DETECTION OF NARROWBAND SIGNALS BY GABOR'S EXPANSION OF SPECTRUM

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ABSTRACT

Gabor expansion is applied to the detection of narrow-band signal under the interference of wide-band colored noise. After the power spectrum of signal is found, this expansion is performed on the spectrum. The width, W, of Gabor's window function is treated as a regularization parameter. Signals with spectral width larger than W are expanded to be DC components in the expansion coefficients. Thus wide-band signals or noise can be suppressed easily, and it results in a robust prewhitening filter.

I INTRODUCTION

Detection of narrow-band signals in noise is an important function in many application areas, such as radar and sonar.Modern Signal processing techniques improve the detection of signal greatly. It is well known that if the covariance matrix of clutter is known, clutter can be suppressed optimally by transversal filters with short order which is the optimal MTI filter [1,2]. However, unless radar is assumed to work under a very simple environment, such as the case that radar is ground based and clutter is non-moving with constant doppler frequency and spread, otherwise the knowledge of clutter is not always available. This problem was solved primarily by adaptive MTI techniques [2].

If clutter is not filtered out by an MTI filter, target detection will be degraded. In this paper, an alternative way to suppress clutter interference in target detection was studied. Since target is usually defined as signals with smaller bandwidth than clutter, Gabor expansion is applied on the estimated spectrum of radar signals to discriminate signal components of different spectral width (or correlation length). After this expansion, a one-dimension spectrum is decomposed into a two dimensional distribution function with coordinates of frequency and correlation length. Thus signal components can be separated from clutter according to their difference in spectral width.

II. GABOR EXPANSION:

Given a signal p(f), it was suggested by Gabor [3] that p(f) can be decomposed into elementary Gaussian signals of

$$g(x - mW) \exp(jnTx)$$

$$= \left(\frac{\sqrt{2}}{W}\right)^{1/2} \exp[-\pi (\frac{f}{W})^2] \exp(jnTx)$$ as

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$$p(f) = \sum_m \sum_n a_{mn}\, g(f - mW) \exp(jnTf) ,$$

where W and T satisfy $WT = 2\pi$. Value of W controls the width of g(x) which is also known as the Gabor's expansion window function. It was given by Bastinnans [4] that the two-dimensional Gabor expansion coefficients a_{mn} can be found through the help of an orthogonal function of g(f) :

$$\gamma(f) = \left(\frac{1}{\sqrt{2}W}\right)^{1/2} \left(\frac{K_0}{\pi}\right)^{-3/2} \exp\left[\pi \left(\frac{f}{W}\right)^2\right] \cdot \sum_{n+1/2 \geq f/W} \exp\left[-\pi \left(n + \frac{1}{2}\right)^2\right] ,$$

and

$$a_{mn} = \int p(f)\, \gamma(f - mW) \exp(-jnTf)\, df$$

III GABOR EXPANSION OF ALL-POLE SPECTRUM

Given N samples of radar signal

$$x(n) = s(n) + c(n) + w(n) \quad ; n = 1 \,..\, N ,$$

where s(n) is the narrow-band signal to be detected, c(n) is the wide-band signal (or colored noise), and w(n) is the white noise. In general x(n) can be modeled by an autoregressive model with order p [5]:

$$x(n) = -\sum_{k=1}^{p} a_k x(n-k) .$$

There are several techniques to find the prediction coefficients a_k [6]. The power spectrum of x(n) is then estimated as

$$p(f) = \frac{\rho}{\left|1 + \sum_{k=1}^{p} a_k \exp(j\, 2\pi k f)\right|^2} ,$$

where ρ is the power of prediction error. p(f) is known as an all-pole spectrum.

The purpose of using Gabor expansion for detecting narrow-band signals in p(f) can be realized by that W in $\gamma(f)$ can be treated as a regularization parameter. Signal components of different spectral widths can be discriminated by $\gamma(f)$ of a specific value of W. A Signal component with spectral width less than W will be decomposed primarily along the n-axises of a_{mn}. A Signal component with spectral width larger than W will be decomposed primarily along the first m-axis of a_{mn}, ie., a_{m0}. Thus, in order to reduce the interferences from wide-band signals, detection of a narrow-band signal must be done based on a_{mn} along n-axises, or equivalently based on an a_{mo} suppressed spectrum :

$$p_s(f) = p(f) - \sum_m a_{m0}\, g(f - mW) .$$

Since the computation of ps(f) does not depend much on the knowledge of, clutter, comparing to the traditional MTI filter or adaptive MTI filter, detection of target based on $p_s(f)$ will be more robust.

Since an all-pole spectrum consists of peaks characterized by

$$p_0(f) = \frac{1}{\left|1 + z_0 \exp(j\, 2\pi f)\right|^2} ,$$

the possibility of detecting narrow-band signals can be shown by the property of Gabor expansion coefficients of $p_0(f)$ with different z_0. The absolute value of z_0 controls the spectral width of $p_0(f)$. Gabor expansions of three single-pole spectrums are given in the following.

Figure 1 shows the Gabor expansion coefficients of $p_0(f)$ with $z_0 = 0.999$, which is a narrow-band spectrum. The width of Gabor window function used is W

= 0.1. It is easy to find that the components of a narrow-band spectrum spreads along the n-axis. Figure 2 is the similar plot for the case of z_o = 0.95. This is a case when spectral width is close to W, it can be found that a_{mn} is approximately a 2-D delta function. Figure 3 is the the similar plot for the case of z_o = 0.5, which is a case of wide -band spectrum. It can be found that the expansion coefficients concentrate along the m-axis

IV EXAMPLE

A numerical example is given in this section. It simulates 32 samples of x(n)= s(n)+c(n)+w(n) with s(n) = A exp(j0.3n), c(n) being a Gaussian clutter with a normalized bandwidth of 0.12 and w(n) being white noise. The SCR is -10 dB and SNR is 20 dB. The spectral width W of $\gamma(f)$ is 1/32. The power spectrum p(f) of x(n) estimated by AR model with p=5 is given in Figure 4. It can be found that this is a difficult case due to the fact that SIR is low. Without MTI filtering, the target can not be detected correctly.

The Gabor expansion of Figure 4 is given in Figure 5. In order to see the detail of the expansion coefficients, the values along a_{m0} were hard-limited at a value of 20.0. The actual peak value of a_{m0} was 627.4. Since x(n) is a composite signal of both narrow and wide band signals, mutual interferences between target and clutter can be observed from the expansion coefficients. Comparing with Figures 1, the target responses are not so concentrate along a single n-axis. However, the clutter response can still concentrate along the a_{m0} axis; this point can be observed by comparing the Gabor expansion coefficients for the same clutter data without target, which is not given here. After the suppression of the a_{m0} components, the resultant spectrum $p_s(f)$ is given in Figure 6. Comparing Figures 4 and 6, it is easy to realize that detection of s(n) based on $p_s(f)$ will be better than based on p(f).

V CONCLUSIONS

A technique for suppressing wide-band noise based on Gabor expansion was presented. Only some preliminary results were given. It is equivalent to a robust filter that does not depend much on the statistic of noise properties. There are some parameters in the calculation of $p_s(f)$ require more detail studies to know their influences on the performance of detection.

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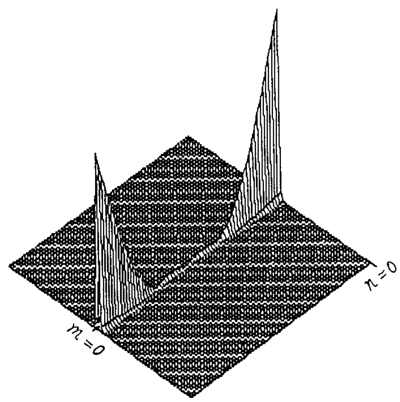

Figure 1. GABOR EXPANSION COEFFICIENTS OF AN ONE-POLE SPECTRUM WITH Z_0=0.999.

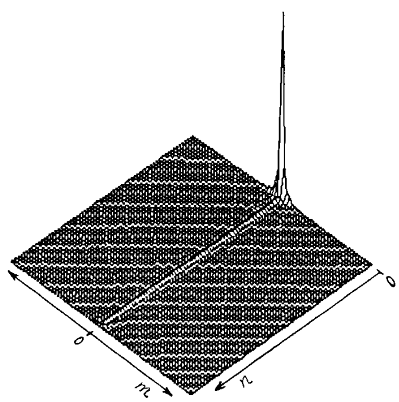

FIGURE 2. GABOR EXPANSION COEFFICIENTS OF AN ONE-POLE SPECTRUM WITH Z_0=0.95.

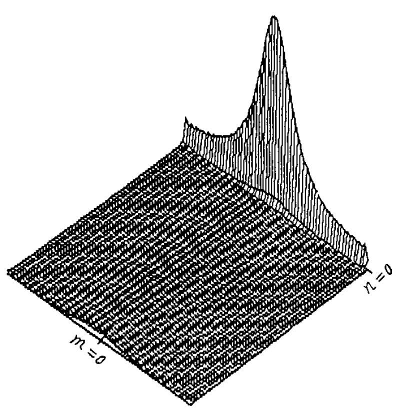

FIGURE 3. GABOR EXPANSION COEFFICIENTS OF AN ONE-POLE SPECTRUM WITH Z_0=0.5.

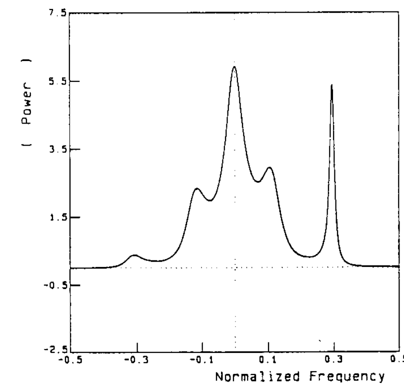

FIGURE 4. AR SPECTRUM OF A TEST SIGNAL WITH SCR=-10DB AND SNR=-10DB.

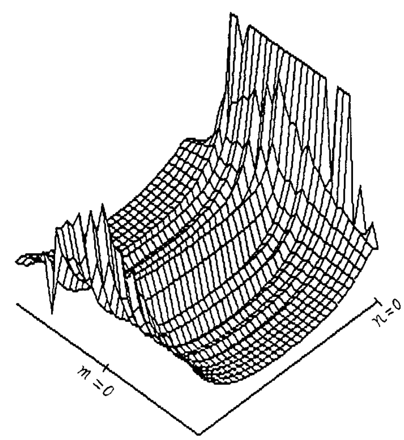

FIGURE 5. GABOR EXPANSION COEFFICIENTS OF SPECTRUM IN FIGURE 4.

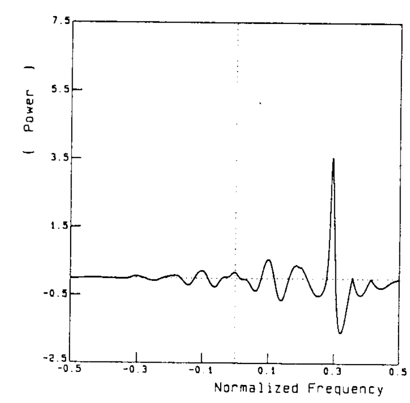

FIGURE 6. THE s_{m0} SUPPRESSED SPECTRUM OF FIGURE 4.