Full-Vectorial Finite Element Modal Analysis of Bounded and Unbounded Waveguides

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1. Introduction

In extending the finite element method (FEM) to analyze open dielectric waveguides, which are basic structures for integrated optical devices and optical communication systems, a variety of approaches have been proposed. The surface integral equations method (SIEM) [1] was used to deal with structures with homogeneous core and cladding. The hybrid methods usually combine the standard FEM with the function expansion methods [2] or integral equation methods [3]. These techniques are used in the exterior region, which is usually homogeneous, while the FEM is applied to an interior region which includes material inhomogeneity. Others applied domain integral methods (DIM) to deal with material inhomogeneity [4]. This method is a different form of the moment method. Unlike the FEM, which results in a generalized linear eigenvalue problem, both the hybrid methods and the domain integral methods result in nonlinear systems of equations for which one has to find solutions by a costly search for the roots of a determinant and the solutions must then be verified carefully to ensure that no roots are missed. Especially for lossy problems, one has to find a root within the entire complex β-plane! To preserve the linearity of the FEM, some authors [5], [6] utilized artificial boundary techniques to deal with such problems. *Yeh et al.* [5] used a perfect conductor at a distance to enclose the entire waveguide structure, and *Mabaya et al.* [6] let the fields vanish at a certain distance from the origin. However, this technique involves a large number of nodal points when the fields extends farther away from the guding region. In this paper, we will discuss how different computational boundaries affect the final computed results obtained by our recently developed FEM-I formulation [7]. We will also compare our results with those obtained by the SIEM in the convergence rate and the memory storage.

2. Theory

In [7], we have derived both the transverse wave equations for the magnetic fields (H formulations) and the electric fields (E formulations). These two forms of formulations are shown, for the first time to our knowledge, to give highly consistent results. The eigenvalue equation for waves propagating in waveguides can be written as

$$[A]\{H\} = \beta^2[B]\{H\} \tag{1}$$

where $\{H\} = \{h_x \ h_y\}^T$ is the transverse magnetic field vector. For the Newmann, the Dirichlet, and the zero conditions [6], the essential boundary conditions $h_{x\,\mathrm{or}\,y} = 0$ or $\partial h_{x\,\mathrm{or}\,y}/\partial(x\,\mathrm{or}\,y) = 0$ are set at the computional boundary. For handling open dielectric waveguides, we incorporate the decay-type infinite element approach [8] to solve (1).

3. Numerical Results

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We consider the structure shown in the inset of Fig. 1 with different boundary conditions including the Newmann condition, the Dirichlet condition, the perfect conductor, and the boundary approximated by infinite elements. Fig. 1 shows the effective index of the fundamental H_{11}^{y} mode for different boundary conditions with the mesh spacing $\Delta x = \Delta y = 0.05t$. The curves marked as 'OPE', 'INF', 'PEC', and 'ZERO' represent the results obtained by using the Newmann condition, infinite elements, the perfect conductor, and the Dirichlet condition, respectively. The curve marked as 'AVE' represents the average results between those of 'OPE' and 'ZERO'. The convergent effective index is about $n_{eff} = 1.35643545$, which is larger than $n_{eff} = 1.35638307$ for the case with the PEC and $r = 0.5t$. For a convergent result with $\Delta n_{eff} \leq 1 \times 10^{-6}$, the proper locations of the artificial boundary are $r = 1.1t$ for 'OPE', $r = 0.85t$ for 'PEC', $r = 1.1t$ for 'ZERO', $r = 0.6t$ for 'AVE', and r is only $0.35t$ for 'INF'. The memory storage utilized with 'INF' is also much smaller than others. It is interesting to note that the propagation constants obtained by 'INF' is approximately equal to those obtained by 'AVE', that is, $\beta_{INF} = (\beta_{OPE} + \beta_{ZERO})/2$. And

$$\beta_{ZERO} \leq \beta_{INF} \leq \beta_{OPE}. \tag{2}$$

Fig. 2(a) and (b) shows the contours of the magnetic field components H_x and H_y, respectively, using infinite elements at $r = 0.5t$, and Fig. 2(c) and (d) shows those of H_x and H_y, respectively, using infinite elements at $r = 0.1t$. The field distributions are almost the same, except that the effective index for $r = 0.1t$ is slightly larger. We also compared our resutls with those obtained by using the surface integral equations method (SIEM). Table 1 shows the effective indexes obtained by using the SIEM and the corresponding memory usage. Table 2 shows the corresponding results obtained by the FEM-I with infinite elments at $r = 0.5t$. Table 3 shows the corresponding results obtained by the FEM-I with perfect conductor at $r = 1.0t$ for acceptable Δn_{eff}. The memory usage for the SIEM is proportional to N^2, where N is the number of total unknowns. While for the FEM-I, the memory usage is approximately proportional to $30N$. It is clear that the memory usage for the FEM-I is much less than that for the SIEM. To check the convergence rate of the calculated effective index, we present the results in Fig. 3. It is also clear that the convergence rate of the FEM-I is higher than that of the SIEM, due to the fact that the order of error of the FEM-I is better than that of the SIEM. Especially at the corner, it is quite difficult for the SIEM to improve the order of the derivatives. Therefore, nonuniform division is required for the SIEM, which again will cost much more computing time.

4. Conclusion

We have developed an efficient method to deal with open dielectric waveguide structures. We have demonstrated the differences in the calculated effective index between different boundary conditions. Compared with the SIEM, the FEM-I is easier to find modes, and due to less memory usage and higher convergence rate, the FEM-I is much more suitable when computing on personal computers.

TABLE 1
EFFECTIVE INDEX AND THE CORRESPONDING MEMORY STORAGE FOR THE STRUCTURE SHOWN IN FIG. 1, OBTAINED BY THE SIEM

Unknowns	Effective index	Memory usage (units)
40	1.3560236	1.6k
120	1.3562974	14.4k
280	1.3563770	78.4k
500	1.3564034	250k
1000	1.3564205	1M
1500	1.3564262	2.25M
2200	1.3564299	4.84M

TABLE 2

Effective Index and the Corresponding Memory Storage for the Structure Shown in Fig. 1 Obtained by the FEM-I with Infinite Elements at $r = 0.5t$.

Unknowns	Effective index	Memory usage (units)
96	1.35628732	2.88k
176	1.35640252	5.28k
280	1.35642397	8.4k
408	1.35643034	12.24k
1408	1.35643545	42.24k
5208	1.35643632	156.24k

TABLE 3

Effective Index and the Corresponding Memory Storage for the Structure Shown in Fig. 1, Obtained by the FEM-I with Perfect Conductor at $r = 1.0t$.

Unknowns	Effective index	Memory usage (units)
176	1.35628693	5.28k
341	1.35640220	10.23k
560	1.35642369	16.8k
833	1.35643008	24.99k
3008	1.35643523	90.24k
11408	1.35643614	342.24k

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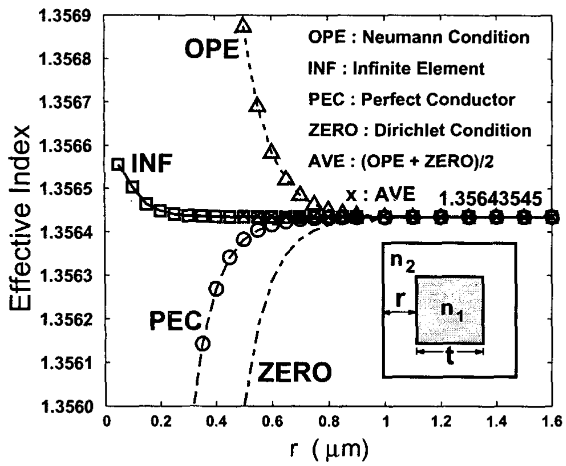

Fig. 1 Effective indexes of the fundamental H_{11}^{y} mode of the square channel waveguide for different boundary conditions with mesh spacing $\Delta x = \Delta y = 0.05t$.

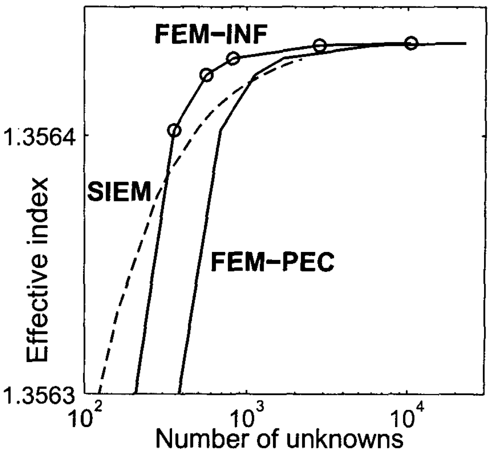

Fig. 3 Effective index for the fundamental mode of the square channel waveguide obtained by the FEM-I with different boundary conditions and by the SIEM.

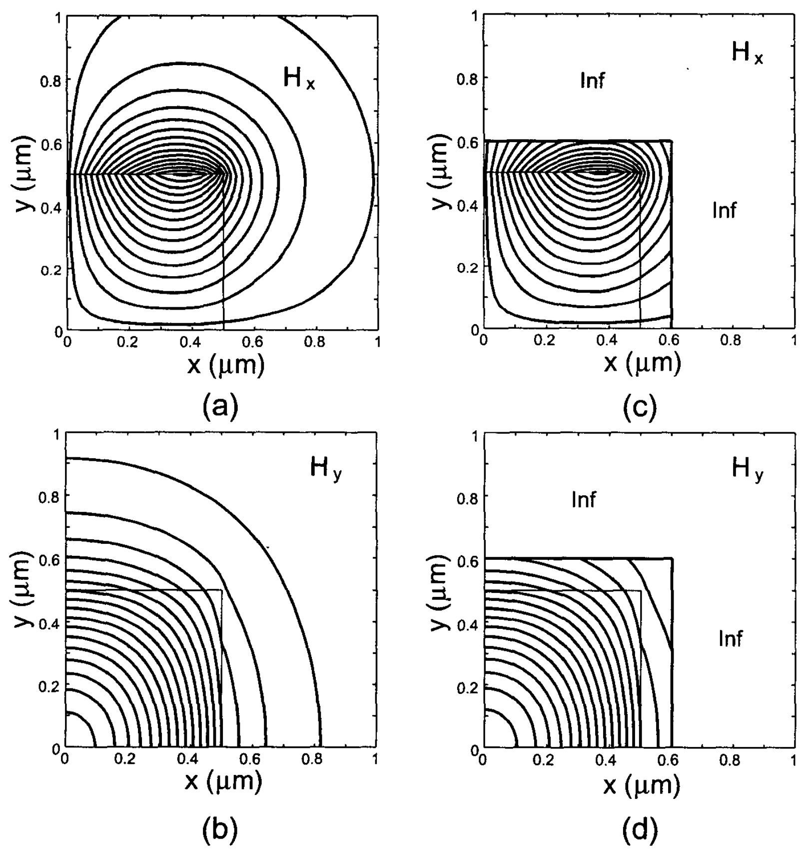

Fig. 2 Contours of the magnetic field components (a) H_x and (b) H_y using infinite elements at r = 0.5t and contours of the magnetic field components (c) H_x and (d) H_y using infinite elements at r = 0.1t.