ARRAY IMAGING WITH SPECTRAL DATA EXTRAPOLATION

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INTRODUCTION

For high resolution array radar imaging, large coherent antenna array imposes the problems of very high data rate and cost for the use of a large number of coherent receiving channels on the imaging system. A possible way to reduce the number of array elements is to use random thinned array[1] in which elements exist at some randomly selected positions only. This approach can achieve high angular resolution, but the array has high sidelobes. To reduce the sidelobe interference, sidelobe reduction process such as CLEAN [2] must be applied.

In this paper, an array data extrapolation algorithm is proposed to increase the angular resolution of an array imaging system. The algorithm applies an augmented Gram-Schmidt procedure to extrapolate the received array data. From simulations, this algorithm may increase the effective size of array to a great extent for the noise free case and to a size of three to five times of the original apperture for high SNR cases.

THE DATA EXTRAPOLATION ALGORITHM

In a far-field array-imaging system, signals received at the array elements can be modeled as the Fourier transformation of a source function or scene $s(u)$, where $u = \sin\theta$ is the reduced direction angle of the imaging system. In general $s(u)$ is assumed to be a complex function of u and the image can be formed as the discrete inverse Fourier transformation of the received array data[1]. Given the case that the number of array elements is N and N is an even number, the array data can be put into a symmetric matrix as

$$B = \begin{pmatrix} I_1 & I_2 & \cdots & I_M \\ I_2 & I_3 & \cdots & I_{M+1} \\ \vdots & & \ddots & \vdots \\ I_M & \cdots & \cdots & I_{2M-1} \end{pmatrix} = \begin{pmatrix} | & | & & | \\ \underline{b}_1 & \underline{b}_2 & \cdots & \underline{b}_M \\ | & | & & | \end{pmatrix} \qquad \text{where } M = N/2.$$

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An augmented matrix of B is defind as

$$A = \left(\begin{array}{cccc|c} & & B & & I_{M+1} \\ & & & & I_{2M+1} \\ & & & & \vdots \\ \hline I_{M+1} & I_{M+2} & \cdots & I_{2M} & I_{2M+1} \end{array}\right) \quad \text{and} \quad \underline{b}_{M+1} = \begin{pmatrix} I_{M+1} \\ I_{M+2} \\ \vdots \\ I_{2M} \end{pmatrix}$$

In the augmented matrix A, the last element I_{2M+1} is unknown since only 2M points of data are available. The algorithm then puts both A and B into an augmented Gram-Schmidt orthonormalization procedure. With vectors g being the interstep vectors and vectors h being the orthonormal basis, the procedure is

$$\underline{g}_1 = \underline{b}_1 \;; \quad \underline{h}_1 = \underline{g}_1 / \langle \underline{g}_1, \underline{g}_1 \rangle \;;$$

$$h'_1 = I_{M+1} / \langle \underline{g}_1, \underline{g}_1 \rangle \qquad \text{and}$$

$$\underline{g}_n = \underline{b}_n - \sum_{m=1}^{n-1} \langle \underline{b}_n, \underline{h}_m \rangle \underline{h}_m \;; \qquad \underline{h}_n = \underline{g}_n / \langle \underline{g}_n, \underline{g}_n \rangle \;;$$

$$g'_n = I_{M+n} - \sum_{m=1}^{n-1} \langle \underline{b}_n, \underline{h}_m \rangle h'_m \qquad h'_n = g'_n / \langle \underline{g}_n, \underline{g}_n \rangle \;;$$

for n = 2..M .

where $\langle \cdot, \cdot \rangle$ is the inner product of vectors. Except for the operations on h'_n , which is the augmented orthonormalization operations, the previous procedure is a standard Gram-Schmidt procedure. After these steps, a set of orthonormal basis of matrix B is obtained. However, the last column of A is not orthonormalized yet since I_{2M+1} is not available. Then the procedure makes a linear decomposition of $\underline{b}_{M+1}$ onto this orthonormal basis { $\underline{h}_i$; i = 1..M }, that is

$\underline{b}_{M+1} = \sum_{i=1}^{M} c_i \underline{h}_i$ and $\underline{c}^T = [c_1, c_2, \ldots, c_M]$ is the coordinate vector. Then I_{2M+1} is calculated as $I_{2M+1} = \sum_{i=1}^{M} c_i h'_i$. This is the right-extrapolated array data. By the same procedure, the array data can be used to do the left-extrapolation for I_0. At this moment there are 2M+2 points of data in total. The algorithm continues to do further right and left extrapolations untill the extrapolation procedure goes unstable.

Then the usual DFT transformation is performed on the measured data as well as the extrapolated data to form an image of the scene.

PROPERTIES OF THE ALGORITHM

Two properties of the algorithm from simulation are given as follows:

1. Data extrapolation is inherently a noise sensitive process, since noise could be treated as signal and extrapolated. For this algorithm, if SNR of data is high (50 dB in the simulations) the data length can be extrapolated to three to five times of the original length. It is equivalent to a three to five times improvement of resolution. After the algorithm goes unstable, usually about 2N points of the data which are extrapolated lastly must be abandoned due to large extrapolation error.

2. The order of the extrapolation procedure (M in the algorithm) can be either fixed or increased as the data being extrapolated, or any combination of them. In general, for the purpose of imaging M should be increased in each step.

COMPUTER SIMULATION :

A set of 10-point test data of two point sources is generated under the following conditions:

(1) target direction : $u_1 = 0.022$ and $u_2 = 0.052$;
(2) target intensity : equal intensity;
(3) target phase : 2.0 and 0.9 radians;
(4) SNR = 50 dB.

The algorithm extrapolated the array data to a length of 83 and went unstable clearly. As stated before, 10 points on each side were abandoned. This made 63 points of data left. A 256-point DFT image of the original 10-point data is given in Figure 1. The DFT image formed with the 63-point extrapolated array data is given in Figure 2. Comparing the figures, resolution improvement is very clear. Sidelobe suppression can be observed in Figure 2 also.

REFERENCES:

[1] B. D. Steinberg, Microwave Imaging With Large Antenna Arrays, Wiley, New York, 1983.
[2] J. Tsao and B. D. Steinberg, " Reduction of Sidelobe and Speckle Artifacts in Microwave Imaging: The CLEAN Technique," IEEE Trans. Antennas Propag., AP-36, April 1988.

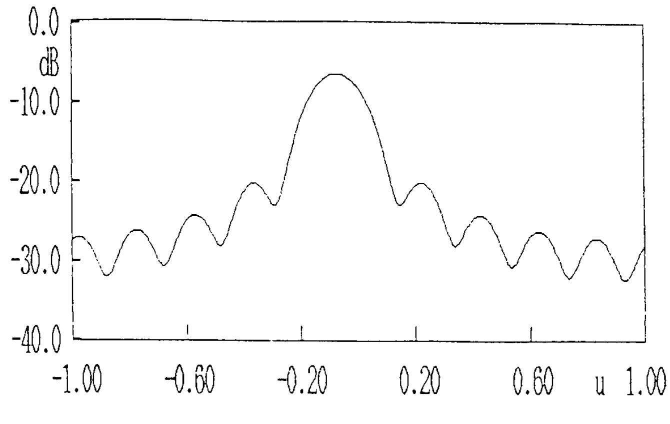

FIG.1 IMAGE OF A 2-TARGET SCENE WITH A 10-ELEMENT ARRAY.

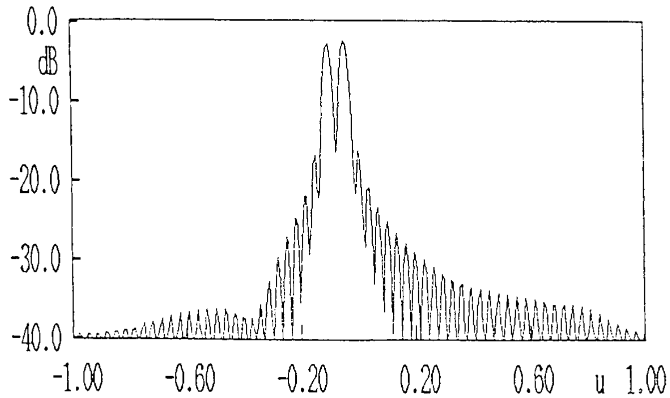

FIG.2 IMAGE FORMED WITH THE EXTRAPOLATED ARRAY DATA.