O(t,t_0) \quad (12)$$

Substitution of (12) into (11) leads to the relation

$$\begin{aligned} \mathbf{y}(t) &= f_y\left[\mathbf{\Phi}(t,t_0) f_x\left[\mathbf{y}(t_0)\right] - \mathbf{w}_O(t,t_0)\right] \\ &= f\left[\mathbf{y}(t_0);t,t_0\right] \end{aligned} \quad (13)$$

(13) is the state equation modeled in polar coordinates. Afterwards, the measurement relation can be easily written directly as follows:

$$\beta(t) = \mathbf{H}\mathbf{y}(t) + \eta(t) \quad (14)$$

where

$$\mathbf{H}=[0 \quad 0 \quad 1 \quad 0], \ \eta(t)\sim N(0,\sigma^2(t))$$

Equations (13) and (14) are the exact MP analogs of (4) and (5), respectively, and the EKF can be used to estimate the optimal state.

2.3 3D Range Estimation

We utilize the results from 2D BOT for 3D range estimations. We adopt the concept as follow:

- Separate 3D space into three 2D planes, that is, *xy*- plane, *yz*-plane, and *xz*-plane, and project the position of target and observer to these planes.
- Estimate 2D range in those three 2D planes.
- Combine the estimated ranges of those 2D planes into a new estimated 3D range.

There are two methods to combine 2D BOT results into 3D range estimation, and we will introduce them below respectively. The first method is to calculate the mean estimates of each plane. We can estimate 2D ranges in *xy*- , *yz*- and *xz*- planes. Then we can obtain two estimates of each state parameter. Therefore, the 3D position estimations can be easily obtained by calculating the mean of these two values:

$$(\hat{r}_x,\hat{r}_y,\hat{r}_z)=(\frac{\hat{r}_x^{(xy)}+\hat{r}_x^{(xz)}}{2},\frac{\hat{r}_y^{(xy)}+\hat{r}_y^{(yz)}}{2},\frac{\hat{r}_z^{(xz)}+\hat{r}_z^{(yz)}}{2}) \quad (15)$$

where $\hat{r}_i^{(jk)}$ means the *i*-position estimated in *jk*-plane.

So the range estimate is given by

$$range_{3D}=\sqrt{\hat{r}_x^{\,2}+\hat{r}_y^{\,2}+\hat{r}_z^{\,2}} \quad (16)$$

Since the EO system can measure the bearings with very high precision (standard deviation is about 400-600 μrad), the ranging method described above may bring out a problem. That is, the estimated 3D position may have larger direction bias than the "measured" bearings. Therefore we introduce a new 3D ranging method. Since the measured target bearings are with high precision, we suppose that the target is located on this direction. So we projecting the 3D bearings to two certain planes, then we can estimate the projected range using 2D BOT algorithms, respectively. Finally, we project two 2D range estimations back to the direction of the measured bearings, and calculate the mean of them as 3D range estimates.

3. Simulation system

3.1 2D simulation methodology

To evaluate the performance effectively, it is necessary to design an environment of relative motion between target and observer to compare the estimation results of the dynamic models in Cartesian coordinate and polar coordinate. We assume that the target and observer are moving with constant speed. But if the target and observer are all moving along with a straight line without changing their courses, it will meet the non-observable problem [5][9]. In our simulation, because the 2D BOT method we adopt is supposed to be moving with constant velocity, we can design the observer moving along with the triangle wave as shown in Fig 2. In order to generate different triangle waves, it needs to introduce some parameters. The first one is "number of maneuvers", which controls how many times a target change its moving course. In Fig 2, the "number of maneuvers" is set to 4 times, and the first one is at A. Once the "number of maneuvers" defined, it also needs to introduce a parameter, "time before the first maneuver", to control when the observer will change the course after starting to move. It controls the length of the edge of the triangle. There still needs some parameters to complete the system. "sampling periods" is determined the time of an observer obtaining a measurement. "duration of simulation" controls the total simulation time. Having defined these parameters of the system, we also need to determine the parameters of the initial motion of observer and target. There are three parameters for target and observer respectively, that is, position, course, and velocity. Course is reference to *y*- axis as θ in Fig 2. These parameters can construct the target and observer motion, and other simulation environments.

Arc and straight model: Because the triangle wave is not realistic for normal observer motion, we design another practical observer trajectory model. This model is composed by two parts: arc parts and straight parts which is shown in Fig 3. In this new model, it is necessary to introduce a new parameter to the system. This parameter, R, controls the radius of arc while the observer flies along with it. If the radius is set bigger, the arc is longer and smoother, and the distance the observer moves increases. Therefore, the whole moving trajectory can be separated into two parts. First part is moving in the course of observer until the time of "time before the first maneuver" is up. The second part is moving along with an arc with the angular speed

$$\omega = \frac{v}{R},$$

where v is the velocity of observer, and R is the radius of partial circle. Then, the observer is moving another uniform motion until another time of "time before the first maneuver" is up to complete a whole maneuver.

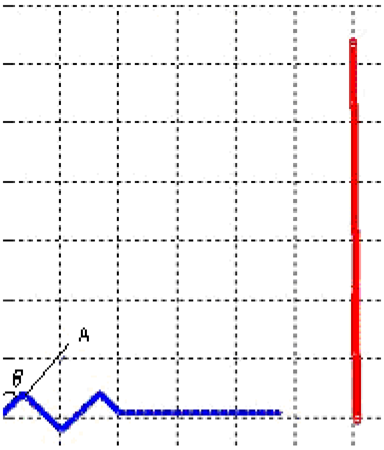


Figure 2 This figure shows the moving trajectories of the observer and the target. The blue points represent the trajectory of the observer, and the red points represent the trajectory of the target.

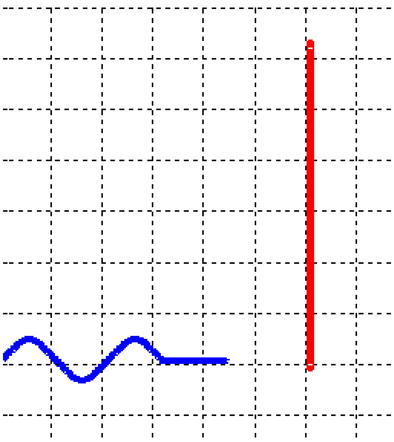


Figure 3 This is the trajectory of arc-and-straight. The blue points represent the trajectory of observer, and the red points represent the trajectory of target.

3.2 3D simulation methodology

In section 2, we have introduced two algorithms of estimating 3D range. In fact, when we want to utilize the projection in 2D plane, the "non-observable" problem will appear on some plane so the estimation obtained from this plane is useless. Therefore, we need to design a mechanism to remove the useless plane and select the useful ones to do the 2D estimation. This mechanism gathers the information of three planes in the time period from the first time the observer changing course to 15 seconds latter, and calculates the range estimated error in this time period. If the errors of one plane converge and lower than some thresholds, this plane is picked up to use. Therefore, the flow of the simulation system is as follow:

- Separate 3D space into three 2D planes, that is, *xy*-plane, *yz*-plane, and *xz*-plane, and project the position of target and observer to these planes.
- Estimate 2D range in those three 2D planes.
- Detect the useful and skip the useless planes.
- Combine the estimated ranges of the chosen 2D planes into a new estimated 3D range.

4. Experimental results

In chapter 4, we show some of our experimental results. First, we compare two different coordinates modeling in 2D BOT. Second, we use three different observer moving trajectories to show the performance of 2D BOT on polar coordinates. Next, we try to estimate ranges by moving along with the arc-and-straight model. Finally, 3D range estimation is simulated to show how the extension of 2D BOT performs.

The standard system parameters are defined as follow:

- Number of maneuvers : 3 times
- Sampling period : 1 second
- Time before the first maneuver : 15th second
- Bearing jitter RMS : 400 μrad
- Duration of simulation : 180 seconds
- Observer parameters :
  - Velocity : 300 m/s
  - Course : 45°
  - Position : (0, 0)
- Target parameters :
  - Velocity : 350 m/s
  - Course : 0°
  - Position : (60, 0)
- Initial estimating state : $\begin{bmatrix}0 & 0 & \beta_m(0) & 10^{-2}\end{bmatrix}^T$
- Covariance matrix : diag $\begin{bmatrix}10^{-4} & 10^{-4} & 10^{-4} & 10^{-8}\end{bmatrix}$

4.1 Comparisons of Cartesian coordinates and polar coordinates modeling

Having defined the parameters in section 3, it is easy to observe the performance by adjusting the parameters. There are two experiments which would directly show the reasons why we choose the polar coordinates modeling to estimate range.

4.1.1 Time before the first maneuver = 5 seconds

In this experiment, the parameter, "time before the first maneuver", is set to 5 seconds. The main idea we do this is to reduce the baseline of the observer, which would show the difference of estimation between these two methods apparently. The rest of the parameters are set to the same values as the standard system parameters. Fig 4 is the comparisons between the estimation methods based on polar coordinates and Cartesian coordinates. It is obvious to observe that the method based on polar coordinates performs better than the one based on Cartesian coordinates.

4.1.2 Observer initial course = 75°

In this experiment, the parameter, “initial course of observer”, is set to 75° from Y axis to X axis. The rest of the parameters are set to the same values as the standard system parameters. The Fig 5 gives us an obvious look that the method based on polar coordinates performs better than the one based on Cartesian coordinates as well.

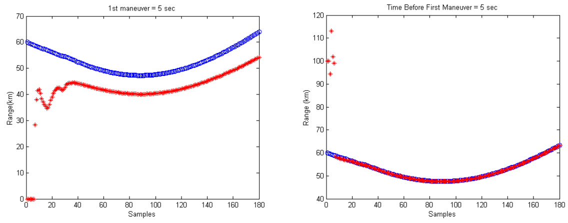


Figure 4 : These figures show the comparison based on the parameter, “Time before the first maneuver”, being set to 5 seconds. The left figure is the experiment result of the method based on Cartesian coordinates, and the right one is the one based on polar coordinates. The blue points represent the true range between observer and target, and the red points represent the estimated range.

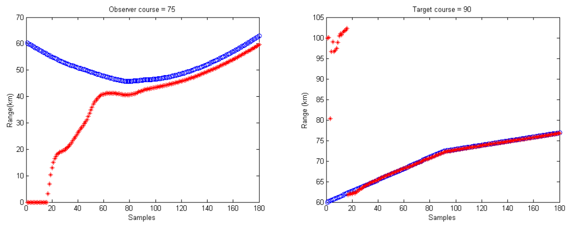


Figure 5 : These figures show the comparison based on the parameter, “course of observer”, being set to 75°. The left figure is the experiment result of the method based on Cartesian coordinates, and the right one is the one based on polar coordinates. The blue points represent the true range between observer and target, and the red points represent the estimated range.

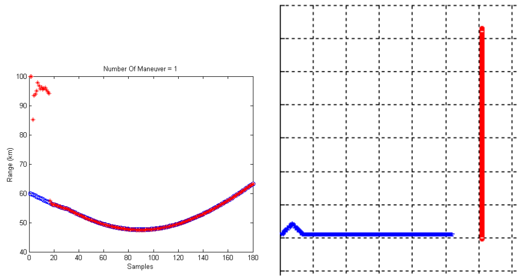
4.2 Experiments in different trajectories of observer using polar coordinates

Now, having demonstrated the performance of the two coordinates systems, it is easy to choose the method based on polar coordinates to do the following experiments. In order to find out the relationship of observer trajectories and ranging results, it is interesting to observe the influence on parameters of “number of maneuver”, “time before the first maneuver”, and “course of observer”.

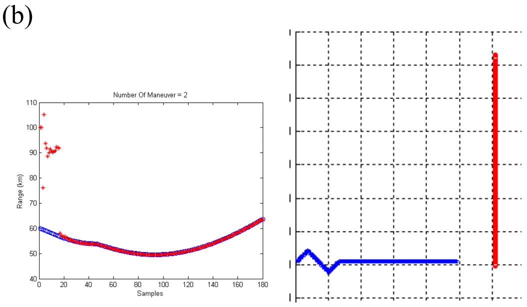
4.2.1 Influence on “number of maneuver”

In this experiment, we will show the influence on the parameter “number of maneuver”. The parameter is set from 1 to 3 times, and the rest of the parameters are set to the same values as the standard system parameters. Fig 6 shows the experimental results. We can conclude that the estimating results are good while the observer changes the course, no matter how many “number of maneuver” is set.

(a)



(b)



(c)

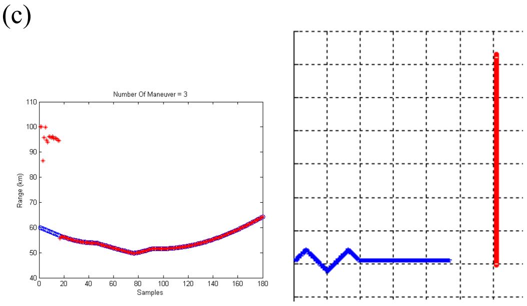
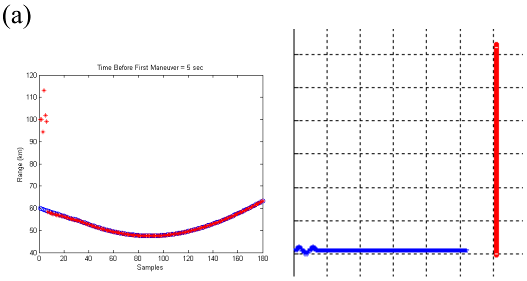


Figure 6 These figures show the results of different values of “number of maneuver” and the corresponding trajectories. In the left part of these figures, blue points represent the true range between target and observer, and the red points represent the estimated range. In the right part of these figures, the blue points represent the trajectory of observer and the red points represent the trajectory of target. (a) “number of maneuver” = 1, (b) “number of maneuver” = 2, (c) “number of maneuver” = 3.

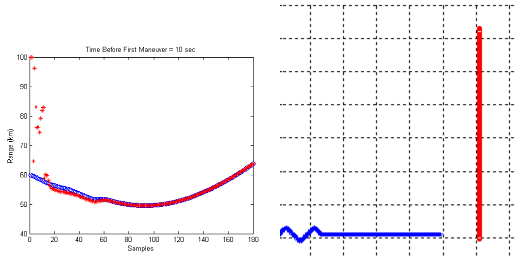
4.2.2 Influence on “time before the first maneuver"

In this experiment, we will show the influence on the parameter of “time before the first maneuver”. The parameter of “time before the first maneuver” is set to 5, 10, 15, and 20 separately, and the rest of the parameters are set to the same values as the standard system parameters. Fig 7 shows the experimental results, and we can easily conclude that no matter what “time before the first maneuver” is set, it can lead to a good estimation result.

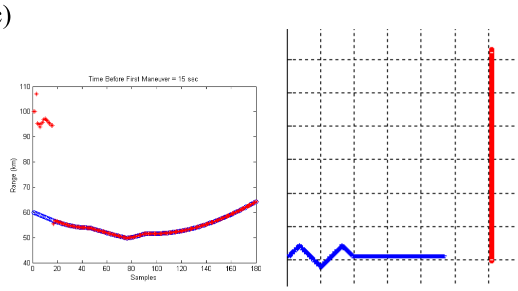
(a)



(b)



(c)



(d)

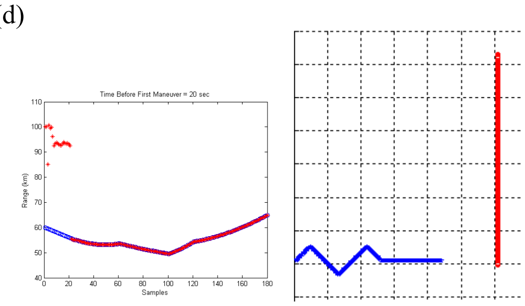
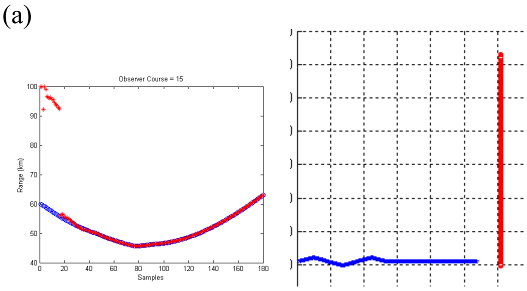


Figure 7 These figures show the results of different values of "time before the first maneuver" and the corresponding trajectories. In the left part of these figures, blue points represent the true range between target and observer, and the red points represent the estimated range. In the right part of these figures, the blue points represent the trajectory of observer and the red points represent the trajectory of target. (a) "time before the first maneuver" = 5, (b) "time before the first maneuver" = 10, (c) "time before the first maneuver" = 15, (d) "time before the first maneuver" = 20.

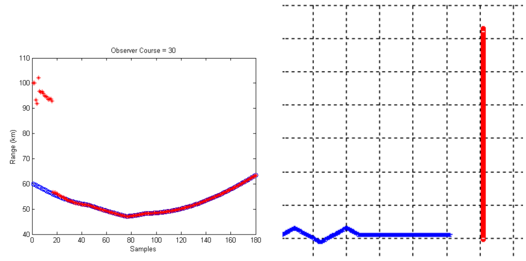
4.2.3 Influence on "initial course of observer"

In this experiment, it will show the influence on the parameter of "initial course of observer". The parameter of "initial course of observer" is set to 15°, 30°, 45°, 60°, and 75° separately, and the rest of the parameters are set to the same values as the standard system parameters. Fig 8 is the experimental results. From the figure, it can conclude that no matter what course observer flies toward, this method estimates well as those previous experiments do.

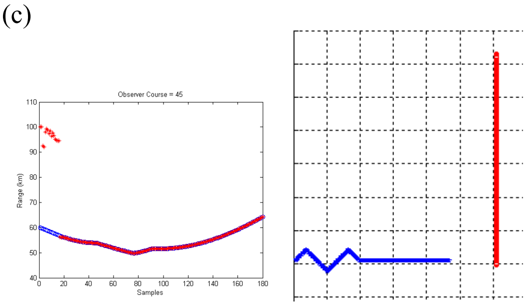
(a)



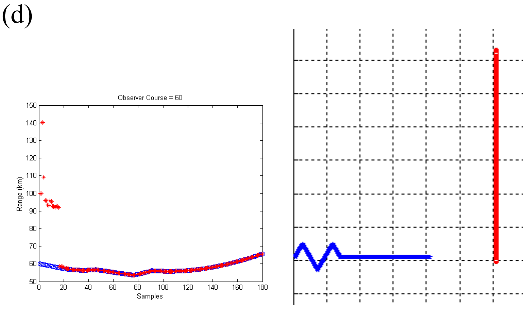
(b)



(c)



(d)



(e)

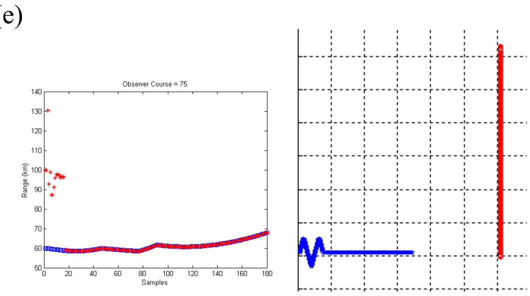


Figure 8 These figures show the results of setting different values to "initial course of observer" and the corresponding trajectories. In the left part of these figures, blue points represent the true range between target and observer, and the red points represent the estimated range. In the right part of these figures, the blue points represent the trajectory of observer and the red points represent the trajectory of target. (a) "initial course of observer" = 15°, (b) "initial course of observer" = 30°, (c) "initial course of observer" = 45°, (d) "initial course of observer" = 60°, (e) "initial course of observer" = 75°.

4.2.4 Influence on "initial course of observer" and "time before first maneuver"

In this experiment, we use carpet plot to observe the influence of "time before the first maneuver" and "initial course of observer". The parameter of "time before the first maneuver" is set to 5, 10, 15 20, and 25 seconds separately, and the parameter of "initial course of observer" is set to 15°, 30°, 45°, 60°, and 75° separately. The rest of the parameters are set to the same values of the standard system parameters. From the experiment, we conclude that if the observer changes the first course too early, it will result in obtaining an unsatisfied estimated result. This is because the baseline is reduced with the "time before the first maneuver" set to a smaller value. Therefore, the stereo can not be established well.

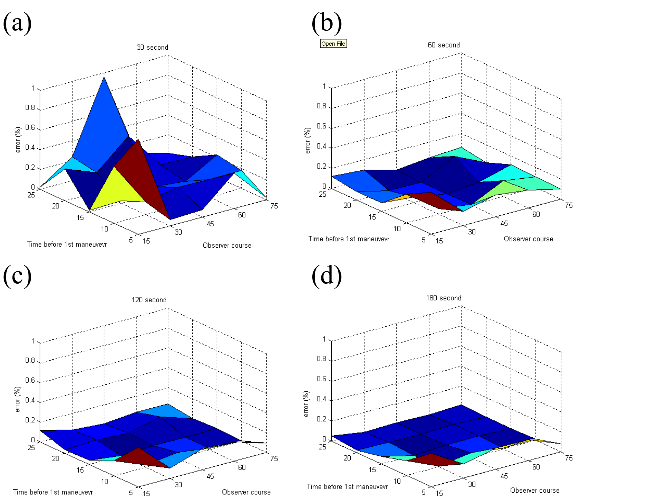


Figure 9 These figures show the results of setting different values to "initial course of observer" and "time before the first maneuver". Fig (a) is the result of the 30th second. Fig (b) is the result of the 60th second. Fig (c) is the result of the 120th second. Fig (d) is the result of the 180th second.

4.3 Experiments on the arc-and-straight model

Having demonstrated the performance of the method based on polar coordinates, it turns to estimate ranges with observer moving along with the trajectory of "arc-and-straight". The traditional way is to move along with the triangle wave as the Fig 2 shows. Now, the observer is designed to move along with the trajectory of arc-and-straight as the sequential blue points in Fig 3.

The new parameter for this trajectory, R, is initialized with 0.05km first. The rest of the parameters the simulation need are set to the same values as the standard system parameters. The experimental results show in the Fig 10. From the Fig 10, it shows that no matter how the observer flies, the estimation results are good in both trajectories. The estimation method based on polar coordinates does quite well in both trajectories of triangle wave and arc-and-straight, but the arc-and-straight model may converge slower.

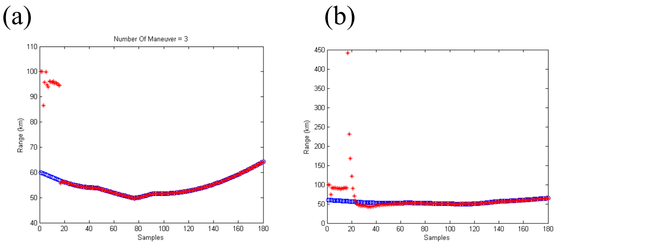


Figure 10 These two figures shows the experimental results estimated under these two moving trajectories. The blue points in both figures represent the true range from observer to target, and the red points represent the estimated range. Fig (a) is the result of observer moving along with triangle wave. Fig (b) is the result of observer moving along with trajectory of arc-and-straight.

4.4 Experiments on 3D range estimation

When extending the 2D estimation to 3D estimation, it is necessary to redefine the parameters of observer and target. The rest of the parameters are the same as the standard system parameters, and this experiment adopts the trajectory of triangle wave.

- Observer parameters :
  - Velocity : 300 m/s
  - Course :
    - 45° from y axis to x axis
    - 45° from x-y plane to z axis
  - Position : (0, 0, 0)
- Target parameters :
  - Velocity : 350 m/s
  - Course :
    - 0° from y axis to x axis
    - 0° from x-y plane to z axis
  - Position : (60, 0, 0)

The results are shown in Fig 11. Because of the good estimation of 2D plane, the 3D range estimates well no matter what method combines these 2D ranges. Therefore, the comparison between method 1 and method 2, mentioned in 11 has a slide difference in Fig 11 (c), which shows the difference of range error between method 1 and method 2. We can conclude that the major problem of 3D range estimation by 2D BOT method is the estimates on non-observable projection plane.

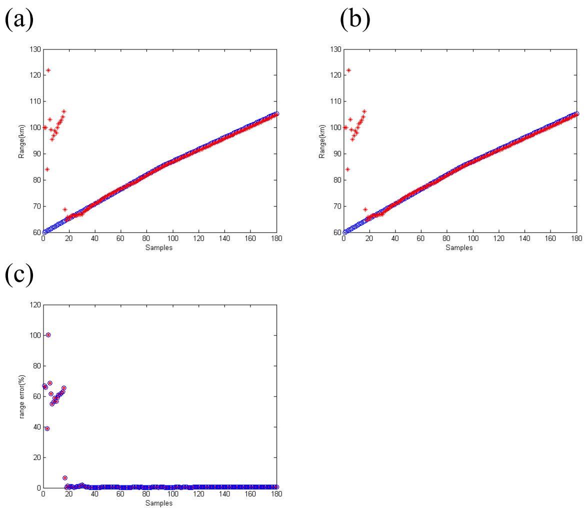


Figure 11 These figures show the results of 3D range estimation. The blue points in Fig 11 (a) and (b) represent the true range between target and observer in 3D space, and the red points represent the estimated range using method 1 and method 2 to combine 3D range, separately. The blue points in Fig 11 (c) represent the estimated error using method 1 to combine 3D range, and the red points represent the estimated error using method 2 to combine 3D range.

5 Conclusions and Discussions

In this paper, we simulate the 3D range estimation algorithms based on the traditional 2-dimensional bearings-only tracking problem. We design two ranging methods and different observer moving models to evaluate the system performance. Polar coordinate system is more suitable than Cartesian systems because it will not produce the bias estimations. Experimental results also show that if observer moves with large baseline with respect to the target, the filter will converge more quickly.

In future work, we will focus on two topics. The first, because of the linear approximation of extended Kalman filter, we want to select another estimation algorithm to estimate highly non-linear state model, especially when it cannot be be approximation linearly locally. The other topic we will focus is how to find an optimal observer motion trajectory so that the filter will converge in the shortest time.

Acknowledgements

This work is supported in part by the National Science Council under the grant of NSC-92-2623-7-002-010.

References

[1] V. J. Aidala, "Kalman-Filter Behavior in Bearings-Only Tracking Applications", *IEEE Transactions on Aerospace and Electronic Systems*,Vol.AES-15, No.1, Jan. 1979

[2] V.J. Aidala and S.C. Nardone, "Biased Estimation Properties of the Pseudolinear Tracking Filter", *IEEE Trans on AES*, V.18, No.4, 1982

[3] A. J. Borstad, "Bearings-Only Target Motion Analysis Estimation Characteristics", *Control and Computers*, Vol. 13, No. 3, 1985

[4] Shao-hui, Cui and Chang-qing, Zhu, "Application of Kalman Filter to Bearing-Only Target Tracking System", *Proceedings of ICSP*, 1996.

[5] Greg, "Applied Optimal Estimation", 1974

[6] S. E. Hammel and V. J. Aidala, "Observability Requirements for Three-Dimensional Tracking via Angle Measurements", *IEEE Transactions on Aerospace and Electronic Systems*,Vol.AES-21, No.2, Mar. 1985

[7] S. E. Hammel and V. J. Aidala, "Utilization of Modified Polar Coordinates for Bearings-Only Tracking", *IEEE Transactions on Automatic Control*, Vol. AC-28, No. 3, Mar. 1983

[8] Allen G. Lindgren and Kai F. Gong, Position and Velocity Estimation via Bearing Observations, *IEEE Transactions on Aerospace and Eletronic Systems,* vol. AES-14, no.4 , 1978.

[9] S.C. Nardone and V.J. Aidala, "Observability criteria for bearings-only target motion analysis", *IEEE Trans on AES*, V.17, No.3, 1981

[10] P. Randall, "Kinematic Ranging for IRSTs", *SPIE Vol. 1950 Acquisition, Tracking, and Pointing VII*, 1993

[11] B. Sridhar and Raymond E. Suorsa, Computer Architectures for a real-time passive ranging algorithm, SPIE, 1993

[12] B. Sridhar and Raymond E. Suorsa, Parallel Processing Systems for Passive Ranging During Helicopter Flight, SPIE 1996