Performance Analysis and Improvement of Decorrelating Detection for Multi-Rate DS/CDMA

Huang Lee, *Member, IEEE,* and Kwang-Cheng Chen, *Senior Member, IEEE*

***Abstract*— We study access strategies for decorrelating detection applied in multi-rate direct-sequence code-division multiple-access (DS/CDMA) systems, including multi-modulation (MM), multi-code (MC), and variable-spreading-length (VSL) schemes by jointly considering signal constellations and multiple-access interference. The mathematical analysis shows that when the number of active users is large, the MM scheme outperforms MC and VSL schemes especially for high-rate transmission. We also conclude that the design of modulation is important in MC and VSL schemes. Numerical analysis demonstrates that applying 4-PSK instead of 2-PSK in MC and VSL schemes can improve about 9 dB performance gain. In addition, by considering cross-correlation of noise components, we propose a detector that minimizes the symbol error probability under the constraint that the complexity grows linearly with the number of active users as decorrelating detectors. Simulations show that about 4 dB performance gain over conventional decorrelating detectors can be achieved for multi-rate DS/CDMA communications.**

***Index Terms*— Decorrelating multiuser detection, code-division multiple-access, multi-rate, multi-code, variable-spreading-length, multi-modulation.**

I. Introduction

IN THE past few years, there has been much research on multi-rate direct-sequence code-division multiple-access (DS/CDMA) systems. According to previous research, multi-code (MC) [1] and variable-spreading-length (VSL) [2] are two widely considered access strategies in multi-rate systems. With conventional detection, it was shown in [3] [4] that MC and VSL schemes offer similar bit-error-rate (BER) performance and always outperform the multi-modulation (MM) scheme [3] for high level modulation (e.g., 16-QAM), where the MM strategy realizes multi-rate transmission by varying the number of elements in signal constellations. However, this result does not necessarily apply to systems considering multiuser detection such as decorrelating detection.

In this letter, we revisited decorrelating detection for MC, VSL, and MM schemes over synchronous additive white Gaussian noise channels. In [4] [5] [6], MC and VSL schemes were analyzed only by evaluating the multiple-access interference (MAI) introduced by spreading sequences, which ignored the fact that multi-rate can also be realized by designing signal constellations as the MM scheme. To thoroughly understand the design of multi-rate communications, a complete analysis of different multi-rate strategies is very much desired.

In addition, previous studies [4] [5] [6] on the low-rate decorrelator (LRD), which detects information bits in a low-rate interval, ignored the cross-correlation of noise components so that the performance of LRD is degraded. By taking the cross-correlation into consideration, we propose a novel detector to improve the performance over LRD.

Manuscript received May 20, 2004. The associate editor coordinating the review of this letter and approving it for publication was Prof. M. Saquib.

The authors are with the Department of Electrical Engineering, National Taiwan University, Taipei, Taiwan (e-mail: hlee@ee.ntu.edu.tw).

Digital Object Identifier 10.1109/LCOMM.2005.02019.

II. System Model

For simplicity, we consider a dual rate DS/CDMA system where the high data rate r_2 is an integer multiple M of the low data rate r_1, and all low-rate users are assigned a spreading sequence with length N and antipodal modulation. Assume that (a) there are K_1 low-rate users (LRUs) and K_2 high-rate users (HRUs) where $K \triangleq K_1+K_2$; (b) the average bit energy E_b of all users are identical; and (c) the spreading sequences used in the system are linearly independent.

After chip-matched filtering, the generalized form of the received signal in low-rate symbol interval can be written as:

$$\underline{r} \triangleq \sum_{k=1}^{K} a_k \sum_{d=1}^{D_k} x_{k,d}\underline{s}_{k,d} + \underline{n} = \mathbf{SA}\underline{x} + \underline{n} \quad (1)$$

where $|a_k|$ and $\measuredangle a_k$ are the received amplitude and phase of user k; D_k is the number of spreading sequences assigned to user k; $x_{k,d}$ and $\underline{s}_{k,d}$ represent the received symbols and spreading sequences of user k; $\underline{n}$ is the $N \times 1$ zero-mean Gaussian random vector (GRV) with covariance matrix $\sigma^2 \mathbf{I}_{N\times N}$; $\mathbf{S} \triangleq [\underline{s}_{1,1} \cdots \underline{s}_{k,1} \cdots \underline{s}_{k,D_k} \cdots \underline{s}_{K,D_K}]_{N\times \widetilde{K}}$; $\mathbf{A} \triangleq \mathrm{diag}\{a_1, \cdots, a_k, \cdots, a_K\}$ is a $\widetilde{K}\times\widetilde{K}$ diagonal matrix; $\underline{x} \triangleq [x_{1,1} \cdots x_{k,1} \cdots x_{k,D_k} \cdots x_{K,D_K}]^T$ is a $\widetilde{K}\times 1$ vector; $\widetilde{K} \triangleq \sum_{k=1}^{K} D_k$; and the superscript $(\cdot)^T$ denotes transpose.

We specify the parameters of (1) for different schemes:

- MM scheme: D_k=1 for all users; $\underline{s}_{k,1}=\frac{1}{\sqrt{N}}[c_{k,1,1} \cdots c_{k,1,N}]^T$; $x_{k,1} \in$ {PAM/PSK/QAM signal constellation}.
- MC scheme: D_k=1 for LRUs, and D_k=M for HRUs. $\underline{s}_{k,d}=\frac{1}{\sqrt{N}}[c_{k,d,1} \cdots c_{k,d,N}]^T$. $x_{k,d} \in \{+\sqrt{E_b}, -\sqrt{E_b}\}$.
- VSL scheme: D_k=1 and $\underline{s}_{k,d}=\frac{1}{\sqrt{N}}[c_{k,d,1} \cdots c_{k,d,N}]^T$ for LRUs. D_k=M and $\underline{s}_{k,d}=\sqrt{\frac{M}{N}}[\underline{0}^T_{(d-1)N/M}\ c_{k,d,1} \cdots c_{k,d,N/M}\ \underline{0}^T_{(M-d)N/M}]^T$ for HRUs where $\underline{0}_n$ represents the $n\times 1$ zero vector. $x_{k,d} \in \{+\sqrt{E_b}, -\sqrt{E_b}\}$.

$c_{k,d,n} \in \{+1, -1\}$ are the chip codes.

III. Decorrelating Detection

For any user k, the received signal $\underline{r}$ can be decorrelated by

$$\underline{y}_k \triangleq \frac{1}{a_k}\left(\mathbf{S}_k^H\mathbf{S}_k - \mathbf{S}_k^H\mathbf{T}_k\left(\mathbf{T}_k^H\mathbf{T}_k\right)^{-1}\mathbf{T}_k^H\mathbf{S}_k\right)^{-1} \cdot \mathbf{S}_k^H\left(\mathbf{I} - \mathbf{T}_k\left(\mathbf{T}_k^H\mathbf{T}_k\right)^{-1}\mathbf{T}_k^H\right)\underline{r} \quad (2)$$

where $\mathbf{S}_k \triangleq [\ \underline{s}_{k,1}\ \ \underline{s}_{k,2}\ \cdots\ \underline{s}_{k,D_k}\ ]_{N\times D_k}$, $\mathbf{T}_k$ is the $N \times \left(\widetilde{K} - D_k\right)$ matrix by striking out all columns related to the user k from the matrix $\mathbf{S}$, and the superscript $(\cdot)^H$ denotes Hermitian operator. After decorrelating, we define

$\underline{n}_k$ the noise vector corresponding to user k's signal $\underline{x}_k \triangleq \begin{bmatrix} x_{k,1} & x_{k,2} & \cdots & x_{k,D_k} \end{bmatrix}^T$, and its $D_k \times D_k$ covariance matrix is given by $E\left[\underline{n}_k \underline{n}_k^H\right] = \sigma^2 \left|a_k\right|^{-2} \mathbf{R}_k$ where $\mathbf{R}_k = \left(\mathbf{S}_k^H \mathbf{S}_k - \mathbf{S}_k^H \mathbf{T}_k \left(\mathbf{T}_k^H \mathbf{T}_k\right)^{-1} \mathbf{T}_k^H \mathbf{S}_k\right)^{-1}$.

The LRD [4] [5] [6] ignores the fact that $\mathbf{R}_k$ is not a diagonal matrix, yielding the following decision for user k:

$$\widehat{\underline{x}}_k^{LRD} \triangleq \arg\max_{\underline{x}_k} \mathrm{Re}\left(2\underline{y}_k^H \underline{x}_k - \underline{x}_k^H \underline{x}_k\right). \tag{3}$$

However, since the noise components in $\underline{n}_k$ are correlated, the matrix $\mathbf{R}_k$ should be considered in the detector to improve the performance. We propose the modified detector

$$\begin{aligned} \widehat{\underline{x}}_k &\triangleq \arg\min_{\underline{x}_k} \left(\underline{y}_k - \underline{x}_k\right)^H \mathbf{R}_k^{-1} \left(\underline{y}_k - \underline{x}_k\right) \\ &= \arg\max_{\underline{x}_k} \mathrm{Re}\left(2\underline{y}_k^H \widetilde{\underline{x}}_k - \underline{x}_k^H \widetilde{\underline{x}}_k\right) \end{aligned} \tag{4}$$

where $\widetilde{\underline{x}}_k = \mathbf{R}_k^{-1} \underline{x}_k$. In Appendix, we prove that the modified detector is optimal in the sense of minimizing the symbol error probability while the receiver of user k is only allowed to utilize statistic $\underline{Y}_k$ as LRD. Therefore, we call the modified decorrelating detector the optimum decorrelating detector (ODD). Note that the performance of ODD is still worse than that of the jointly optimum multiuser detector [7] because the receiver of user k observes only $\underline{Y}_k$ not $\underline{Y}_{k'}$ for $k' = 1, \ldots, K$, and suffers the noise enhancement effect. Besides, because the ODD improves the performance by considering the cross-correlation of $\underline{n}_k$, it is easy to show that LRD and ODD have the same performance when D_k is equal to one.

IV. System Analysis

We prove in Appendix that when the noise power is small enough, the performance can be evaluated by the normalized minimum distance

$$d_{\min}^2 \triangleq \min_{\underline{x}_k \neq \underline{x}_k'} \frac{1}{E_b} \left(\underline{x}_k' - \underline{x}_k\right)^H \mathbf{R}_k^{-1} \left(\underline{x}_k' - \underline{x}_k\right). \tag{5}$$

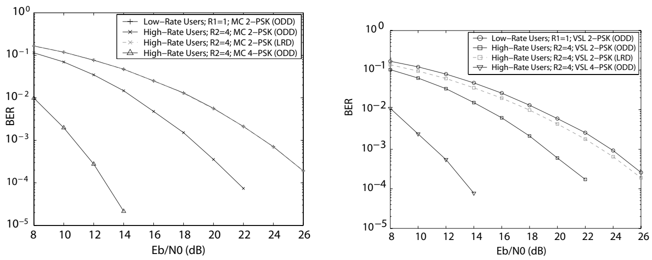
In addition to the minimum distance, MAI is also characterized in (5) since MAI can be regarded as the noise enhancement effect represented by $\mathbf{R}_k^{-1}$ while applying decorrelating detector. Moreover, because $\mathbf{R}_k^{-1}$ is composed of spreading sequences, the performance is often dependent on the use of spreading sequences [6] [8], thus making global comparison of access schemes difficult. To remove the influence of spreading sequences in the comparison, we employ the random spreading sequence approach to analyze performance as [3] [4] [8]. The expectation value of the normalized minimum distance is

$$\overline{d}_{\min}^2 \triangleq \min_{\underline{x}_k \neq \underline{x}_k'} E\left[\frac{1}{E_b} \left(\underline{x}_k' - \underline{x}_k\right)^H \mathbf{R}_k^{-1} \left(\underline{x}_k' - \underline{x}_k\right)\right]. \tag{6}$$

The $\overline{d}_{\min}^2$ of LRUs can be obtained from the first LRU without loss of generality and is given by

$$\overline{d}_{\min}^2 = \gamma_{low} E\left[1 - \underline{s}_{1,1}^H \mathbf{T}_1 \left(\mathbf{T}_1^H \mathbf{T}_1\right)^{-1} \mathbf{T}_1^H \underline{s}_{1,1}\right] = \gamma_{low} \eta_{low}$$

where $\gamma_{low} = \min_{\underline{x}_1 \neq \underline{x}_1'} \frac{1}{E_b} \left|\underline{x}_1' - \underline{x}_1\right|^2$ is the normalized minimum distance of the signal constellation without MAI. By the similar procedure, the expectation value of the normalized minimum distance of HRUs is given by $\overline{d}_{\min}^2 = \gamma_{high} \eta_{high}$ where $\gamma_{high} = \min_{\underline{x}_k \neq \underline{x}_k'} \frac{1}{E_b} \left|\underline{x}_k' - \underline{x}_k\right|^2$ and k indicates any HRU.



(a) MC scheme. (b) VSL scheme.

Fig. 1. BER vs SNR. $K_1 = 6$, $K_2 = 6$, $r_1 = 1$, and $r_2 = 4$.

The noise enhancement effect and the pattern of signal constellations can be characterized by η_{low} (η_{high}) and γ_{low} (γ_{high}), respectively. Further, the coefficient η_{low} (η_{high}) is equal to the optimum near-far resistance [7] and can be regarded as a coefficient that shortens the normalized minimum distance. Therefore, the performance analysis of different schemes is achieved by jointly considering MAI and signal constellations.

The $\overline{d}_{\min}^2$ of the MM scheme is given by

$$\overline{d}_{\min}^2 = \begin{cases} \gamma_{low}\left(1 - \left(K_1 + K_2 - 1\right)/N\right) & \text{LRU} \\ \gamma_{high}\left(1 - \left(K_1 + K_2 - 1\right)/N\right) & \text{HRU.} \end{cases} \tag{7}$$

The γ_{low} and γ_{high} of MC and VSL schemes are equal to 4, and the η_{low} and η_{high} of MC and VSL schemes can be obtained by Propositions in [8]. For the MC scheme, we get

$$\overline{d}_{\min}^2 = \begin{cases} 4\left(1 - \left(K_1 + MK_2 - 1\right)/N\right) & \text{LRU} \\ 4\left(1 - \left(K_1 + MK_2 - 1\right)/N\right) & \text{HRU.} \end{cases} \tag{8}$$

For VSL (with general random code [8] applied), we obtain

$$\overline{d}_{\min}^2 = \begin{cases} 4\left(1 - \left(K_1 + MK_2 - 1\right)/N\right) & \text{LRU} \\ 4\left(1 - \left(K_1 + MK_2 - M\right)/N\right) & \text{HRU.} \end{cases} \tag{9}$$

In the MM scheme, the coefficients γ_{low} and γ_{high} are equal to 4 for 2-PSK and 4-PSK modulation, and γ_{high} is smaller than 4 for higher level modulation (e.g., $\gamma_{high} = \frac{8}{5}$ for 16-QAM). Hence, for high level modulation, when the number of active users is small, MC and VSL schemes have better performance than MM. However, once the number of active users increases, $\overline{d}_{\min}^2$ of MC and VSL decreases more rapidly than that of MM scheme so that the MM scheme can outperform MC and VSL.

V. Simulation Results

In all simulations, we apply random spreading sequences [3] [4] [8] with length N equal to 32.

We demonstrate that the proposed ODD achieves better performance than LRD. In our simulation, ODD improves the performance about 5 and 4 dB over LRD for high-rate MC and VSL users, respectively, as shown in Fig. 1.

We investigate the BER performance for different access strategies and transmission rates as the number of HRUs K_2 increases in Fig. 2. We apply ODD in simulations where the signal-to-noise ratio (SNR $= \frac{E_b}{2\sigma^2}$) is fixed to be 10 dB. The numerical analysis indicates that when the number of active users increases, the performance of MC and VSL schemes degrades more rapidly than that of MM scheme.

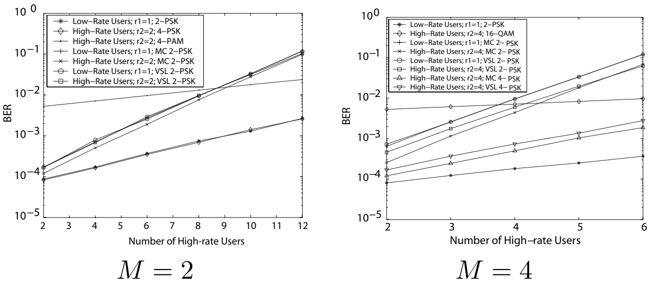


Fig. 2. BER vs number of high-rate users K_2. $K_1 = 6$.

Above analyses show that modulation plays an important role in multi-rate systems, which leads us to modify modulation schemes in MC and VSL strategies. We verify that utilizing 4-PSK instead of 2-PSK in MC and VSL schemes improves performance, even though applying 4-PSK and 2-PSK gives the same performance in single-rate [9] and MM multi-rate systems. Because 4-PSK MC and VSL schemes utilize less spreading sequences, they suffer less MAI than 2-PSK MC and VSL schemes. The performance therefore degrades slower than that of 2-PSK MC and VSL schemes as show in Fig. 2 (b). It is also shown in Fig. 1 that applying 4-PSK in MC and VSL schemes improves the performance about 9 dB performance gain.

VI. Conclusions

The performance of MM, MC, and VSL access strategies was analyzed by the normalized minimum distance such that the structure of signal constellations and MAI are jointly considered. Both the mathematical and numerical analyses demonstrated that the noise enhancement effect does more harm to MC and VSL strategies especially for high-rate users. Because the performance of MC and VSL degrades more rapidly as the number of active users increases, the MM scheme outperforms MC and VSL schemes when the number of active users is large.

In addition, a modified decorrelating detector for multi-rate systems was proposed. We proved that the proposed detector minimizes the symbol error probability while maintaining reasonable complexity, which grows linearly with the number of active users as LRD. The cross-correlation of noise components ignored by LRD was considered to achieve around 4 dB performance gain over LRD.

We showed that the design of modulation is critical in multi-rate systems. The MM scheme usually provides smaller minimum distance than MC and VSL schemes. However, the MM scheme also induces less MAI in systems. When applying MC and VSL schemes, we can design their signal constellations to improve the performance. In our simulations, we applied 4-PSK to obtain about 9 dB performance gain. The trade-off on the design of modulation schemes depends on the number of active users and spreading sequences. Since the normalized minimum distance reflects the effect of modulation schemes and MAI, we can use it to evaluate the performance when designing multi-rate communications.

Appendix

We prove that the proposed detector minimizes the symbol error probability. The HRU k observes statistic $\underline{Y}_k$ as LRD, and makes decision among $M = 2^{r_2}$ possible transmitted vectors $\{\underline{\theta}_1, \underline{\theta}_2, \cdots, \underline{\theta}_M\}$. The statistic $\underline{Y}_k$ is a GRV with covariance $\widetilde{\mathbf{R}} = \sigma^2 |a_k|^{-2} \mathbf{R}_k$. Define the observation space Ω that contains all possible value $\underline{Y}_k$ and is partitioned into M disjoint subspace Ω_n where $\Omega = \cup_{n=1}^{M} \Omega_n$. Denote $P(\underline{Y}_k \in \Omega_n \mid \underline{\theta}_m)$ the probability that the receiver chooses $\underline{\theta}_n$ when $\underline{\theta}_m$ is transmitted. The symbol error probability is given by

$$P_e = \frac{1}{M} \sum_{m=1}^{M} \sum_{n=1, n\neq m}^{M} P\left(\underline{Y}_k \in \Omega_n \mid \underline{\theta}_m\right) = 1 - \frac{1}{M} \sum_{m=1}^{M} \int \cdots \int_{\Omega_m} f_{\underline{\theta}_m}\left(\underline{y}_k\right) d\underline{y}_k$$

where $f_{\underline{\theta}_m}\left(\underline{y}_k\right)$ is the probability density function of GRV with mean $\underline{\theta}_m$ and covariance $\widetilde{\mathbf{R}}$. The optimum solution that minimizes P_e is given by choosing $\Omega_m = \left\{\underline{y}_k \mid f_{\underline{\theta}_m}(\underline{y}_k) \geq f_{\underline{\theta}_n}(\underline{y}_k),\, m \neq n\right\}$. Hence, it suffices to say that the optimum receiver should be $\widehat{\underline{x}}_k = \arg\max_{\underline{x}_k} f_{\underline{x}_k}(\underline{y}_k)$ equal to (4).

We derive the upper bound of P_e. Define the random variable

$$T_{n,m} = \mathrm{Re}\left[\left(\underline{x}_n - \underline{x}_m\right)^H \widetilde{\mathbf{R}}^{-1} \left(\underline{Y}_k - \frac{1}{2}\left(\underline{x}_n + \underline{x}_m\right)\right)\right].$$

Since $T_{n,m}(\underline{Y}_k)$ is the linear transformation of GRV, $T_{n,m}(\underline{Y}_k)$ is Gaussian random variable. We can get its mean $\overline{t_{n,m}} = E\left[T_{n,m}(\underline{Y}_k) | \underline{\theta}_m\right] = \frac{-1}{2}\left(\underline{x}_n - \underline{x}_m\right)^H \widetilde{\mathbf{R}}^{-1}\left(\underline{x}_n - \underline{x}_m\right)$ and variance $d_{n,m}^2 = E\left[\left(T_{n,m}(\underline{Y}_k) - \overline{t_{n,m}}\right)^2 | \underline{\theta}_m\right] = -2\overline{t_{n,m}}$ under $\underline{\theta}_m$ transmitted. The P_e is upper bounded by

$$P_e \leq \frac{1}{M} \sum_{m=1}^{M} \sum_{n=1, n\neq m}^{M} \int_0^{\infty} \frac{1}{\sqrt{2\pi} d_{n,m}} e^{\frac{-\left(t_{n,m} + \overline{t_{n,m}}\right)^2}{2 d_{n,m}^2}} dt_{n,m} = \frac{1}{M} \sum_{m=1}^{M} \sum_{n=1, n\neq m}^{M} Q\left(\frac{|a_k| \sqrt{\left(\underline{x}_n - \underline{x}_m\right)^H \mathbf{R}^{-1} \left(\underline{x}_n - \underline{x}_m\right)}}{\sigma}\right).$$

When the noise power is small enough, the performance is consequently dominated by the minimum distance (5).

References

[1] C.-L. I and R. D. Gitlin, “Multi-code CDMA wireless personal communications networks,” in *Proc. IEEE ICC*, vol. 2, June 1995, pp. 1060–1064.
[2] C.-L. I and K. K. Sabnani, “Variable spreading gain CDMA,” *AT&T Lab Notebook*, no. 76949, pp. 9–11, Aug. 1993.
[3] T. Ottosson and A. Svensson, “Multi-rate schemes in DS/CDMA systems,” in *Proc. IEEE VTC*, vol. 2, July 1995, pp. 1006–1010.
[4] Z. Guo and K. B. Letaief, “Performance of VSG-CDMA and MC-CDMA in multirate systems,” in *Proc. IEEE VTC*, vol. 1, 2001, pp. 501–505.
[5] M. Saquib, R. Yates, and N. Mandayam, “Decorrelating detectors for a dual rate synchronous DS/CDMA system,” *Wireless Personal Communications*, vol. 9, pp. 197–216, May 1999.
[6] J. Chen and U. Mitra, “Analysis of decorrelator-based receivers for multirate DS/CDMA communications,” *IEEE Trans. Veh. Technol.*, vol. 48, pp. 1966–1983, Nov. 1999.
[7] S. Verdu, *Multiuser Detection*, 1st ed. Cambridge Univ. Press, 1998.
[8] J. Chen and U. Mitra, “Optimum near-far resistance for dual-rate DS/CDMA signals: Random signature sequence analysis,” *IEEE Trans. Inform. Theory*, vol. 45, pp. 2434–2447, Nov. 1999.
[9] J. G. Proakis, *Digital Communications*, 3rd ed. McGraw-Hill, 1995.