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Adaptive Decentralized Compliant Control of Robot Manipulators

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Abstract

In this paper, we develop an adaptive decentralized compliant control of robot manipulator. The robot manipulator is in contact with the environment and the contact force is normal to the contact plane. When the environment stiffness is given, an adaptive decentralized compliant controller is designed based on a Lyapunov based method such that both the force tracking error and the complementary position tracking error are bounded and will converge to zero asymptotically. On the other hand, when the environment stiffness is not given, the compliant controller above can be used with a motion modification generator such that the force tracking error will converges to a residual set whose size is small if the environment stiffness is large, and the complementary position tracking error is bounded and converges to zero asymptotically.

Keywords: Decentralized control, adaptive control, compliant control, robot manipulators.

1. Introduction

In many application areas where robot manipulators are employed to handle complex tasks, such as edge following, grinding, painting, and deburring, contact between the end-effector and the environment is almost always inevitable. For such robotic tasks, it is necessary to control not only the position of manipulator but also the contact force control at the contact. Among several existing control approaches of handling the tasks, a controlled interaction with the environment can be sought by imposing a suitable dynamic behavior or impedance between contact force and end-effector position [1]. Explicit force feedback measurements are not strictly required in such a case, but a desired force cannot generally be specified. Some other strategies have been proposed to achieve control of the contact force as well as the position, for examples, hybrid position/force control [2], constraint-based position/force control [3], [4], inner-outer position/force control [5], parallel force/position control [6], and scaling-factor position/force control [7], [8]. One common characteristic of all the control schemes above is that they are centralized techniques. However, such characteristic may not be suitable for real-time simple-hardware implementation due to need of intensive.

To resolve the above difficulty, based on the scheme by Fu [9] a new control scheme for compliant control of robot manipulators is developed in this paper. The controller is implemented in a decentralized manner, i.e., a subcontroller is independently and locally equipped at the servo loop of each joint. Implication of these features is then the high promise of real-time low cost implementation. In the paper, with some usual assumption, the model of the contact force considered here is assumed to be in a linear relation between the contact depth δ and the normal contact force f_n. There are two kinds of decentralized compliant controllers presented in this paper: one is the case when the environment stiffness is given whereas the other is the case when the environment stiffness is unknown.

The paper is organized as follows. In Section 2, dynamics formulation of the manipulator environment, several important properties, and the control objective are presented. In Section 3, two kinds of decentralized compliant controllers are proposed. Finally, some concluding remarks are given in Section 4.

2. Problem Statement

For a general open-chain, n-link rigid manipulator interacting with the environment, the dynamic model can be derived by using Lagrangian-Euler method and can be expressed in a symbolic form [10]:

$$M(q)\ddot{q}+C(q,\ \dot{q})\dot{q}+g(q)=\tau-J^{T}(q)f+d, \qquad (2.1)$$

where $q, \dot{q} \in R^n$ are the joint configuration and the joint velocity of the manipulator, $\tau \in R^n$ is the actuator torque input, $M(q)$: $R^n \to R^{n\times n}$ is the manipulator inertia matrix which is symmetric positive definite, $C(q,\dot{q})\dot{q}$: $R^n \times R^n \times R^n \to R^n$ is the vector of coriolis and centrifugal generalized force, and $g(q)$: $R^n \to R^n$ is the vector of gravitational generalized force, $J^{T}(q)$: $R^n \to R^{n\times 6}$ is the manipulator Jacobian matrix, $f \in R^6$ is the vector of contact forces and moments exerted by the manipulator on the environment, and $d \in R^n$ is the input disturbance vector. To simply the underlying control problem, we will assume that the end-effector is in contact with the part only at a single point so that the moment exerted by the end-effector equals zero. Therefore, the contact force and moment f can be denoted as $f = [f_t^T + f_n^T,\ 0,\ 0,\ 0]^T$, where $f_n \in R^3$ is the vector of contact force perpendicular to the contact surface and $f_t \in R^3$ is the vector of contact force tangential to the contact surface of the part. In this study, f_t is negligible by assuming that the contact depth is small enough and the contact surface is approximately frictionless. In many applications, the disturbance introduced into the ith actuator will usually depend only on the activity associated with the ith joint. In this study, we consider the disturbances $d = d(t, q, \dot{q}) = (d_1, \ldots, d_n)^{\mathrm{T}}$ which satisfy the following:

$$| d_i | \le d_i^1 + d_i^2 | q | + d_i^3 | \dot{q} | \tag{2.2}$$

for some $d_i^j \ge 0$, $i = 1, \ldots, n, j = 1, 2, 3$. It is noteworthy that there is an important feature about the following:

$$\dot{q}^{\mathrm{T}}(\frac{1}{2}\frac{d}{dt}M(q) - C(q, \dot{q}))\dot{q} = 0 \tag{2.3}$$

for all $(q^{\mathrm{T}}, \dot{q}^{\mathrm{T}}) \in \mathrm{R}^{2n}$, i.e., $(\frac{1}{2}\dot{M} - B)$ is a skew-symmetric matrix over the space of joint velocity.

Accurate modeling of the contact between the manipulator and the environment is usually difficult to obtain in terms of an analytic form, due to complexity of the physical phenomena involved during the interaction. It is then reasonable to resort to a simple but major model, relying on the robustness of the control system to absorb the effects of inaccurate modeling part. Following the guidelines, the environment in physical contract with the robot end-effector can be viewed as a semi-rigid and frictionless surface with slight elasticity since it serves a good local approximation of contact plane with regular curvature. Of course, one should realize that the aforementioned elasticity might be due to the compliance nature of the force sensor equipped at the robot end-effector, or due to compliance of the plane.

Consider the end-effector position vector $r \in \mathrm{R}^3$ on a compliant plane and satisfies a continuously differentiable scalar function $\phi(r) = \delta$ where $\delta \in \mathrm{R}^1$ is a parameter accounting for the contact depth [8]. The vector of the force exerted by the surface on the effector is

$$f_n = -k_f \delta\, n, \tag{2.4}$$

where $n = (\partial\, \phi(r)/\partial\, r \,/\, \|\partial\, \phi(r) \,/\partial\, r\|) \in \mathrm{R}^3$ is an unit vector orthogonal to the surface at the contact point r and $k_f \in \mathrm{R}^1$ is the environment stiffness which is a positive constant. For convenience, a new position coordinate vector is defined as $x_p = [x_f, x_c^{\mathrm{T}}]$ where the scalar $x_f = \delta$ allowing us to specify the contact depth and the complimentary vector $x_c = \varphi(r) \in \mathrm{R}^2$ allowing us to assign a point on the compliant surface for a given x_f. Note that the existence of the vector $\varphi(r)$ is always guaranteed by the implicit function theorem.

A typical task for a robot manipulator in contact with the environment is to follow the desired force profile $f_{nd}(t)$ in some pre-specified subspace and the desired position profile $x_{cd}(t)$ in another complementary subspace.

3. Controller Design

There are two kinds of decentralized compliant controllers designed now: one is the case when the environment stiffness is given whereas the other is the case when the environment stiffness is unknown.

A. The environment stiffness is given.

If the environment stiffness k_f is given, the desired contact depth corresponding to the desired force profile in the sense of the above force model, the exact contact depth, $\delta_d^*(t)$, can be found as follows

$$\delta_d^* = k_f^{-1} \| f_{nd} \| \tag{3.1}$$

Then, the desired position trajectory of the end-effector $x_{pd}(t)$ can be defined as $x_{pd} = [\delta_d^*, x_{cd}^{\mathrm{T}}]^{\mathrm{T}}$ where $x_{cd} = x_{cd}(t)$ is the desired complementary position trajectory. Similarly, we let the desired orientation the end-effector be given as x_{od} so that the desired trajectory can be denoted as $x_d = [x_{pd}^{\mathrm{T}}, x_{od}^{\mathrm{T}}]^{\mathrm{T}}$. Based on this definition, the desired joint trajectory q_d can be determined as follows

$$q_d = \Psi^{-1}(x_d) \tag{3.2}$$

where $\Psi(q)$: $\mathrm{R}^n \to \mathrm{R}^6$ is the kinematic relation. Note that the ability to determine the desired joint trajectory is based on the assumptions that the inverse of kinematic relationship exists.

Now, consider the error states e_1 and e_2 as $e_1 = q - q_d$ and $e_2 = \dot{q} - \dot{q}_d$, so that the error dynamics can be expressed as

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & M^{-1}C \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} (\tau - v - d) \tag{3.3}$$

where

$$\begin{aligned} v &= v(t, q, \dot{q}) \\ &= M(q)\ddot{q}_d + C(q, \dot{q})\dot{q}_d + g(q) - k_f \delta J(q)^{\mathrm{T}} \overline{n} \end{aligned} \tag{3.4}$$

where $\overline{n} = [n^{\mathrm{T}}, 0, 0, 0]^{\mathrm{T}}$. Let the joint torque input τ be designed as some appropriate feedforward compensation, to be specified later, integrated with proportional feedback and derivative feedback, i.e.,

$$\tau(t) = u(t) - K_P e_1(t) - K_D e_2(t), \tag{3.5}$$

where $u(t)$ indicates the feedforward compensation part, and K_P and K_D are proportional and derivative feedback gain matrices, respectively. The control scheme given above will also be the so-called decentralized control if $u_i(t)$, the ith element of the compensation signal $u(t)$, is only a function of position and velocity of the ith joint, for $i = 1, \ldots, n$, and all gain matrices are diagonal, i.e., the torque input of each actuator will receive feedback only from the information of the joint controlled by that actuator. After substituting the control law into the error dynamics model, it can be rewritten as

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K_P & -M^{-1}(K_D + C) \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} (u - v - d), \tag{3.6}$$

or simply

$$\dot{e} = A(\xi)e + B(\xi)(u - v - d), \tag{3.7}$$

where $e^{\mathrm{T}} = (e_1^{\mathrm{T}},\ e_2^{\mathrm{T}})$ and $\xi^{\mathrm{T}} = (\xi_1^{\mathrm{T}},\ \xi_2^{\mathrm{T}}) = (q^{\mathrm{T}},\ \dot{q}^{\mathrm{T}})$ with $A(\xi)$ and $B(\xi)$ being defined appropriately. Apparently, the error dynamics model consists of a nominal system and a forcing perturbation term.

Before we processed with the stability analysis of the above proposed control scheme, the following assumptions are given to make the problem more tractable and to facilitate subsequence.

Assumptions:

A1) The joints in the robot manipulator are revolute.

A2) The feedback gain matrices K_P and K_D are diagonal, positive definite matrices, i.e., $K_P = \mathrm{diag}(k_{p1}, \ldots, k_{pn})$ and $K_D = \mathrm{diag}(k_{d1}, \ldots, k_{dn})$ for $k_{pi} > 0$ and $k_{di} > 0$, $i = 1, \ldots, n$.

A3) The desired force trajectory $f_{nd}(t)$ and the desired complementary position trajectory $x_{cd}(t)$ are smooth enough and bounded functions.

A4) The compliant surface is smooth enough.

A5) The parameters inside the model subject to uncertainty are assumed unknown except their related bounds. Moreover the input disturbance d satisfies the condition of (2.2) but with the constants d_i^j, $i = 1, \ldots, n$ and $j = 1, 2, 3$, unknown. A very conservative upper bound is $d_{\max}$, however in advance.

A6) $\Psi(q)$: $\mathrm{R}^n \rightarrow \mathrm{R}^6$ is Lipschitz continuous such that $\|\Psi(q_1) - \Psi(q_2)\| \le L\|q_1 - q_2\|$ for q_1, $q_2 \in \mathrm{R}^n$, where L is a constant.

Remarks:

R1) The assumption A1 implies that the induced norm of the manipulator Jacobian matrix $J(q)$ is bounded for all q.

R2) The assumption A2 is needed for implementing the control algorithm in an IJC context.

R3) Since the gain matrices together determine the linearized closed-loop dynamics, these values usually will be not independently specified. Instead, there will be certain relationship among these matrices according to the desired frequency, for example, like

$$K_P = \kappa K_D \tag{3.8}$$

for some suitable $\kappa > 0$.

Now, we are ready to propose the control law, which adapts the compensation signal u during the control process. Recall that v is an unknown signal due to the model uncertainty. Its bound, however, can be computed using the following facts:

$$\|M(\xi_1)\| \le k_1,\quad |g(\xi_1)| \le k_2,\quad \forall \xi_1 \in R^n \tag{3.9}$$

and

$$\|C(\xi_1,\ \xi_2)\| \le k_3|\xi_2| \quad \forall(\xi_1^{\mathrm{T}},\ \xi_2^{\mathrm{T}}) \in R^{2n} \tag{3.10}$$

for some k_1, k_2, and $k_3 \ge 0$, and, by using the assumption A1 and A6,

$$|k_f\ \delta J^{\mathrm{T}}(\xi_1)\overline{n}| \le k_4|\xi_1| + k_5 \quad \forall \xi_1 \in R^n \tag{3.11}$$

for some k_4 and $k_5 \ge 0$ so that the following bounded can be obtained

$$|v(t,\ \xi_1,\ \xi_2)| \le \beta_1 + \beta_2\,|\,\xi_1\,| + \beta_3|\xi_2| \equiv \beta^{\mathrm{T}}w(\xi) \tag{3.12}$$

for some suitable positive constants β_1, β_2 and β_3, where $\beta = (\beta_1, \beta_2, \beta_3)^{\mathrm{T}}$ and $w(\xi) = (1, |\xi_1|, |\xi_2|)^{\mathrm{T}}$.

The existence of the constant vector β is guaranteed but unknown from the assumption A3 and A4, whereas the signal vector $w(\xi)$: $\mathrm{R}^{2n} \rightarrow \mathrm{R}^n$ is available for all time $t \ge t_0$. The basic idea of designing the controller is to construct the following compensation $u(t)$:

$$u_i(t) = \begin{cases} [\hat{\beta}_i^{\mathrm{T}} w(\xi_d) + \hat{d}_i^{\mathrm{T}} \phi_i(\xi_1^i,\ \xi_2^i)]^2 \dfrac{z^i(e)}{\varepsilon_i} \\ \quad \text{if } |\,z^i(e)\,| \le \dfrac{\varepsilon_i}{(\hat{\beta}_i^{\mathrm{T}} w + \hat{d}_i^{\mathrm{T}} \phi_i)} \\ [\hat{\beta}_i^{\mathrm{T}} w(\xi_d) + \hat{d}_i^{\mathrm{T}} \phi_i(\xi_1^i,\ \xi_2^i)] \dfrac{z^i(e)}{|\,z^i(e)\,|} \\ \quad \text{if } |\,z^i(e)\,| > \dfrac{\varepsilon_i}{(\hat{\beta}_i^{\mathrm{T}} w + \hat{d}_i^{\mathrm{T}} \phi_i)} \end{cases} \tag{3.13}$$

for $i = 1, \ldots, n$ with the following notation.

Notations:

N1) For any vector $y \in \mathrm{R}^m$, y^i denotes the ith element of y;

N2) $\xi_d^{\mathrm{T}} = (\xi_{d1}^{\mathrm{T}},\ \xi_{d2}^{\mathrm{T}}) = (q_d^{\mathrm{T}},\ \dot{q}_d^{\mathrm{T}})$;

N3) For $i = 1, 2, \ldots, n$, $\hat{\beta}_i$ represents the estimate of the vector $\beta = (\beta_1,\ \beta_2,\ \beta_3)^{\mathrm{T}}$ from the ith actuator part;

N4) $\phi(\xi_1^i,\ \xi_2^i) = (\,1,\ |\,\xi_1^i\,|,\ |\,\xi_2^i\,|\,)^{\mathrm{T}}$, for $i = 1, 2, \ldots, n$;

N5) For $i = 1, 2, \ldots, n$, d_i represents the estimate of the vector $d_i = (d_i^1,\ d_i^2,\ d_i^3)^{\mathrm{T}}$;

N6) $z(e) = e_1 + \gamma e_2$ for some $\gamma > 0$;

N7) Consider the auxiliary signals ε_i, $i = 1, 2, \ldots, n$, which satisfy

$$\dot{\varepsilon}_i = -g_i\varepsilon_i,\quad \varepsilon_i(t_0) > 0, \tag{3.14}$$

where $g_i > 0$, $i = 1, 2, \ldots, n$; along with the following adaptive law on the adaptive parameter vectors $\hat{\beta}_i$ and $\hat{d}_i$, for $i = 1, 2, \ldots, n$:

$$\dot{\hat{}}_i = \Gamma_i\,|\,z^i(e)\,|\,w(\xi_d) \tag{3.15}$$

and

$$\hat{d}_i = \overline{\Gamma}_i \,|\, z^i(e) \,|\, \phi_i(\xi_1^i,\ \xi_2^i), \qquad (3.16)$$

where Γ_i and $\overline{\Gamma}_i > 0$, $i = 1, 2, \ldots, n$, are the adaptive gain matrices. From the definition of the forward compensation $u(t)$, and N6, it can be easily seen that the ith term of $u(t)$, namely, $u_i(t)$, a function of only position and velocity of the ith joint. Therefore, the control input τ is indeed implemented as an IJC algorithm.

The following is one main results of the paper, which summarizes the stability properties of the manipulator system under the adaptive control law.

Theorem 3.1. *Consider the error dynamics model* (3.6) *of the robot manipulator subject to the adaptive decentralized controller* (3.5) *and* (3.13)-(3.16) *satisfies the assumptions* A1-A6. *Given any $h > 0$, with the corresponding initial condition $e(t_0) \in B_h$, there exist sufficient large $\lambda_{\min}(K_P)$ and γ such that*
1) *all signals are bounded,*
2) *the tracking error $e(t)$ will converge to zero asymptotically as t tends to infinity, and*
3) *the force tracking error $f_n(t) - f_{nd}(t)$ as well as the complementary position tracking error $x_c(t) - x_{cd}(t)$ will converge to zero asymptotically as t tends to infinity.*

Proof. Define $v^*(t)$ by $v^*(t) = v(t, q_d, \dot{q}_d)$ and, hence, the bound on v^* can be easily found as $|\, v^* \,| \leq \beta^T w(\xi_d)$. Rewrite the error dynamics model into the following form

$$\dot{e} = A(\xi)e + B(\xi)(v^* - v) + B(\xi)(u - v^* - d) \qquad (3.17)$$

with $e(t_0) = e_0$ where $t_0 \geq 0$ and the 2nd term on the right hand side (RHS) satisfies

$$\left|\, v^* - v \,\right| \leq l_v \left|\, e \,\right| \qquad (3.18)$$

for some suitable $l_v \geq 0$. Now construct a Lyapunov function candidate V_c as follows

$$V_c = \frac{1}{2} e^T P(\xi) e + \frac{1}{2} \sum_{i=1}^{n} v_{ci} \qquad (3.19)$$

where $P(\xi) \in R^{2n \times 2n}$ is defined as

$$P(\xi) = \begin{bmatrix} \gamma K_P & M \\ M & \gamma M \end{bmatrix} \qquad (3.20)$$

and v_{ci}, $i = 1, 2, \ldots, n$, are defined by

$$\begin{aligned} v_{ci} &= (\hat{\beta}_i - \beta)^T \Gamma_i^{-1} (\hat{\beta}_i - \beta) \\ &\quad + (\hat{d}_i - d_i)^T \overline{\Gamma}_i^{-1} (\hat{d}_i - d_i) + g_i^{-1} \ldots_i \end{aligned} \qquad (3.21)$$

Then, take the time derivative of V_c along the solution trajectories of the error dynamics model with the former adaptive controller to show that

$$\begin{aligned} \frac{d}{dt} V_c &\ \ e^T Q(\ )e \ \ e_1^T \dot{M} e_2 \ \ z^T (v^* \ \ v) \ \ z^T u \\ &+ \sum_{i=1}^{n} [(\hat{\beta}_i^T w + \hat{d}_i^T \phi_i) \,|\, z^i \,| - \varepsilon_i] \end{aligned} \qquad (3.22)$$

where $Q(\xi) \in R^{2n \times 2n}$ is defined by

$$Q(\xi) \ \ \begin{bmatrix} K_P & (K_D + B)/2 \\ (K_D + B)/2 & \gamma K_D - M \end{bmatrix} \qquad (3.23)$$

Note that, given some $h > 0$ (to be specified later), $P(\xi)$ and $Q(\xi)$ can be made uniformly positive definite over B_h i.e., for some strictly positive α_i, $i = 1, 2, 3$, and 4, we can prove that $\alpha_1 I \leq P(\xi) \leq \alpha_2 I$ and $\alpha_3 I \leq Q(\xi) \leq \alpha_4 I$.

Let h be chosen as that there exists $T > 0$ such that, starting from $e(t_0)$, $|\, e(t) \,| \leq h - h'$ for all $t \in [t_0, t_0 + T]$ and, hence, $(e_1^T, e_2^T)^T \in B_{h-h'}$ for all $t \in [t_0, t_0 + T]$. Then, from the fact that

$$\xi \ \ \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} q_d \\ \dot{q}_d \end{bmatrix} \qquad (3.24)$$

and the assumption A3, $\xi \in B_h$ for all $t \in [t_0, t_0 + T]$ so that the second term on the RHS of (3.33) can bounded by

$$e_1^T \dot{M} e_2 \leq l_m h \,|\, e_1 \,|\,|\, e_2 \,| \ \leq l_m h \,|\, e \,|^2, \ t \in [t_0,\ t_0 + T] \qquad (3.25)$$

for some $l_m > 0$. Therefore,

$$\begin{aligned} &-e^T Q(x) e + e_1^T \dot{M} e_2 + z^T (v^* - v) \\ &\leq -(\alpha_3 - l_m h - l_v) \,|\, e \,|^2 \equiv -\alpha_5 \,|\, e \,|^2 \end{aligned} \qquad (3.26)$$

for some $\alpha_5 > 0$. α_3 can be increased so that $\alpha_5 > 0$ if $\lambda_{\min}(K_p)$ and γ are increased sufficiently such that

$$\begin{aligned} \frac{d}{dt} V_c &\leq -\alpha_5 \,|\, e \,|^2 + z^T u + \\ &\sum_{i=1}^{n} [(\hat{\beta}_i^T w + \hat{d}_i^T \phi_i) \,|\, z^i \,| - \varepsilon_i] \end{aligned} \qquad (3.27)$$

for $\xi \in B_h \subset R^{2n}$ for some $h >> h'$.

After evaluating the time derivative of V_c for all cases, we can conclude that for all $t \in [t_0, t_0 + T]$

$$V_c\big|_t \leq V_c\big|_{t_0} \qquad (3.28)$$

so that

$$\frac{1}{2} p_1 \,|\, e(t) \,|^2 \leq V_c\big|_{t,\,t_0} \leq \frac{1}{2} p_2 \,|\, e(t_0) \,|^2 + \frac{1}{2} \sum_{i=1}^{n} v_{ci}\big|_{t,\,t_0} \qquad (3.29)$$

where

$$p_1 \ \ \min_{R^{2n}} \lambda_{\min} P(\xi),\ p_2 \ \ \max_{\in R^{2n}} \lambda_{\max} P(\xi). \qquad (3.30)$$

Note that the upper bound on $\sum_{i=1}^{n} v_{ci}\big|_{t=t_0}$ can be estimated given the initial conditions $\hat{\beta}_i(t_0)$ and $\hat{d}_i(t_0)$. Hence, if h is further chosen such that

$$\left\lfloor \frac{1}{p_1}(p_2 \mid e(t_0) \mid + v_{ci}\big|_{t\ t_0}) \right\rfloor^{1/2} + h'' < h, \tag{3.31}$$

where $\xi_d \in B_{h''}$, then it can be easily verified through an argument of contradiction that. Consequently, it follows that $e \in L_2 \cap L_\infty$ and for $i = 1, 2, \ldots, n$,

$$\hat{\beta}_i \in L_\infty, \quad \hat{d}_i \in L_\infty, \quad \varepsilon_i \in L_\infty, \tag{3.32}$$

and, hence, the compensation signal $u \in L_\infty$. This further implies that from Barbalet's Lemma [12] $e \to 0$ as $t \to \infty$. Let $x = [x_p^T, x_o^T]^T$. Since $x(t) - x_d(t) = \psi(q(t)) - \psi(q_d(t))$, $x \to x_d$ as $t \to \infty$ such that $f_n \to f_{nd}$ and $x_c \to x_{cd}$ as $t \to \infty$.

Note that without any prior knowledge of the manipulator of parameters and possibly under deterioration of parameter variation with time or state-dependent input disturbances, the force tracking error and the position tracking error are bounded and will converge to zero asymptotically.

B. The environment stiffness is unknown.

When the environment stiffness constant k_f is not given, the exact contact depth δ_d^* cannot be determined by using (3.1) and hence the desired joint trajectory cannot be determined. In order to find the desired joint trajectory, a motion modification generator to which determines a scheduled contact depth δ_d is devised as follows

$$\delta_d = -\lambda_f \delta_d + (\ \|f_{nd}\| - \|f_n\|\ ) = -\lambda_f \delta_d + k_f(\delta_d^* - \delta) \tag{3.33}$$

with $\delta_d(t_0) = 0$ where λ_f is a positive constant. Let $J_x^+(q)$ be the pseudo-inverse of $J_x(q)$ which is the Jacobian matrix from q to x where $x = [x_p^T, x_o^T]^T$. Therefore, the desired path in the joint space is determined as

$$q_d = J_x^+(q_d)x_d \tag{3.34}$$

with $q(t_0) = \Psi^{-1}(x(t_0))$ where $x_d = [x_{pd}^T, x_{od}^T]^T$.

The following is the other main result, which summarizes the stability properties of the manipulator system under the adaptive control law when the environment stiffness is unknown.

Theorem 3.2. *Consider the error dynamics model* (3.6) *of the robot manipulator with the adaptive decentralized controller* (3.5) *and* (3.13)-(3.16) *and motion modification generator* (3.33) *satisfying the assumptions* A1-A6. *Given a* $h > 0$ *small enough, with the corresponding initial condition* $e(t_0) \in B_h$, *there exist sufficient large* $\lambda_{\min}(K_P)$, γ *and* λ_f *such that*

1) *all signals are bounded,*

2) *the tracking error* $e(t)$ *will converge to zero asymptotically as* t *tends to infinity, and*

3) *the force tracking error* $f_n(t) - f_{nd}(t)$ *will converges to a residual set whose size will be small if the environment stiffness* k_f *is large. Furthermore, the complementary position tracking error* $x_c(t) - x_{cd}(t)$ *will converge to zero asymptotically as* t *tends to infinity.*

Proof. First of all, we prove that the desired joint trajectory q_d and its associated derivatives q_d and q_d are bounded under the motion modification generator (3.33) such that the assumption A3 can be satisfied. Let us define $\varepsilon = 1/\lambda_f$ and obtain the following equations

$$\begin{aligned}
\dot{\delta}_d &= -\delta_d - k_f(\delta - \delta_d) \\
\ddot{\delta}_d &= -\dot{\delta}_d - k_f(\dot{\delta} - \dot{\delta}_d^*) \\
\dddot{\delta}_d &= -\ddot{\delta}_d - k_f(\ddot{\delta} - \ddot{\delta}_d^*) \\
\delta_\delta^{(4)} &= -\dddot{\delta}_d - k_f(\dddot{\delta} - \dddot{\delta}_d^*)
\end{aligned} \tag{3.35}$$

Neglect ε, we obtain

$$\delta_d = \dot{\delta}_d = \ddot{\delta}_d = \dddot{\delta}_d = 0 \tag{3.36}$$

which is the only root of (3.35). Substituting (3.36) to the robotic manipulator, then the resulting reduced robotic manipulator system with the responding initial conditions has a unique and bounded solution defined on $[t_0, \infty)$ if sufficiently large $\lambda_{\min}(K_P)$ and γ are provided. On the other hand, the boundary-layer system as follows

$$\frac{d}{d\tau}\begin{bmatrix} {}_d \\ \dot{}\,{}_d \\ \ddot{}\,{}_d \\ \dddot{}\,{}_d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} {}_d \\ \dot{}\,{}_d \\ \ddot{}\,{}_d \\ \dddot{}\,{}_d \end{bmatrix} \tag{3.37}$$

is exponential stable, uniformly in (t, e_1, e_2, $\hat{\beta}_1, \ldots, \hat{\beta}_n$, $\hat{d}_1, \ldots, \hat{d}_n$, $\varepsilon_1, \ldots, \varepsilon_n$, q_d, $\dot{q}_d$, $\ddot{q}_d$). Hence, according to the singular perturbation theorem [13], q_d, $\dot{q}_d$, $\ddot{q}_d$ will be bounded with a sufficiently small ε, and their bounds can be conservatively estimated by using the order of ε. In other words, with sufficiently large $\lambda_{\min}(K_P)$ and γ, q_d, $\dot{q}_d$, $\ddot{q}_d$ will be bounded with a sufficiently large λ_f and their bounds can be conservatively estimated by using the order of λ_f. Now the following proof is similar to that of Theorem 3.1. That is, construct a Lyapunov function candidate as in (3.19), and its time derivative along the solution of trajectories of the error dynamics model (3.6) subject to the adaptive decentralized controller (3.5), (3.13)-(3.16) with the motion modification generator (3.33). In a similar fashion, we can shown that given a $h > 0$ small enough, with the corresponding initial condition $e(t_0) \in B_{h}$,, for sufficiently large sufficient large $\lambda_{\min}(K_P)$ and γ, $e \in L_2 \cap L_\infty$ and $\hat{\beta}_i \in L$, $\hat{d}_i \in L_\infty$, $\varepsilon_i \in L$, for $i = 1, 2, \ldots, n$. As a result, $u \in L_\infty$, and hence $\dot{e} \in L_\infty$ such that from Barbalet's Lemma it follows $e \to 0$ as $t \to \infty$ so that the force tracking error $f_n(t) - f_{nd}(t)$ will converges to a residual set whose size will be small if the environment stiffness k_f is large.

4. Conclusion

In this paper, we develop an adaptive decentralized control of compliance robot manipulator. The robot manipulator is contact with the environment and there exists a force between the end-effector and the contact plane. According to the desired force trajectory and the position trajectories as well as the environment stiffness, the corresponding joint trajectory can be calculated, and, then, the compliant controller is designed based on a Lyapunov based method, which consists of a PD feedback part and dynamic compensation part. It is shown that without any prior knowledge of the manipulator of payload parameters and possibly under deterioration of parameter variation with time or state-dependent input disturbances, both the force tracking error and complementary position tracking error are bounded and converge to zero asymptotically. In particular, the controller is implemented in a decentralized manner such that real-time implementation can be of high of promise. When the environment stiffness is not given, the compliant controller above can be used with a motion modification generators such that the force tracking error converges to a residual set whose size is small if the environment is large, and the complementary position tracking error are bounded and converge to zero asymptotically.

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