The Feasibility of Combating Multipath Interference by Chirp Spread Spectrum Techniques over Rayleigh and Rician Fading Channels

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Abstract— **This paper studies the feasibility of using chirp spread spectrum signals to combat multipath interference in binary digital communication systems over Rayleigh and Rician fading channels. The chirp system can be implemented using surface acoustic wave devices and can be made compact in size. Under also the influence of additive white Gaussian noise, the bit error probability has been successfully derived. Through numerical examples it is witnessed that chirp system can be used to greatly reduce the effect of multipath interference. The arrangement is quite attractive in deploying e.g. indoor wireless communications.**

I. Introduction

Multipath interference results from reflection/refraction in the course of signal transmission. It has been a problem of historical interest. It certainly has adverse effect on the quality of indoor and outdoor mobile communications. Many techniques have been proposed to cope with multipath interference, e.g. diversity reception[1], channel equalization[2]-[4], and spread spectrum signaling[5]-[7].

The concept of chirp signals has been in existence for several decades. Prior works related to chirp signals can be found in [8]-[18]. It has been extensively used in radar systems for pulse compression[8]-[10]. In [11]-[14], possible applications in data communications were discussed. Application such as performing Fourier transformation was addressed in [15]-[16]. Chirp signals were also used in conjunction with direct sequence spread spectrum signal to combat the Doppler effect and jamming signal[17]-[18]. Whether it is feasible to use chirp signals to combat multipath interference has been very rarely studied. In [12]-[13] it was only mentioned that chirp signals have the ability to cope with multipath interference, but so far there has been no quantitative analysis done in the past.

Sketched in Fig. 1 is the block diagram of a chirp transmitter. The input signal is binary, i.e. mark(1) or space(0). A chirp signal is a sinusoid whose frequency is a linear function of time. The frequency of a chirp signal may vary in an increasing manner from a low frequency to a high frequency. We shall call such signal an upchirp. It can also vary in the opposite manner. In this paper we focus on the discussion of upchirp signals.

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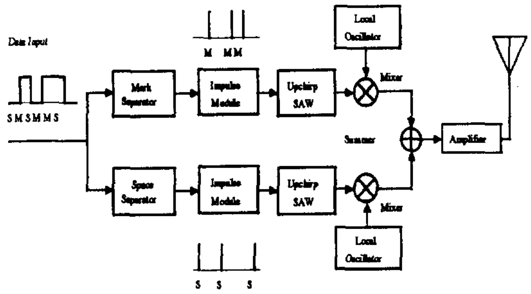

Fig. 1 Block Diagram of Transmitter

Mathematically a upchirp signal can be represented as follows [10]

$$s(t) = Asin[2\pi f_m t(1 + \frac{1}{2}\mu_m t)], \text{ for } 0 \le t < T_b \quad (1)$$

where m=0 or 1, T_b is the bit duration, and A is the amplitude. In (1), f_0 and f_1 are clearly the starting frequencies of space and mark, respectively. The channel used to send space ranges from f_0 to $f_0(1+\mu_0 T_b)$ and f_1 to $f_1(1+\mu_1 T_b)$ for mark. The two channels must be nonoverlapping.

The concept of chirp signal is easy to understand. It can be implemented using surface acoustic wave (SAW) devices and can be made compact in size which is particularly useful in handheld telephone set. The purpose of this study is to mathematically analyse the feasibility of using chirp signals to reduce the effect of multipath interference in a binary digital communication system over Rayleigh/Rician fading channel. One possible application of our proposed system is low power indoor wireless digital transmissions.

II. Channel Model

A. Rayleigh Fading Channel

In a Rayleigh fading channel, the signal $r(t)$ received by

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the receiver can be represented as follows

$$r(t) = \sum_{j=0}^{M} \beta_j s(t - t_j) + n(t) \tag{2}$$

where M denotes the number of delay paths, $s(t)$ the signal transmitted by the transmitter, and $n(t)$ the channel noise. In (2), β_j and t_j are used to denote the "signal strength" and the "propagation time from transmitter to receiver" of the j-th path.

The path strengths β_j, for $j = 0 \cdots M$, are Rayleigh random variables, i.e.

$$f(\beta_j) = \begin{cases} \frac{\beta_j}{\sigma_j^2} exp[-\frac{\beta_j^2}{2\sigma_j^2}], & \beta_j \geq 0 \\ 0, & \beta_j < 0 \end{cases} \tag{3}$$

In a Rayleigh fading channel there is no direct or line-of-sight (LOS) path. All paths fade, but independently.

B. Rician Fading Channel

In a Rician fading channel, the zeroth path is in general a LOS path. The signal $r(t)$ can be represented as follows

$$r(t) = As(t - t_j) + \sum_{j=1}^{M} \beta_j s(t - t_j) + n(t) \tag{4}$$

which is a combination of a strong direct or LOS path and some reflected paths. The LOS path has a constant signal strength A, while the other reflected signal strengths β_j, for $j = 1 \cdots M$, are Rayleigh random variables with distribution given in (3).

Another important parameter to characterize Rician fading channel is the ratio of the LOS power to the sum of all the reflected powers. Let us denote this parameter by ρ, i.e.

$$\rho = \frac{A^2}{2\sum_{j=1}^{M} \sigma_j^2} \tag{5}$$

When $\rho = \infty$, i.e. $\sum_{j=1}^{M} \sigma_j = 0$, we have nothing but a single path channel. In this case the received signal is only contaminated by the additive white Gaussian noise. When $\rho = 0$, i.e. $A = 0$, the LOS path vanishes and the channel becomes a Rayleigh fading channel of Sec. II.A.

C. Basic Assumptions

A1. The signal on each path is Rayleigh faded (except the zeroth path of the Rician channel). Different paths fade independently. As in [6]-[7], we assume that all paths have the same mean power level, i.e. $\sigma_j^2 = \sigma^2$, for $j = 1 \cdots M$.

A2. Fading is slow with respect to T_b and frequency-nonselective with respect to the signal bandwidth. As a result, the strength of the received signal does not vary significantly within a bit duration.

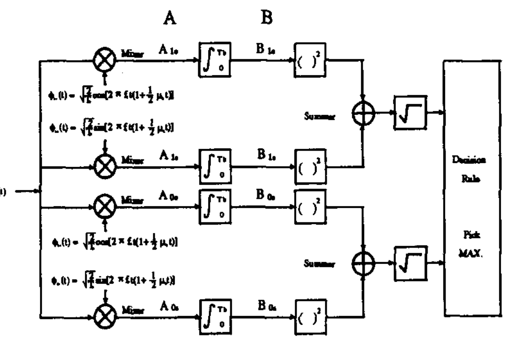

Fig. 2 Simplified Block Diagram of Receiver

A3. $\{t_j - t_0\}_{j=0}^{M}$ is assumed to be uniformly distributed over one bit duration, i.e. $(0,T_b)$[6]-[7].

A4. $\{\beta_j\}_{j=0}^{M}$ and $\{t_j - t_0\}_{j=0}^{M}$ are independent.

A5. The channel noise is additive white Gaussian with two-sided power spectral density $N_0/2$.

A6. The receiver is always synchronized to the zeroth path.[19]

III. Results

A. Rayleigh Fading Channel

For the convenience of our discussion, the block diagram of receiver can be simplified and drawn in Fig. 2. We examine the reception of a specific bit. We also assume $t_0 = 0$, then t_j in (2) becomes also the relative delay between the j-th path and the zeroth path, since $t_j - t_0 = t_j$. After mathematical derivation we obtain the following bit error probability P_e when $t_1, \cdots, t_M$ are given

$$P_e(t_1, \cdots, t_M) = \frac{\sigma_b^2 + N_0}{T_b\sigma^2 + 2\sigma_a^2 + 2\sigma_b^2 + 2N_0} \tag{6}$$

where

$$\sigma_a^2 = \sum_{j=1}^{M} \frac{T_b}{2}[\frac{sin(\pi W t_j)}{\pi W t_j}]^2\sigma^2$$

$$\sigma_b^2 = \sum_{j=1}^{M} \frac{T_b}{2}[\frac{sin(\pi W t_j)}{\pi W(T_b - t_j)}]^2\sigma^2$$

and $W = f_i\mu_i T_b = f_{i'}\mu_{i'} T_b$.

Finally

$$P_e = \frac{1}{T_b^M}\int_0^{T_b}\int_0^{T_b}\cdots\int_0^{T_b} P_e(t_1, \cdots, t_M)dt_1 \cdots dt_M \tag{7}$$

B. Rician Fading Channel

For Rician fading channel, there is now a LOS path. From (5), the relation between the LOS signal strength A

and the mean power of a delay path σ^2 becomes

$$A = \sqrt{2\rho \sum_{j=1}^{M} \sigma_j^2} = \sqrt{2M\rho\sigma^2} \quad (8)$$

After mathematical derivation we obtain the following P_e when $t_1, \cdots, t_M$ are given

$$P_e(t_1, \cdots, t_M) = \frac{\sigma_b^2 + N_0}{2(\sigma_a^2 + \sigma_b^2 + N_0)} exp[\frac{-\frac{T_b}{2}A^2}{2(\sigma_a^2 + \sigma_b^2 + N_0)}] \quad (9)$$

where

$$\sigma_a^2 = \sum_{j=1}^{M} \frac{T_b}{2}[\frac{sin(\pi W t_j)}{\pi W t_j}]^2 \sigma^2$$

$$\sigma_b^2 = \sum_{j=1}^{M} \frac{T_b}{2}[\frac{sin(\pi W t_j)}{\pi W (T_b - t_j)}]^2 \sigma^2$$

Finally

$$P_e = \frac{1}{T_b^M} \int_0^{T_b} \int_0^{T_b} \cdots \int_0^{T_b} P_e(t_1, \cdots, t_M) dt_1 \cdots dt_M \quad (10)$$

Here we can notice that when $\sigma_a = \sigma_b = 0$, i.e. when there is no fading component, (6) reduces to the bit error rate of a pure additive white Gaussian noise channel, i.e.

$$P_e = \frac{1}{2} exp[-\frac{\frac{T_b}{2}A^2}{2N_0}] \quad (11)$$

IV. Numerical Examples and Discussions

In the following examples for the Rayleigh fading channel the number of delay paths M is assumed to be 0, 1, 2 or 3. We use SNR to denote the average signal-to-noise ratio, i.e. $SNR = E[\beta^2 cos^2(2\pi f_i t)] \cdot T_b/N_0 = T_b\sigma^2/N_0$. SNR is taken to be 10, 20, 30, or 40 dB. The per channel bandwidths $W = f_i \mu_i T_b$ considered are 1M, 2M, 3M and 4M Hz. For Rician fading channel, the number of delay paths M is taken to be 1 which is the case considered in most studies. The influence from fading component depends mainly on the power ratio ρ of the LOS to the fading component, and is assumed to be 1, 5 or 10. We define the SNR of Rician fading channel as the bit energy of LOS path to the channel noise, i.e. $SNR = \frac{T_b}{2}A^2/N_0$.

Fig. 3 plots P_e versus SNR under different values of W for $M = 2$. In Fig. 3 we witness the followings

1) $W = 0.01$M Hz = 10K Hz corresponds to the unspread case, i.e. $\mu_i = 0$. We observe an unacceptably high value of P_e no matter how big the value of SNR is. The unspread case is provided to demonstrate the improvement which can be offered by our chirp system.

2) In the limiting case when $W \to \infty$ we observe that the multipath interference can be almost completely removed which is reasonable. This curve also gives us the lower bound of P_e that our chirp system can achieve.

3) For each given finite value of W, we observe that P_e can only be lowered to a limit no matter how high the signal-to-noise ratio is. This limit of course is due to the existence of multipath interference and the insufficiently large value of W. This means that if the offered bandwidth is finite there exists a limit on P_e which the system is never able to exceed.

Fig. 4 presents another view by plotting P_e versus SNR under $W = 4$MHz for various values of M. In this figure $M = 0$ corresponds to the situation in which multipath channel is nonexistent. The result of $M = 0$ is independent of the value of W and also coincides with the $P_e - SNR$ curve of the well known BFSK over a Rayleigh fading channel. The curve $M = 0$ has a meaning equivalent to the curve of $W = \infty$ in Fig. 3. Both of them serve the purpose of telling us the limit on the capability of our chirp system.

In Fig. 5 we plot P_e versus W for $M = 2$ under various values of SNR. In this case $SNR = \infty$ corresponds to a noiseless channel. The reason that P_e still shows a value of $P_e > 10^{-4}$ is due to the existence of multipath interference and W being not large enough. If W is pushed to ∞ then P_e should approach zero eventually.

The remaining figures are done for Rician channel.

In Fig. 6, we plot P_e versus SNR under different values of W for $\rho = 5$ in a Rician fading channel. Here we witness that

1) In terms of P_e when compare to Fig. 3, Rician fading is less damaging than Rayleigh fading. This is very reasonable because in Rician channel there is a strong and constant strength LOS path.

2) In the limiting case when $W \to \infty$, we observe that the multipath interference can be almost completely removed and is again reasonable. This curve also coincides with the $P_e - SNR$ curve of the well known BFSK in an additive white Gaussian noise channel and gives us the lower bound of P_e that our chirp system can achieve in a Rician fading channel.

3) For each given value of W, we again observe that P_e can only be lowered to a limit no matter how high the signal-to-noise ratio is. And this limit occurs at a SNR lower than that in Rayleigh channel. This limit of course is due to the existence of multipath interference and the insufficiently large value of W.

In Fig. 7 we plot P_e versus W for $\rho = 5$ under various values of SNR. In this case $SNR = \infty$ corresponds to a noiseless channel. Trends similar to those in Fig. 5 of Rayleigh channel are observed.

Fig. 8 presents another view by plotting P_e versus SNR

under $W = 2\text{MHz}$ for various values of ρ. In this figure $\rho = \infty$ corresponds to the situation in which fading component is nonexistent and this curve also coincides with the $P_e - SNR$ curve of the well known BFSK in an additive white Gaussian noise channel.

V. Conclusions

We have in this paper proposed and analyzed the performance of a chirp spread spectrum system in a Rayleigh/Rician fading channel. The validity of our analysis is not only verified by computer simulations but also supported by intuitive reasonings. Our key result obtained in this paper is the bit error probability P_e.

Through extensive numerical examples we indeed observe that the effect of multipath interference can be greatly reduced at the cost of sufficient bandwidth. This should pose no problem in indoor environment in which the radiated power can be very well controlled in order to use as much bandwidth as one desires. When considering the limitation of technology, the bandwidth of chirp signals generated by SAW devices can exceed 100MHz[21], which should be wide enough for most applications. The simplicity and compactness of the system offers yet additional attractions. Error control coding techniques can be further employed to improve the performance when necessary.

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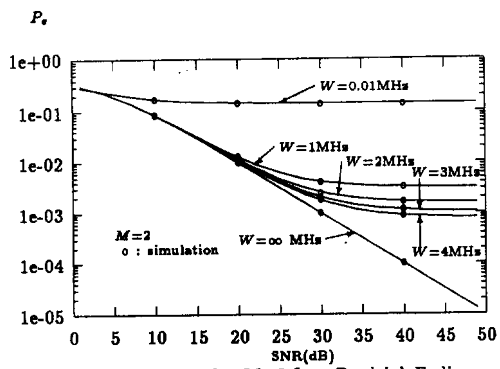

Fig. 3 P_e Versus SNR under M=2 for a Rayleigh Fading Channel

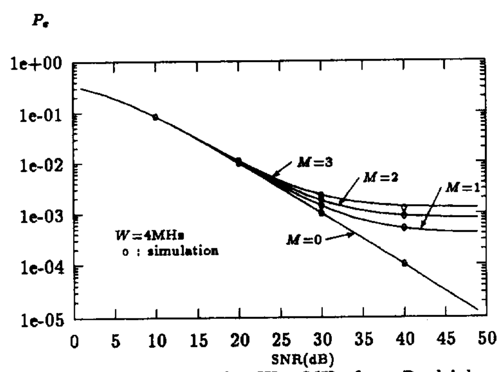

Fig. 4 P_e Versus SNR under W=4MHz for a Rayleigh Fading Channel

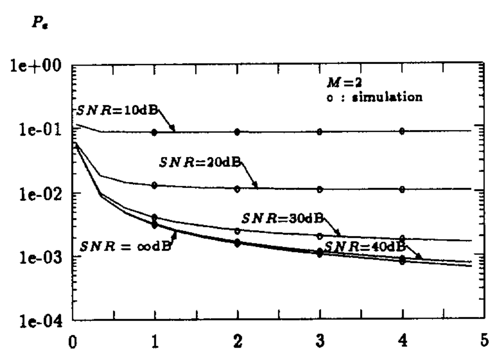

Fig. 5 P_e Versus W under M=2 for a Rayleigh Fading Channel

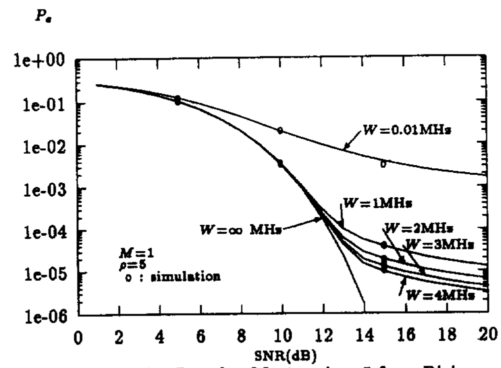

Fig. 6 P_e Versus SNR under M=1 and ρ=5 for a Rician Fading Channel

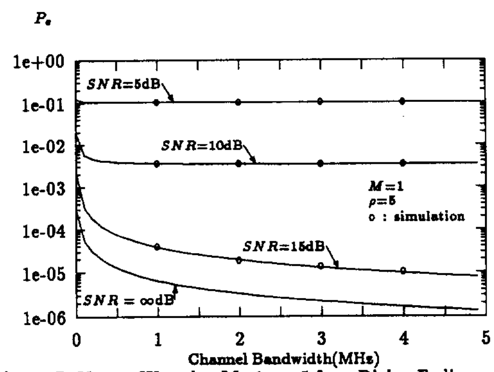

Fig. 7 P_e Versus W under M=1, ρ=5 for a Rician Fading Channel

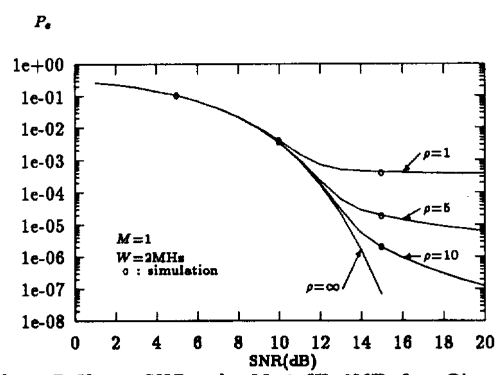

Fig. 8 P_e Versus SNR under M=1, W=2MHz for a Rician Fading Channel