# CALIBRATION OF POLARIMETRIC RADAR SYSTEM USING 

 THREE PERFECTLY POLARIZATION-ISOLATED CALIBRATORSTzong-Jyh Chen and Tah-Hsiung Chu*
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## I. INTRODUCTION

Recently several calibration techniques using different calibrators for the polarimetric radar system are reported $[1-2]$. These methods in general require three calibrators with known polarimetric scattering matrices (PSM's). Reduction in complexity of calibration process is possible, as the characteristics of calibrators is considered. Chen et al. [3] use three perfectly polarization-isolated calibrators: a flat plate, a dihedral corner reflector, and an arbitrarily rotated dihedral corner reflector, by assuming the magnitudes of co-polarization terms in PSM of each calibrator to be equal.

In this paper, the calibration method in [3] is generalized such that the requirement for each polarization-isolated calibrator to have the same co-polarized RCS is not needed. Only the co-polarized terms in PSM of the first calibrator is required. The range and co-polarized RCS's of the other two calibrators and the rotation angle of the third calibrator can be derived in the calibration process and used to verify the calibration.

## II. CALIBRATION ALGORITHM

The relation between the target PSM S and the measured target PSM $S^{m}$ is expressed as [3]

$$
\left[\begin{array}{ll}
S_{\mathrm{Vv}}^{\mathrm{w}} & S_{\mathrm{Vh}}^{\mathrm{m}}  \tag{1}\\
S_{\mathrm{hv}}^{\mathrm{v}} & S_{\mathrm{hh}}^{\mathrm{R}}
\end{array}\right]=\left[\begin{array}{cc}
I_{\mathrm{vv}} & I_{\mathrm{vh}} \\
I_{\mathrm{hv}} & I_{\mathrm{hh}}
\end{array}\right]+\left[\begin{array}{ll}
R_{\mathrm{vv}} & R_{\mathrm{vh}} \\
R_{\mathrm{hv}} & R_{\mathrm{hh}}
\end{array}\right]\left[\begin{array}{ll}
S_{\mathrm{vv}} & S_{\mathrm{vh}} \\
S_{\mathrm{hv}} & S_{\mathrm{hh}}
\end{array}\right]\left[\begin{array}{lll}
T_{\mathrm{vv}} & T_{\mathrm{vh}} \\
T_{\mathrm{hv}} & T_{\mathrm{hh}}
\end{array}\right]
$$

where I is an additive isolation error matrix which can be directly measured without locating any scattering target. $\mathbf{T}$ and $\mathbf{R}$ are the transfer matrices of transmitting and receiving antennas to be characterized. With the isolation error matrix I suppressed, the measured PSM's of three perfectly polarization-isolated calibrators are

$$
\begin{align*}
& \mathbf{A}_{1}=\mathbf{R}\left[\begin{array}{ll}
\alpha_{1} & 0 \\
0 & \alpha_{2}
\end{array}\right] \mathbf{T}  \tag{2}\\
& \mathbf{A}_{\mathbf{2}}=\mathbf{R}\left[\begin{array}{ll}
\beta_{1} & 0 \\
0 & \beta_{2}
\end{array}\right] \mathbf{T}  \tag{3}\\
& \mathbf{A}_{\mathbf{3}}=\mathbf{R}\left[\begin{array}{ll}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{array}\right]^{-1}\left[\begin{array}{ll}
\gamma_{1} & 0 \\
0 & \gamma_{2}
\end{array}\right]\left[\begin{array}{cc}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{array}\right] \mathbf{T}
\end{align*}
$$

where $\alpha_{1}, \alpha_{2}, \beta_{1}, \beta_{2}, \gamma_{1}$, and $\gamma_{2}$ are complex constants to account for the different ranges and co-polarized RCS's of these calibrators. The third calibrator is rotated by $\theta$ angle with respect to the radar range direction to provide cross-polarized information for use in the calibration. Equation (2) can be rewritten as

$$
\mathbf{R}=\mathbf{A}_{1} \mathbf{T}^{-1}\left[\begin{array}{ll}
\alpha_{1} & 0  \tag{5}\\
0 & \alpha_{2}
\end{array}\right]^{-1}
$$

which indicates that $\mathbf{R}$ can be found, as $\alpha_{1}$ and $\alpha_{2}$ are given and $\mathbf{T}$ is derived as the following. Let

$$
\mathbf{T}=T_{\mathrm{hh}}\left[\begin{array}{cc}
x w & w  \tag{6}\\
y & 1
\end{array}\right]=T_{\mathbf{h h}} \mathbf{T}^{\prime}
$$

where $x=T_{\mathrm{vv}} / T_{\mathrm{vh}}, y=T_{\mathrm{hv}} / T_{\mathrm{hh}}$, and $w=T_{\mathrm{vh}} / \mathrm{T}_{\mathrm{hh}}$ are parameters to be determined. After proper matrix manipulation, the following four equations can be derived from (2)-(4)

$$
\begin{equation*}
E_{12} x^{2}+\left(E_{22}-E_{11}\right) x-E_{21}=0 \tag{7}
\end{equation*}
$$

$$
\begin{equation*}
E_{12} y^{2}+\left(E_{22}-E_{11}\right) y-E_{21}=0 \tag{8}
\end{equation*}
$$

$G_{5}(u \cot \theta)^{2}-G_{6} w \cot \theta+G_{7}=0$

$$
\begin{equation*}
G_{5}(u \tan \theta)^{2}+G_{6} u \tan \theta+G_{7}=0 \tag{9}
\end{equation*}
$$ $\left(\alpha_{1} x+\alpha_{2} y\right) F_{11}-\left(\alpha_{1}+\alpha_{2}\right)\left(x y F_{12}-F_{21}\right)-\left(\alpha_{1} y+\alpha_{2} x\right) F_{22}$, and $G_{7}=-\alpha_{2}\left[F_{12} y^{2}+\right.$ $\left.\left(F_{22}-F_{11}\right) y-F_{21}\right]$. With the constraint $|x| \gg|y|, x$ and $y$ can solved from (7) and (8). As the range of $\theta$ is known a priori, values of $\theta$ and $w$ can be obtained from (9) and (10). Then $\beta_{1}, \beta_{2}, \gamma_{1}$, and $\gamma_{2}$ can be solved in terms of $\alpha_{1}$ and $\alpha_{2}$. After $x, y$, and $w$ in (6) derived, $\mathbf{R}$ in (5) can be expressed in normalized form as

$$
\mathbf{R}=\frac{1}{T_{\mathrm{hh}}} \mathbf{A}_{\mathbf{1}} \mathrm{T}^{-1-1}\left[\begin{array}{cc}
\alpha_{1} & 0  \tag{11}\\
0 & \alpha_{2}
\end{array}\right]^{-1}=\frac{1}{T_{\mathrm{hh}}} \mathbf{R}^{\prime}
$$

Using (1), (6), and (11) the correct target PSM can be calculated as

$$
\begin{equation*}
\mathbf{S}=\mathbf{R}^{-1}\left(\mathrm{~S}^{\mathrm{m}}-\mathrm{I}\right) \mathrm{T}^{-1}=\mathbf{R}^{1-1}\left(\mathrm{~S}^{\mathrm{m}}-\mathbf{I}\right) \mathrm{T}^{1-1} \tag{12}
\end{equation*}
$$

## III. EXPERIMENTAL RESULTS

A flat metal plate with dimension 17 cm by 17 cm is used as the first calibrator. Since the scattering responses of a square plate for vertical and horizontal polarizations are the same, $\alpha_{1}$ is equal to $\alpha_{2}$. A dihedral corner reflector with dimension 12 cm by 17 cm for each side is used as the second and third calibrators. Results of the rotation angle of dihedral corner reflector (third calibrator) are shown in Fig. 1 in comparison with the method in [3]. The deviation of calculated rotation angle over operation band is shown significantly improved. Figure 2 shows the magnitude of co-polarization ratio $\beta_{1} / \beta_{2}$ of dihedral corner reflector (second calibrator) in comparison with the theoretical results.

Measurement results of co-polarization ratio ( $S_{\mathrm{vv}} / S_{\mathrm{hh}}$ ) of a conducting cylinder with diameter 2.54 cm and length 128.5 cm is shown in Fig. 3 in comparison with the method in [3] and the theoretical data. The discrepancy for magnitude and phase are shown about within 0.5 dB and 9 degrees over 7 to 17 GHz .

REFERENCES:
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Fig. 1 Results of rotation angle $\theta$ of the third calibrator.


Fig. 2 Results of magnitude of co-polarization ratio $\beta_{1} / \beta_{2}$ of the second calibrator.


Fig. 3 Results of (a) magnitude and (b) phase of $S_{\mathrm{vv}} / S_{\mathrm{hh}}$ of a conducting cylinder with diameter 2.54 cm and length 128.5 cm .

