Analysis of Dielectric-Loaded Waveguide

CHENG-CHEH YU AND TAH-HSIUNG CHU, MEMBER, IEEE

Abstract—In this paper, the calculation of eigenvalues including propagation constants and cutoff wavelengths of LSE_{nm} and LSM_{nm} modes in homogeneous and inhomogeneous lossless dielectric-slab-loaded rectangular waveguides using a modified variation–iteration method is presented. The initial eigenvalues used in the iteration are selected on the basis of a physical consideration. Numerical examples shown are capable of efficiently calculating propagation constants and cutoff wavelengths of LSE_{nm} and LSM_{nm} modes given values of the free-space propagation constant and the lower bound of the cutoff wavelength, respectively.

I. Introduction

SHOWN in Fig. 1 is the structure of a rectangular waveguide loaded with lossless dielectric slab (homogeneous or inhomogeneous). The propagation modes in the waveguide are the longitudinal section electric (LSE) and magnetic (LSM) modes, which can be derived from magnetic (π_h) and electric (π_e) Hertzian potential functions [1]. Theoretical analyses of the dielectric-loaded waveguide can be found in [2]–[8], where eigenvalues are calculated by solving the characteristic equations containing transcendental functions. The eigenvalues are then calculated numerically by using, for example, the variation method [1], [9]–[11], the finite element method [12], the mode-matching method [13], the variation–iteration method [14], and the method involving hypergeometric functions [15]. These numerical methods are efficient for calculating eigenvalues of the fundamental mode. The variation–iteration method, originally developed in atomic physics, has been shown to be able to calculate eigenvalues of the higher order modes. But in the process exact field distributions of all the lower order modes have to be calculated so that they can be subtracted from the initial trial field for the higher order mode of interest. The resulting trial field is then used to calculate the higher order mode eigenvalues. However, errors of the resulting higher order mode eigenvalues are accumulated, and the computation is time consuming using this approach [16].

In this paper, a modified variation–iteration method is formulated in Section II to solve the higher order mode eigenvalues efficiently without calculating the exact field distributions of all the lower order modes. In the iteration process, an adaptive update factor is introduced to stabilize the numerical behavior. In Section III, formulations

Manuscript received November 27, 1989; revised April 9, 1990. This work was supported by the Republic of China under National Science Fund Grant NSC78-0404-E002-55.

The authors are with the Electrical Engineering Department, National Taiwan University, Taipei, Taiwan, Republic of China.

IEEE Log Number 9036756.

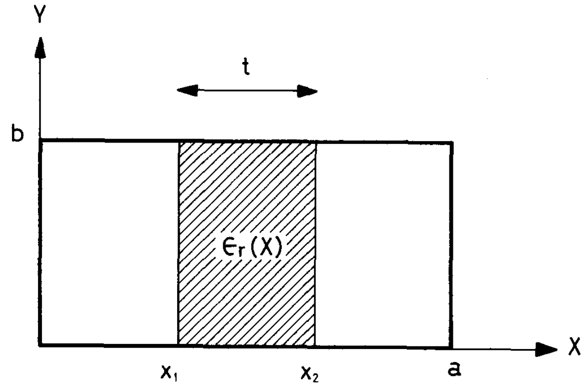


Fig. 1. Dielectric-slab-loaded waveguide structure.

of the propagation constant and cutoff wavelength of LSE and LSM modes of different orders (i.e., LSE_{nm} and LSM_{nm} modes) using this approach are developed. Numerical simulation results are given in Section IV. Initial values of propagation constants and cutoff wavelengths of LSE_{nm} and LSM_{nm} modes used in the iteration are also discussed on the basis of a physical consideration. Lastly, findings of this method are summarized in Section V.

II. Formulation of Modified Variation–Iteration Method

In general the eigenvalue problem can be characterized as

$$q_i = F(\phi_i) \tag{1}$$

where $F(\cdot)$ is the variational formulation, ϕ_i is an unknown eigenfunction, q_i is the corresponding unknown eigenvalue, and the subscript i is the order of the mode of interest.

In the $(n-1)$th iteration, the trial field $\phi_i^{(n-1)}$ can be found by substituting a trial eigenvalue $q_i^{(n-1)}$ into the related wave equation with appropriate boundary conditions. Therefore, by substituting $\phi_i^{(n-1)}$ into (1), one can obtain

$$q_i'^{(n)} = F\left(\phi_i^{(n-1)}\right) \tag{2}$$

where the prime indicates the resulting eigenvalue from (1).

The new eigenvalue $q_i^{(n)}$ in the (n)th iteration of the proposed modified variation–iteration approach is defined as

$$q_i^{(n)} = q_i^{(n-1)} + \alpha \delta q_i^{(n-1)} \tag{3}$$

where $\delta q_i^{(n-1)} = q_i'^{(n)} - q_i^{(n-1)}$ and α is an adaptive update factor with value $0 < \alpha \leqslant 1$. The criterion for adaptively assigning α its value is that when $|\delta q_i^{(n-1)}|$ is small, a larger value of α is chosen and vice versa. During the iteration, the convergence of eigenvalue q_i is reached as $|\delta q_i^{(n-1)}|$ becomes very small. The iteration is then terminated.

In the next section, we will use the modified variation–iteration method described above to calculate the propagation constants and cutoff wavelengths of LSE_{nm} and LSM_{nm} modes of the waveguide structure given in Fig. 1.

III. Formulation on Lossless-Dielectric-Slab-Loaded Waveguide

The permittivities of the air and dielectric regions in Fig. 1 are ϵ_0 and $\epsilon_r(x)\epsilon_0$, and the permeabilities are both μ_0. The stationary formulas for the propagation constants and cutoff wavelengths of LSE_{nm} and LSM_{nm} modes are given [1] as follows.

A. Stationary Formulas for LSE_{nm} Modes

The electric field distribution inside the dielectric-loaded waveguide is

$$\boldsymbol{E} = -j\omega\mu_0 \nabla \times \boldsymbol{\pi}_h \tag{4}$$

where the magnetic Hertzian potential is $\boldsymbol{\pi}_h = \boldsymbol{a}_x \phi_h(x, y)\cos(py)e^{-j\beta z}$, and $\phi_h(x, y)$ satisfies the differential equation

$$\frac{d^2\phi_h}{dx^2} + \left[\epsilon_r(x)k^2 - h^2 - \beta^2\right]\phi_h = 0 \tag{5}$$

with the boundary conditions $\phi_h = 0$ at $x = 0$ and a, and ϕ_h continuous at $x = x_1$ and x_2. In addition, $k = \omega\sqrt{\mu_0\epsilon_0}$, β is the propagation constant, $p = m\pi/b$ is the wavenumber in air and in dielectric regions in the y direction, with $m = 1, 2, \cdots$, and

$$h = \sqrt{k^2 - p^2 - \beta^2} \tag{6}$$

is the wavenumber in the air region in the x direction.

The stationary formula for the propagation constant, β, is

$$\beta^2 = \frac{\int_0^a \left[\left(\epsilon_r(x)k^2 - h^2\right)\phi_h^2 - \left(\frac{d\phi_h}{dx}\right)^2\right]dx}{\int_0^a \phi_h^2\, dx} \tag{7}$$

whereas the stationary formula for the cutoff wavelength, λ_c, is

$$\lambda_c^2 = \frac{4\pi^2 \int_0^a \epsilon_r(x)\phi_h^2\, dx}{\int_0^a \left[h^2\phi_h^2 + \left(\frac{d\phi_h}{dx}\right)^2\right]dx}. \tag{8}$$

B. Stationary Formulas for LSM_{nm} Modes

The magnetic field distribution inside the dielectric-loaded waveguide is

$$\boldsymbol{H} = j\omega\epsilon_r(x)\epsilon_0 \nabla \times \boldsymbol{\pi}_e \tag{9}$$

where the electric Hertzian potential is $\boldsymbol{\pi}_e = \boldsymbol{a}_x \phi_e(x, y)\sin(py)e^{-j\beta z}$, and $\phi_e(x, y)$ satisfies the differential equation

$$\frac{d}{dx}\left(\frac{1}{\epsilon_r(x)}\frac{d\phi_e}{dx}\right) + \frac{1}{\epsilon_r(x)}\left[\epsilon_r(x)k^2 - h^2 - \beta^2\right]\phi_e = 0 \tag{10}$$

with $d\phi_e/dx = 0$ at $x = 0$ and a, and $d\phi_e/dx$ continuous at $x = x_1$ and x_2.

The stationary formula for the propagation constant, β, is

$$\beta^2 = \frac{\int_0^a \frac{1}{\epsilon_r(x)}\left[\left(\epsilon_r(x)k^2 - h^2\right)\phi_e^2 - \left(\frac{d\phi_e}{dx}\right)^2\right]dx}{\int_0^a \frac{1}{\epsilon_r(x)}\phi_e^2\, dx} \tag{11}$$

whereas the stationary formula for the cutoff wavelength, λ_c, is

$$\lambda_c^2 = \frac{4\pi^2 \int_0^a \phi_e^2(x)\, dx}{\int_0^a \frac{1}{\epsilon_r(x)}\left[h^2\phi_e^2 + \left(\frac{d\phi_e}{dx}\right)^2\right]dx}. \tag{12}$$

C. Iteration Algorithm

The iteration algorithm used in the proposed modified iteration–variation method for calculating propagation constants and cutoff wavelengths involves the following three steps. Note in the following description that q is used to represent eigenvalue β or λ_c for convenience.

1) Choose the initial value $q^{(0)}$ using the procedure to be described in subsection III-D.
2) Use the approach to be described in subsection III-E to find the corresponding trail field $\phi_h^{(0)}$ or $\phi_e^{(0)}$. Then $q'^{(1)}$ is calculated using the stationary formulas given above in subsections III-A and III-B. This step can be generalized to the (n)th iteration.
3) In the (n)th iteration $q^{(n)}$ is calculated using (3).

If the absolute value of $\delta_q^{(n)}$ is within the specified value of accuracy, the convergence is reached. The iteration is then terminated.

D. Initial Values of $\beta^{(0)}$ and $\lambda_c^{(0)}$

Since the dielectric-loaded waveguide can support an infinite number of modes, there are an infinite number of propagation constants and cutoff wavelengths corresponding to these modes. However, the number of propagation constants becomes finite if the waveguide is under the excitement of a given value of free-space propagation constant. Similarly, the number of cutoff wavelengths above a given lower bound value of the cutoff wavelength also becomes finite. The description for selecting initial propagation constants and cutoff wavelengths used in the iteration is given in the following.

When the wavenumber h given in (6) is real, the transversal wave in the air region is a standing wave in the x direction. The energy propagated (for calculating β) or stored (for calculating λ_c) in the air region then becomes noticeable. The propagation constants and cutoff wavelengths of the dielectric-loaded waveguide can be treated as those of an empty waveguide with dimensions a and b perturbed by a dielectric slab. Therefore, the propagation constants and cutoff wavelengths of the empty waveguide are suitable as initial values for the LSE_{nm} and LSM_{nm} modes. Equations for these initial values are

$$\beta^{(0)} = \sqrt{k^2 - \left(\frac{n\pi}{a}\right)^2 - \left(\frac{m\pi}{b}\right)^2} \tag{13}$$

$$\lambda_c^{(0)} = \frac{2}{\sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2}} \tag{14}$$

where $n = 1, 2, \cdots$ for both LSE_{nm} and LSM_{nm} modes, $m = 0, 1, 2, \cdots$ for the LSE_{nm} mode, and $m = 1, 2, 3, \cdots$ for the LSM_{nm} mode.

When the wavenumber h in (6) is purely imaginary, the transversal wave in the air region becomes evanescent in the x direction. The energy propagated (for calculating β) or stored (for calculating λ_c) in the dielectric region is then dominant. In this case, the propagation constants and cutoff wavelengths of the dielectric-loaded waveguide can be treated as those of a waveguide completely filled with the dielectric with dimensions t and b. Therefore, initial values of the propagation constants and cutoff wavelengths are selected as those of the waveguide completely filled with homogeneous dielectric of dimensions t and b, and the dielectric constant is the maximum value $\epsilon_{r,\max}$ of the dielectric slab considered. Initial values are given as

$$\beta^{(0)} = \sqrt{\epsilon_{r,\max} k^2 - \left(\frac{n\pi}{t}\right)^2 - \left(\frac{m\pi}{b}\right)^2} \tag{15}$$

$$\lambda_c^{(0)} = \frac{2\sqrt{\epsilon_{r,\max}}}{\sqrt{\left(\frac{n}{t}\right)^2 + \left(\frac{m}{b}\right)^2}} \tag{16}$$

where $n = 1, 2, \cdots$ for both LSE_{nm} and LSM_{nm} modes, $m = 0, 1, 2, \cdots$ for the LSE_{nm} mode, and $m = 1, 2, 3, \cdots$ for the LSM_{nm} mode.

In general, two sets of initial values for the propagation constants and cutoff wavelengths of LSE_{nm} and LSM_{nm} modes according to the approaches described above are selected for the iteration algorithm. From the simulation results obtained in the next section, these initial values will be shown to converge to all the correct propagation constants and cutoff wavelengths of the selected modes.

E. Approach to Calculate $\phi^{(n)}$ from Given $\beta^{(n)}$ and $\lambda_c^{(n)}$

Analytic expressions for the homogeneous-dielectric-slab-loaded waveguide can be found in [1]. Therefore $\phi_h^{(n)}$ and $\phi_e^{(n)}$ can be calculated for given $\beta^{(n)}$ or $\lambda_c^{(n)}$. For inhomogeneous-dielectric-slab-loaded waveguide, one can substitute $\beta^{(n)}$ into (5) for the LSE mode or into (10) for the LSM mode and then use the finite element method to solve the differential equations with the appropriate boundary conditions to calculate $\phi_h^{(n)}$ and $\phi_e^{(n)}$. If $\lambda_c^{(n)}$ is given, by setting β to zero in (5) and (10) one can also solve $\phi_h^{(n)}$ and $\phi_e^{(n)}$ using the finite element method.

Numerical results obtained using the described modified variation–iteration method of homogeneous- and inhomogeneous-dielectric-slab-loaded waveguides will be discussed in the next section.

IV. Simulation Results

In all the following numerical examples, the waveguide dimension a is set at 2, and the adaptive update factor α in (3) is given as

$$\alpha = \begin{cases} 0.05, & 1.00 \leqslant |\delta q^{(n)}| \\ 0.10, & 0.10 \leqslant |\delta q^{(n)}| < 1.00 \\ 0.50, & 0.01 \leqslant |\delta q^{(n)}| < 0.10 \\ 1.00, & |\delta q^{(n)}| < 0.01. \end{cases} \tag{17}$$

The values of α and ranges of $|\delta q^{(n)}|$ given in (17) are chosen on the basis of the criterion given in Section II for adequate convergence speed in the calculation of eigenvalues. However, their values are not critical.

1) LSE_{10} Mode in a Waveguide Centrally Loaded with a Homogeneous Dielectric Slab with Dimensions $b/a = 1/2$, $x_1 = (a-t)/2$, $x_2 = (a+t)/2$: Fig. 2 shows the results of the propagation constants of the LSE_{10} mode for various dielectric thicknesses with $\epsilon_r(x) = 9$. The cutoff wavelengths of the LSE_{10} mode for various dielectric ϵ_r constants and dielectric thicknesses are shown in Fig. 3; these are in good agreement with the values obtained by Vartanian [3].

2) LSE_{10} Mode in a Waveguide Centrally Loaded with an Inhomogeneous Dielectric Slab with Dimensions $b/a = 1/2$, $x_1 = (a-t)/2$, $x_2 = (a+t)/2$, and $\epsilon_r(x) = 1 + 4(\epsilon_{r,\max} - 1)(x - x_1)(x_2 - x)/(x_2 - x_1)^2$: For $\epsilon_{r,\max} = 9$, results of the propagation constants of the LSE_{10} mode for various dielectric thicknesses are shown in Fig. 4, which are in good agreement with the values given by Chen [11]. The cutoff wavelengths of the LSE_{10} mode for various values of $\epsilon_{r,\max}$ and various dielectric thicknesses are shown in Fig. 5.

3) LSE and LSM Modes in a Waveguide Centrally Loaded with a Homogeneous Dielectric with Dimensions $b/a = t/a = 1/2$, $x_1 = (a-t)/2$, $x_2 = (a+t)/2$, $\epsilon_r(x) = 2.0$: For the propagation constant calculation, the free-

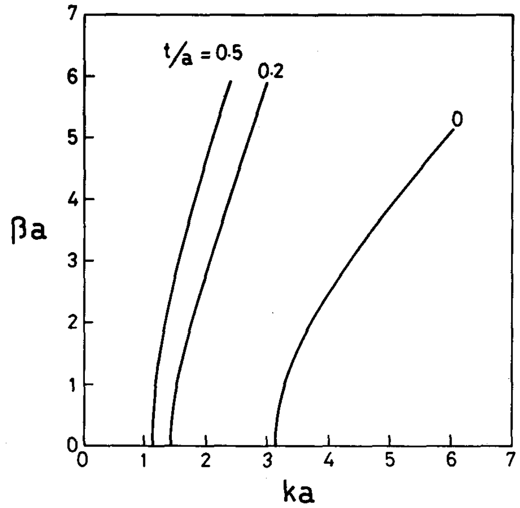


Fig. 2. Plot of propagation constant of LSE_{10} mode in a waveguide centrally loaded with homogeneous dielectric with $b/a = 1/2$, $x_1 = (a-t)/2$, $x_2 = (a+t)/2$, and $\epsilon_r(x) = 9$.

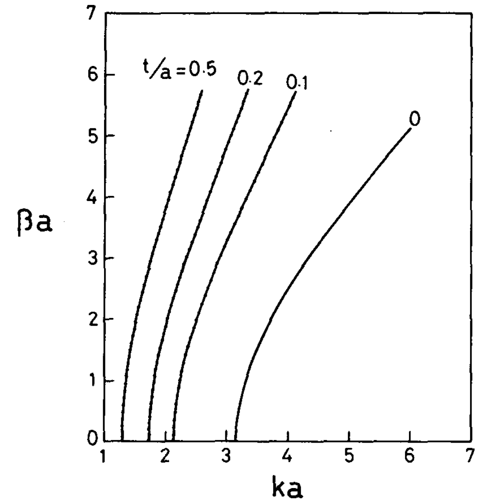


Fig. 4. Plot of propagation constant of LSE_{10} mode in a waveguide centrally loaded with inhomogeneous dielectric with $b/a = 1/2$, $x_1 = (a-t)/2$, $x_2 = (a+t)/2$, $\epsilon_r(x) = 1 + 4(\epsilon_{r,\max} - 1)(x - x_1)(x_2 - x)/(x_2 - x_1)^2$, and $\epsilon_{r,\max} = 9$ (results of solid line obtained by the modified variation–iteration method and dotted line by Chen [11]).

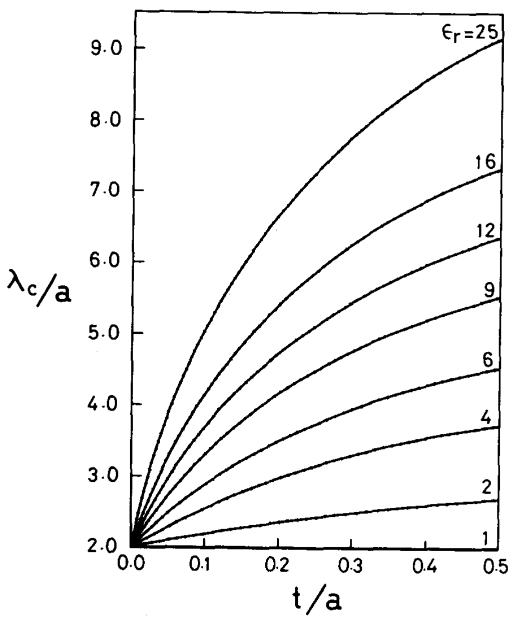


Fig. 3. Plot of cutoff wavelength of LSE_{10} mode in a waveguide centrally loaded with homogeneous dielectric with $b/a = 1/2$, $x_1 = (a-t)/2$, and $x_2 = (a+t)/2$ (results of solid line obtained by the modified variation–iteration method and dotted line by Vartanian [3]).

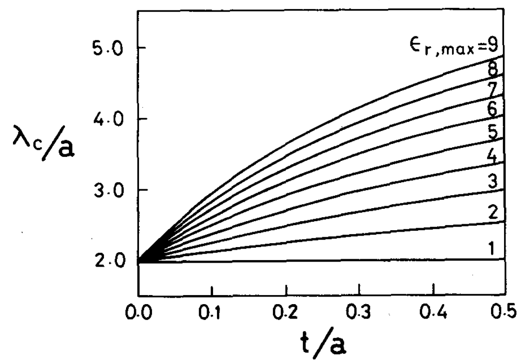


Fig. 5. Plot of cutoff wavelength of LSE_{10} mode in a waveguide centrally loaded with inhomogeneous dielectric with $b/a = 1/2$, $x_1 = (a-t)/2$, $x_2 = (a+t)/2$, and $\epsilon_r(x) = 1 + 4(\epsilon_{r,\max} - 1)(x - x_1)(x_2 - x)/(x_2 - x_1)^2$.

space propagation constant is selected as $4\pi/a$. For the cutoff wavelength calculation, the lower bound of the cutoff wavelength is selected as $a/2$. Numerical results are tabulated in Table I.

4) LSE and LSM Modes in a Waveguide Offset Loaded with a Homogeneous Dielectric with Dimensions $b/a = t/a = 1/2$, $x_1 = t$, $x_2 = a$, $\epsilon_r(x) = 2.0$: For the calculation of the propagation constant and cutoff wavelength, the free-space propagation constant and the lower bound of the cutoff wavelength are both selected as in case 3. Numerical results are tabulated in Table II. Note that, during the iteration, there may be more than one initial eigenvalue (β or λ_c) converging to the same correct final value.

In order to verify the numerical results for the propagation constants and cutoff wavelengths of the LSE_{nm} and LSM_{nm} modes obtained using the modified variation–iteration method, the numerical results given in Tables I and II are verified with the related characteristic equations, in which the positions of zero crossing correspond to the correct eigenvalues. For homogeneous-dielectric-slab-loaded waveguide, the characteristic equations can be formulated using the transverse resonant method [1]. For inhomogeneous-dielectric-slab-loaded waveguide, the inhomogeneous dielectric slab can be ap-

TABLE I

RESULTS OF PROPAGATION CONSTANTS AND CUTOFF WAVELENGTHS OF LSE_{nm} AND LSM_{nm} MODES IN A HOMOGENEOUS DIELECTRIC CENTRALLY LOADED WAVEGUIDE WITH $b/a = t/a = 1/2$, $x_1 = (a-t)/2$, $x_2 = (a+t)/2$, AND $\epsilon_r(x) = 2$

mode	βa	λ_c/a
LSE_{10}	17.127	2.702
LSE_{20}	15.147	1.241
LSE_{30}	11.821	0.785
LSE_{40}	7.602	0.589
LSE_{11}	15.933	1.216
LSE_{21}	13.783	0.888
LSE_{31}	10.013	0.656
LSE_{41}	4.280	0.524
LSE_{12}	11.637	0.668
LSE_{22}	8.457	0.577
LSM_{11}	15.739	1.215
LSM_{21}	13.203	0.838
LSM_{31}	9.450	0.662
LSM_{41}	4.989	0.529
LSM_{12}	11.369	0.656
LSM_{22}	7.476	0.563

proximated as a stack of many thin homogeneous slabs. The characteristic equations can then be obtained by the matrix method proposed by Gardiol [6].

The numerical examples given above show that values of the propagation constants and cutoff wavelengths of the LSE_{nm} and LSM_{nm} modes calculated using the modified variation–iteration method developed in this paper are in good agreement with the exact values calculated from the characteristic equations with errors less than 10^{-2}. Moreover, the calculation results of the propagation constants and cutoff wavelengths of the LSE_{nm} and LSM_{nm} modes contain all the existing modes, which are also verified using the characteristic equations.

In the following, a few observations are made on the convergence of the iteration algorithm. First, the initial eigenvalues (β and λ_c) obtained from (13)–(16) are generally enough to cover all the correct eigenvalues; in other words, there may be a few selected initial eigenvalues that will converge to the same final eigenvalues after the computation. In the simulation, the value of m is given first; values of n can then be specified to determine the corresponding LSE_{nm} and LSM_{nm} modes as all the eigenvalues are obtained. Second, in some cases, the initial eigenvalue may converge to the value satisfying the characteristic equation (or (5) and (10)), but the resulting wavenumber h (in the air region) becomes zero. This resulting eigenvalue is then discarded because it corresponds to the null field, as can be seen from (4) and (9). For example, in Table I, if the initial propagation constant $\beta^{(0)}$ is assigned to be $10.419/a$ in evaluating the propagation constant of the LSM_{n1} mode for the structure given in subsection IV-C, the resulting propagation

TABLE II

RESULTS OF PROPAGATION CONSTANTS AND CUTOFF WAVELENGTHS OF LSE_{nm} AND LSM_{nm} MODES IN A HOMOGENEOUS DIELECTRIC OFFSET LOADED WAVEGUIDE WITH $b/a = t/a = 1/2$, $x_1 = t$, $x_2 = a$, AND $\epsilon_r(x) = 2$

mode	βa	λ_c/a
LSE_{10}	16.932	2.482
LSE_{20}	14.286	1.178
LSE_{30}	11.193	0.820
LSE_{40}	8.312	0.597
LSE_{11}	15.723	1.154
LSE_{21}	12.832	0.823
LSE_{31}	9.264	0.674
LSE_{41}	5.441	0.535
LSE_{12}	11.349	0.654
LSE_{22}	6.797	0.546
LSM_{11}	16.368	1.312
LSM_{21}	14.254	0.841
LSM_{31}	9.644	0.668
LSM_{41}	5.441	0.536
LSM_{12}	12.226	0.690
LSM_{22}	9.205	0.590

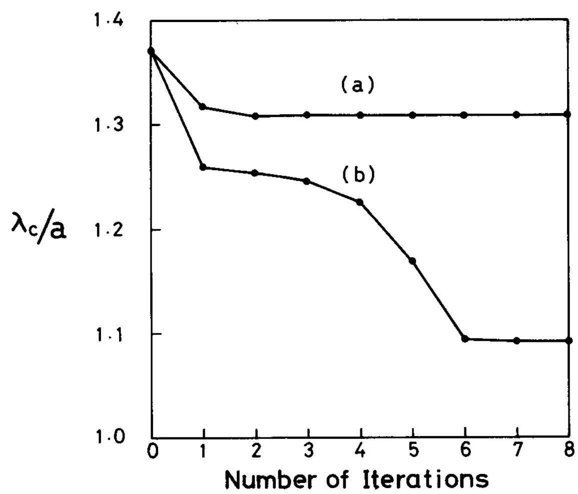


Fig. 6. Effect of adaptive update factor upon the cutoff wavelength calculation of the LSE_{n2} mode in a waveguide that is offset loaded with homogeneous dielectric with $b/a = t/a = 1/2$, $x_1 = t$, $x_2 = a$, and $\epsilon_r(x) = 2$. Curve (a) is obtained with adaptive update given in eq. (17), and curve (b) is obtained without adaptive update factor (i.e., $\alpha = 1$).

constant will converge to $10.883/a$, and h will become zero as calculated from (6). Third, the rate of convergence of the modified variation–iteration method developed in this paper is very efficient. The numbers of iterations for the results given in Tables I and II are all less than 15. Fourth, the adaptive update factor α is able to stabilize the numerical behavior of the iteration algorithm. This factor helps the initial eigenvalue used to converge to the nearest correct result, as illustrated in Fig. 6. In this figure, $\lambda_c^{(0)}$ is $0.686a$ in evaluating the cutoff wavelength of the LSE_{n2} mode for the structure used in Table II. For curve (a), the adaptive update factor α is given by (17). The convergence is shown to be very smooth, and efficiently reaches to the nearest mode with $\lambda_c = 0.654a$. However, for curve (b) the adaptive update factor α is not used (i.e., $\alpha = 1$), and the resulting cutoff wavelength shown converges to the other mode with $\lambda_c = 0.546a$. In general, α is small when the absolute value of the increment of eigenvalue is large and vice versa. However, a larger update factor α usually leads to faster convergence but may skip some eigenvalues that exist. Therefore, properly selecting the adaptive update factor for different ranges of eigenvalue increment is necessary.

V. Conclusion

The use of modified variation–iteration with an adaptive update factor in the iteration algorithm permits us to evaluate the eigenvalues (including propagation constants and cutoff wavelengths) of LSE_{nm} and LSM_{nm} modes in a rectangular waveguide loaded with a homogeneous or inhomogeneous lossless dielectric slab in an efficient manner with the initial eigenvalues given in subsection III-D based on a physical consideration.

As shown in the numerical examples, this method is very efficient for finding all the eigenvalues of LSE_{nm} and LSM_{nm} modes by a given free-space propagation constant for the propagation constant calculation and a given lower bound of the cutoff wavelength for the cutoff wavelength calculation. In addition, the importance of the adaptive update factor in preventing the possibility of missing existing eigenvalues is also discussed.

References

[1] R. E. Collin, *Field Theory of Guided Waves*. New York: McGraw-Hill, 1960, ch. 6.

[2] L. Pincherle, "Electromagnetic waves in metal tubes filled longitudinally with two dielectrics," *Phys. Rev.*, vol. 66, pp. 118–130, Sept. 1944.

[3] P. H. Vartanian, W. P. Ayres, and A. L. Helgesson, "Propagation in dielectric slab loaded rectangular waveguide," *IRE Trans. Microwave Theory Tech.*, vol. MTT-6, pp. 215–222, Apr. 1958.

[4] R. Seckelmann, "Propagation of TE modes in dielectric loaded waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-14, pp. 518–527, Nov. 1966.

[5] N. Eberhardt, "Propagation in the off center *E*-plane dielectrically loaded waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-15, pp. 282–289, May 1967.

[6] F. E. Gardiol, "Higher-order modes in dielectrically loaded rectangular waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 919–924, Nov. 1968.

[7] K. A. Zaki and A. Atia, "Modes in dielectric-loaded waveguides and resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 1039–1045, Dec. 1983.

[8] K. A. Zaki and C. Chen, "Intensity and distribution of hybrid mode fields in dielectric-loaded waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 1442–1447, Dec. 1985.

[9] A. D. Berk, "Variational principles for electromagnetic resonators and waveguides," *IRE Trans. Antennas Propagat.*, pp. 104–111, Apr. 1956.

[10] R. E. Collin and R. M. Vaillancourt, "Application of Rayleigh–Ritz method to dielectric steps in waveguides," *IRE Trans. Microwave Theory Tech.*, vol. MTT-7, pp. 177–184, July 1957.

[11] C. T. Liu and C. H. Chen, "A variational theory for wave propagation in inhomogeneous dielectric slab loaded waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 805–812, Aug. 1981.

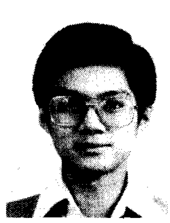
[12] Z. J. Csendes and R. Sivester, "Numerical solution of dielectric loaded waveguides: I—finite element analysis," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 1124–1131, Dec. 1970.

[13] Z. J. Csendes and R. Sivester, "Numerical solution of dielectric loaded waveguides: II—modal approximation technique," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 504–509, June 1971.

[14] A. S. Vander Vorst and R. J. M. Govaerts, "Application of variation iteration method to inhomogeneously loaded waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 468–475, Aug. 1970.

[15] W. G. Lin, "Electromagnetic wave propagation in uniform waveguides containing inhomogeneous dielectric," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 339–348, Apr. 1980.

[16] P. H. Morse and H. Feshbach, *Methods of Theoretical Physics*. New York: McGraw-Hill, 1953, pt. II, pp. 1026–1030, pp. 1137–1158.



Cheng-Cheh Yu was born on January 1, 1964, in Taipei, Taiwan, Republic of China. He received the M.S. degree from the National Taiwan University, Taipei, Taiwan, in 1988, and is now working toward the Ph.D. degree there. His research interests include microwave and millimeter-wave passive devices and circuits, antennas, optoelectronics, optical signal processing, and electronic circuit design.

Tah-Hsiung Chu (M'87) was born in Taiwan on July 30, 1953. He received the B.S. degree from the National Taiwan University, Taipei, Taiwan, in 1976, and the M.S. and Ph.D. degrees from the University of Pennsylvania in 1980 and 1983, respectively, all in electrical engineering.

From 1983 to 1986 he was a member of the technical staff of the Microwave Technology Center at the RCA David Sarnoff Research Center in Princeton, NJ. Since 1986 he has been on the faculty of the Department of Electrical Engineering at the National Taiwan University, where he is now a Professor of Electrical Engineering. His research interests include microwave imaging systems, electromagnetic theory, microwave circuit and subsystem design, microwave measurement techniques, and digital and optical signal processing.